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THEORETICAL AND MATHEMATICAL PHYSICS

Periodic Intermittent Regime of a Boundary Flow

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Abstract—Melting of an ultrathin lubricant film during friction between two atomically smooth surfaces is investigated using the Lorentz model for approximating the viscoelastic medium. Second-order differential equations describing damped harmonic oscillations are derived for three boundary relations between the shear stresses, strain, and temperature relaxation times. In all cases, phase portraits and time dependences of stresses are constructed. It is found that under the action of a random force (additive uncorrelated noise), an undamped oscillation mode corresponding to a periodic intermittent regime sets in, which conforms to a periodic stick–slip regime of friction that is mainly responsible for fracture of rubbing parts. The conditions in which the periodic intermittent regime is manifested most clearly are determined, as well as parameters for which this regime does not set in the entire range of the friction surface temperature.

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INTRODUCTION

Interest in friction in molecular-thin lubricant layers between two atomically smooth solid surfaces [1–3] for various values of the applied load, pressure, and layer thickness, upon a change in the shear rate, temperature, etc., has considerably increased in recent years. The studies in this field are aimed at determining tribological and rheological properties of ultrathin lubricant layers, which differ qualitatively from the properties of bulk lubricants. The necessity of studying such differences is dictated by the development of nanotechnologies and the increasing need in obtaining low-dimensional rubbing systems. Such tribological units are used in modern information tanks, aerospace devices, miniature motors, positioning systems in microelectronics, etc. It should be noted that analogous behavior of lubricants is observed in practically each working mechanism and is associated with lubricant squeezing from under the surfaces under the action of an applied load.

Along with experiments [1–3], theoretical description of boundary friction [4–7] and its computer simulation [8] were carried out. It was found that, with decreasing the thickness of a lubricant layer in the course of friction, its physical properties first change quantitatively, and then, the changes become strongly qualitative [9]. It was shown in particular that in the steady-state regime of friction, the behavior of the lubricant layer may correspond to a multiphase state leading to the stick-slip regime (i.e., transitions between solid-like and liquid-like states of the lubricant). However, the liquid-like state sharply differs from the common state of the liquid especially in the case of chain molecules of the hexadecane type. Dynamic properties of layers in the boundary regime, in which transitions between different types of dynamic states occur during slips, are of special importance. Recent studies of slip of mica and quartz surfaces, in which a number of substances (monolayers of metals, organic liquids, and water) were used as lubricants, have shown that such phase transitions are observed as a rule rather than as an exception. These transitions are manifested in the presence of intermittent motion characterized by transitions between two or more dynamic states. The knowledge of the stick–slip flow regime is important in tribology since this regime is mainly responsible for damage and ware of rubbing parts. However, the stick–slip friction regime is even more common and is the reason for sounds extracted from a violin and creaking noise of doors, automobiles, etc.

Analysis of the properties of ultrathin layers requires special high-technology equipment because these layers are nanosize objects. However, experimental setups and corresponding techniques [10] have been developed and constructed in spite of their complexity and make it possible to measure, for example, the thickness of the molecular layer, viscosity, friction coefficient, and shear components of viscous and elastic stresses.

In [7], a rheological description of a viscoelastic medium conducting heat was used to derive a system of kinetic equations describing the mutually correlated behavior of shear stresses σ and strain ε, as well as temperature T in an ultrathin lubricant film during friction between atomically smooth solid surfaces. In the framework of this theory, the effect of additive uncorrelated noise on lubricant melting was analyzed [11], melting due to dissipative effects was considered [12], temperature correlations [13] and various temperature dependences of viscosity were taken into account [14], and hysteretic effects during melting were investigated [15, 16]. However, these studies revealed an intermittent stochastic regime, in which the static and kinetic friction forces vary at random with time. Although this regime was observed experimentally [17] using methods of molecular dynamics [8] also, the stick–slip mode of boundary friction is periodic by nature [1, 5, 6, 8]. This study is aimed at clarifying the reasons for the emergence of this regime and its features using the rheological model [7].

1. PERIODIC STICK–SLIP REGIME

The model proposed in [7] for describing melting of a thin lubricant layer between solids is based on a system of dimensionless equations of the type

$$\tau_\sigma \dot{\sigma} = -\sigma + g\varepsilon, \quad (1)$$

$$\tau_\varepsilon \dot{\varepsilon} = -\varepsilon + (T-1)\sigma, \quad (2)$$

$$\tau_T \dot{T} = (T_e - T) - \sigma\varepsilon + \sigma^2, \quad (3)$$

where σ is the shear component of stresses emerging in the lubricant, ε is the shear component of strains, and T is the lubricant temperature. Here, we have also introduced constant $g < 1$, which is equal numerically to the ratio of the shear modulus of the lubricant to its characteristic value, and rubbing surface temperature T_e. It was shown that zero stationary stresses σ_0 correspond to a solid-like structure of the lubricant; for $\sigma_0 \neq 0$, the lubricant melts and passes to a liquid-like state.

Let us consider stationary states, in which all derivatives in Eqs. (1)–(3) are zero and the parameters of the lubricant do not change with time. Analysis of the system of equations shows that if temperature T_e of rubbing surfaces is lower than the critical temperature

$$T_{c0} = 1 + g^{-1} \quad (4)$$

shear stresses assume steady-state value $\sigma_0 = 0$, while for $T_e > T_{c0}$, the value

$$\sigma_0 = \sqrt{\frac{gT_e - (g+1)}{1-g}}, \quad (5)$$

corresponding to the liquid regime of flow is observed. The behavior of the system prior to the establishment of the stationary state strongly depends on the relation between relaxation times. Let us consider the cases when the value of one of these times can be regarded as small.

1.1. The Case of $\tau_T \ll \tau_\sigma, \tau_\varepsilon$

In this case, we set $\tau_T \dot{T} \approx 0$ in Eq. (3), express T from this equation, and obtain two-parametric system (1), (2) substituting T into Eq. (2). We can now reduce two resultant first-order differential equations in stress σ and strain ε to a single second-order equation in σ. For this purpose, we must express ε in terms of σ from Eq. (1) and write the time derivative of this expression. Substituting resultant dependences $\varepsilon(\sigma, \dot{\sigma})$ and $\dot{\varepsilon}(\sigma, \dot{\sigma})$ into Eq. (2), we obtain the sought equation. If we measure time in the units of τ_σ, this equation acquires the form

$$\ddot{\sigma} + \frac{1 + \tau + \sigma^2}{\tau}\dot{\sigma} + \frac{\sigma}{\tau}[1 + \sigma^2 - g(T_e - 1 + \sigma^2)] = \xi(t), \quad (6)$$

where $\tau = \tau_\varepsilon/\tau_\sigma$. In Eq. (6), we additionally take into account the effect of random force $\xi(t)$, which has moments

$$\langle \xi(t) \rangle = 0, \quad \langle \xi(t)\xi(t') \rangle = 2D\delta(t-t'), \quad (7)$$

where D plays the role of the stochastic source's intensity.

Let us write Eq. (6) in the canonical form

$$\ddot{\sigma} + 2\beta\dot{\sigma} + \omega_0^2\sigma = \xi(t), \quad (8)$$

where damping factor β and natural frequency σ_0 of oscillations depend on stresses. Since a constant value of stresses ($\dot{\sigma} = 0$) sets in the system in the stationary state ($D = 0$), it can be found by equating the last term on the right-hand side of Eq. (6) to zero and coincides with Eq. (5). The damping factor in the vicinity of stationary point σ_0 can easily be found from Eq. (6). To this end, we must substitute the value of (5) into Eq. (6) and compare the resultant relation with Eq. (8). The sought damping factor can be written in the form

$$\beta = 0.5\tau^{-1}(g^{-1} - 1)^{-1}(\tau(g^{-1} - 1) + T_e - 2). \quad (9)$$

However, to determine the type of oscillations unambiguously, we must also find ω_0 and analyze the frequency of damped oscillations

$$\omega = \sqrt{\omega_0^2 - \beta^2}.$$

A detailed analysis of this system of equations was performed in [18] and is beyond the scope of this work.

To solve Eq. (8) numerically, we use the substitution $y = \dot{\sigma}$, which gives

$$\begin{aligned} \dot{\sigma} &= y, \\ \dot{y} &= -2\beta y - \omega_0^2\sigma + \xi(t), \end{aligned} \quad (10)$$

where β and ω_0 follow from comparison of Eqs. (6) and (8). We will further use the Euler method for integration. The iterative procedure in this case has the form [12, 13]

$$\begin{aligned} \sigma_2 &= \sigma_1 + y_1\Delta t, \\ y_2 &= y_1 + (-2\beta y_1 - \omega_0^2\sigma_1)\Delta t + \sqrt{\Delta t}W_n. \end{aligned} \quad (11)$$

To simulate random force W_n, we use the Box–Muller model [19],

$$W_n = \sqrt{2D}\sqrt{-2\ln r_1}\cos(2\pi r_2), \quad r_i \in (0, 1], \quad (12)$$

where pseudorandom numbers r_1 and r_2 have uniform distributions.

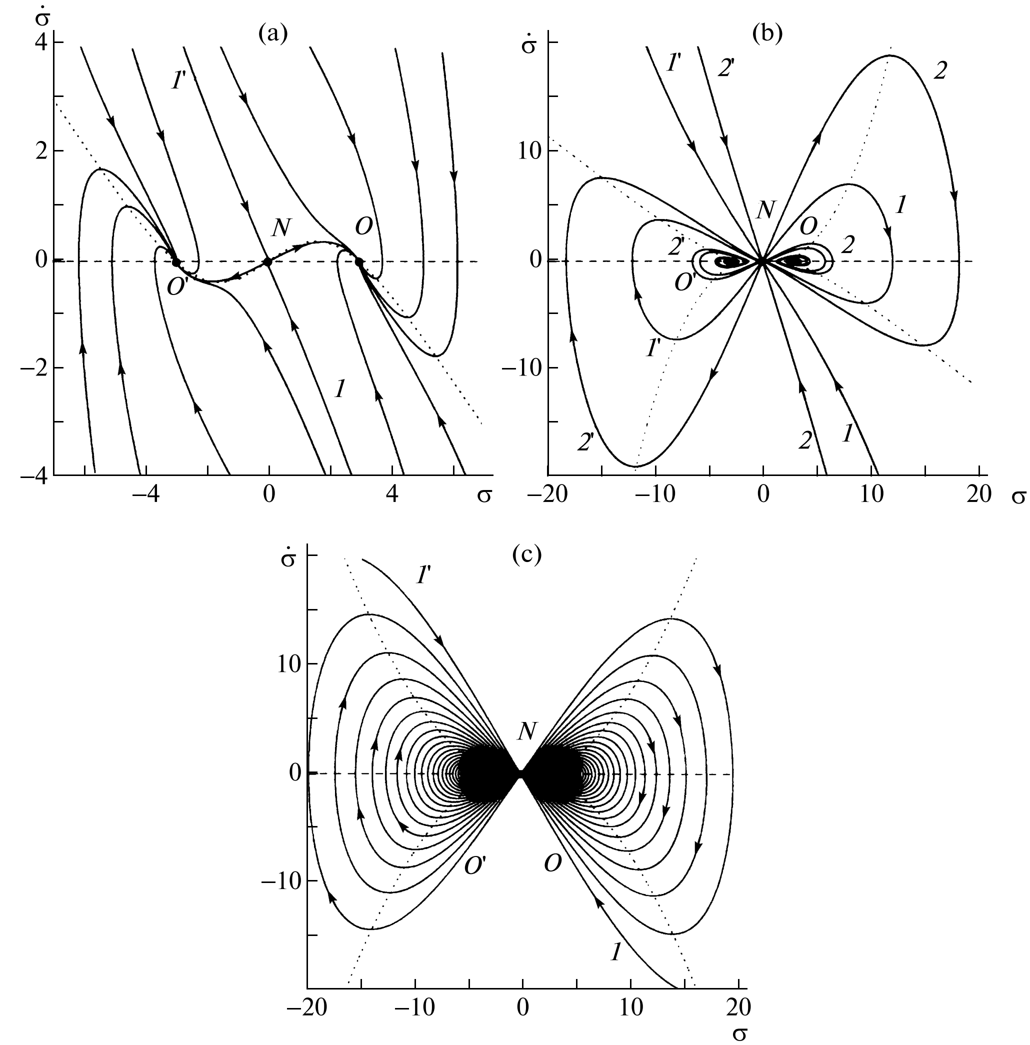

Fig. 1. Phase portraits of the system with parameters $g = 0.3$ and $T_e = 25$ for noise intensity $D = 0$, which are (a) solution to Eq. (6) for $\tau = \tau_\varepsilon/\tau_\sigma = 15$; (b) solution to Eq. (13) for $\tau = \tau_T/\tau_\sigma = 120$; and (c) solution to Eq. (15) $\tau = \tau_T/\tau_\varepsilon = 120$.

Figure 1a shows the result of numerical solution of Eq. (6) (using (11) and (12)) in the form of a phase portrait. The dashed line is the isoclinal on which $\dot{\sigma} = 0$ and phase trajectories have a vertical tangent. In the coordinates considered here, the isoclinal line is the abscissa axis. The dotted curve is isoclinal $\ddot{\sigma} = 0$, on which phase trajectories have a horizontal tangent. The expression for this isoclinal $\dot{\sigma}(\sigma)$ can be derived from Eq. (6). It can be seen from the figure that three singular points exist: saddle point N at the origin, which is an unstable point since it corresponds to the maximum value of the synergic potential [7], and two points, O and O' symmetric relative to $\sigma = 0$. In the vicinity of these points a tendency towards oscillatory process (which, however, is not realized in view of the large value of the damping factor) can be observed during establishment of stationary value σ_0. For the given relation between the relaxation times, such a behavior is manifested in the entire range of parameters [18]. It should be noted that positive and negative values of stresses correspond to motion of the upper rubbing surface in opposite directions. For instance, for the positive initial value of σ (proportional to the shear rate) and negative value of $\dot{\sigma}$ (acceleration), bidirectional motion is possible in accordance with the figure.

Figure 2a shows the time dependences of stresses corresponding to the trajectories in Fig. 1a. These dependences describe an aperiodic transient stick–slip regime in which the values of stress vary until slip at a constant velocity ($\sigma = \mathrm{const}$) sets in.

Figure 3a shows the solution of the same equation as in Fig. 2a for $D \neq 0$. It can be seen that the values of stress vary chaotically with time, but within small limits (since the noise intensity is low), which corresponds to the slip regime at an almost constant velocity. The curve begins at instant $t = 1000$ since we are

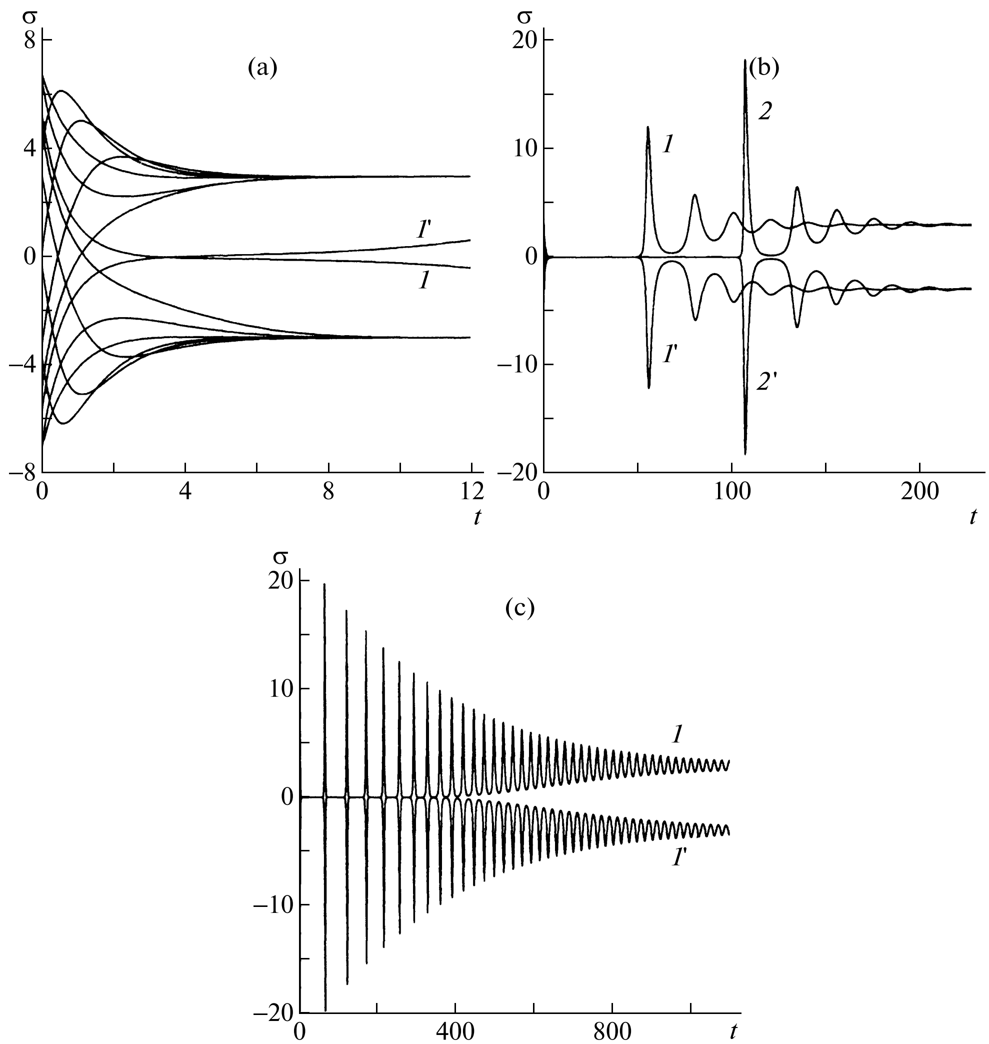

Fig. 2. Time dependences of stresses $\sigma(t)$ corresponding to Fig. 1.

interested here not in the transient, but in the steady-state friction regime.

Figure 4a shows the dependence of the intensity of the signal depicted in Fig. 3a on its frequency. It can be seen that $S(\nu)$ = const for low frequencies, and then this function decreases. The decrease is due to the fact that Eq. (6) for white noise $\xi(t)$ is a filter that does not transmit high frequencies and imparts a certain color to the noise. The $S(\nu)$ dependence has no clearly manifested peaks, which indicates the absence of a periodic component in the $\sigma(t)$ dependence. Thus, the slip regime with an insignificantly fluctuating shear rate sets in with time in the case under consideration.

1.2. The Case of $\tau_\varepsilon \ll \tau_T, \tau_\sigma$

In this case, the approximation $\tau_\varepsilon \dot{\varepsilon} \approx 0$ is used in the initial system; for the time measured in the units of τ_σ, this approximation leads to the equation

$$\ddot{\sigma} + \left[\frac{1+\sigma^2}{\tau} - \frac{\dot{\sigma}}{\sigma}\right]\dot{\sigma} + \frac{\sigma}{\tau}[1 + \sigma^2 - g(T_e - 1 + \sigma^2)] = \xi(t), \tag{13}$$

in which we have introduced the ratio $\tau = \tau_T/\tau_\sigma$. Equation (13) describes oscillations with the damping factor

$$\beta = 0.5\tau^{-1}(g^{-1} - 1)^{-1}(T_e - 2). \tag{14}$$

Figure 1b shows the phase portrait obtained by solving Eq. (13). It can be seen that the same singular points exist, the only difference being that points O and O' are transformed into stable foci and damped oscillations take place in the system. Phase trajectories are marked by digits in the figure. The isoclinal shown by the dotted curve differs from the previous case. This is due to the fact that the damping factor in Eq. (13) now depends on $\dot{\sigma}$; for $\ddot{\sigma} = 0$, the expression for the

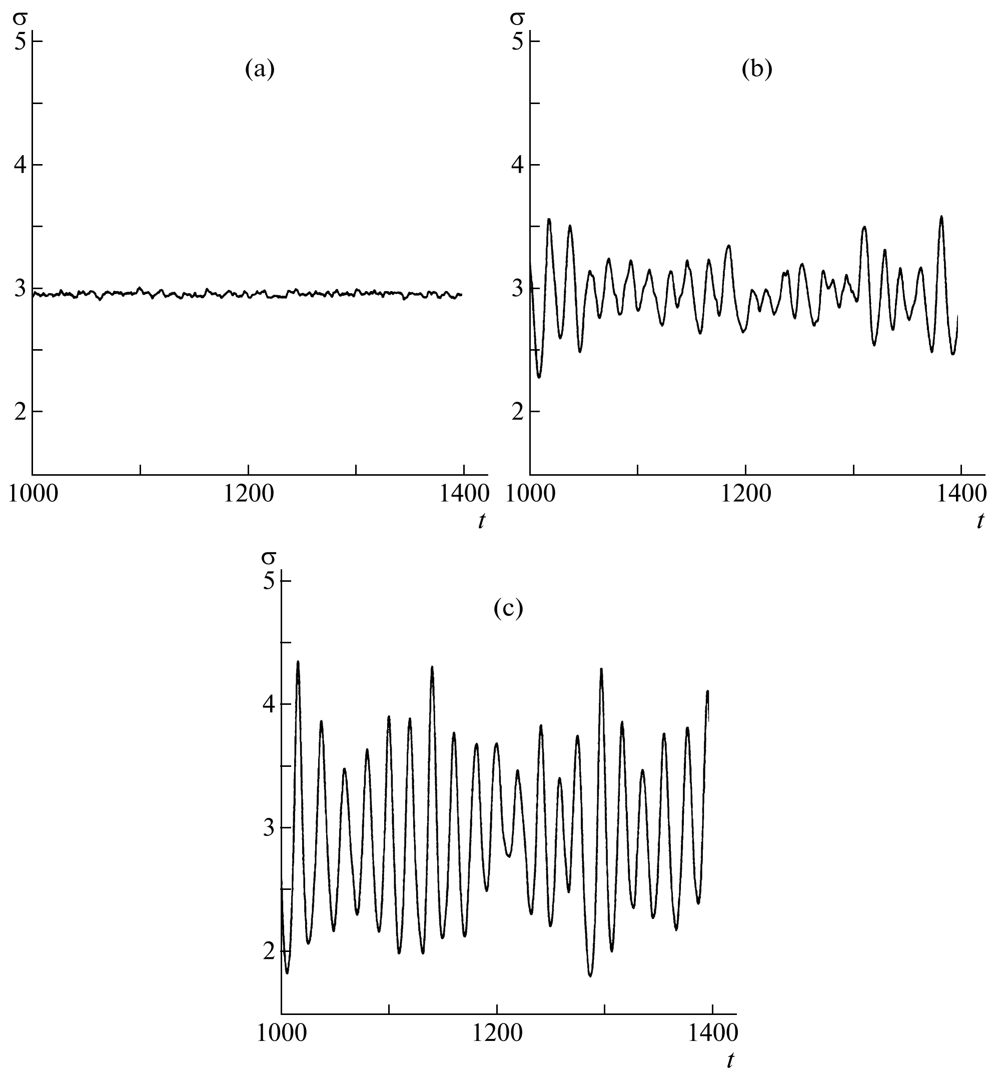

Fig. 3. Time dependences of stresses $\sigma(t)$ corresponding to Figs. 1 and 2 for noise intensity $D = 4 \times 10^{-4}$.

isoclinal $\dot{\sigma}(\sigma)$ is the solution to a quadratic equation: consequently, the dependence becomes double-valued.

The corresponding time dependences (Fig. 2b) contain extended segments on which $\sigma \approx 0$, which corresponds to slow motion of rubbing surfaces. This is due to the fact that the configurative point on the phase portrait of the evolving system is near the origin ($\sigma \approx 0$) with a low rate of stress variation ($\dot{\sigma} \approx 0$). However, a nonzero value of stresses corresponding to slip always sets in. The transition to bidirectional motion does not take place, indicating a higher value of the potential maximum at the origin.

The $\sigma(t)$ dependence obtained under the action of noise and depicted in Fig. 3b represents, as before, a steady-state regime with parameters that do not change with time. The dependence is visually periodic. To confirm this fact, we additionally carried out the Fourier analysis. The $S(\nu)$ dependence in Fig. 4b shows that if the frequency of the signal component increases, the signal intensity decreases; however, at $\nu \approx 0.05$, a peak is observed, which indicates the presence of a periodic component in $\sigma(t)$. Thus, a periodic variation of stresses is realized in Fig. 3b, which corresponds to an oscillatory process in the system. In this case, transitions between the liquid-like to solid-like structure of the lubricant leading to the stick–slip regime take place.

Earlier, we assumed that the solid-like structure corresponds to zero stresses; when the stresses assume nonzero values upon an increase in temperature T_e above the critical value, the lubricant melts. In this case, the following situation takes place. Let us suppose that stresses are initially minimal, which corresponds to a solid-like lubricant as before. If we set the surfaces in motion, the values of σ increase (any ascending segment on the dependence shown in Fig. 3b). When the stresses exceed the critical value, melting takes place, after which elastic component σ_{el} relaxes, and the total

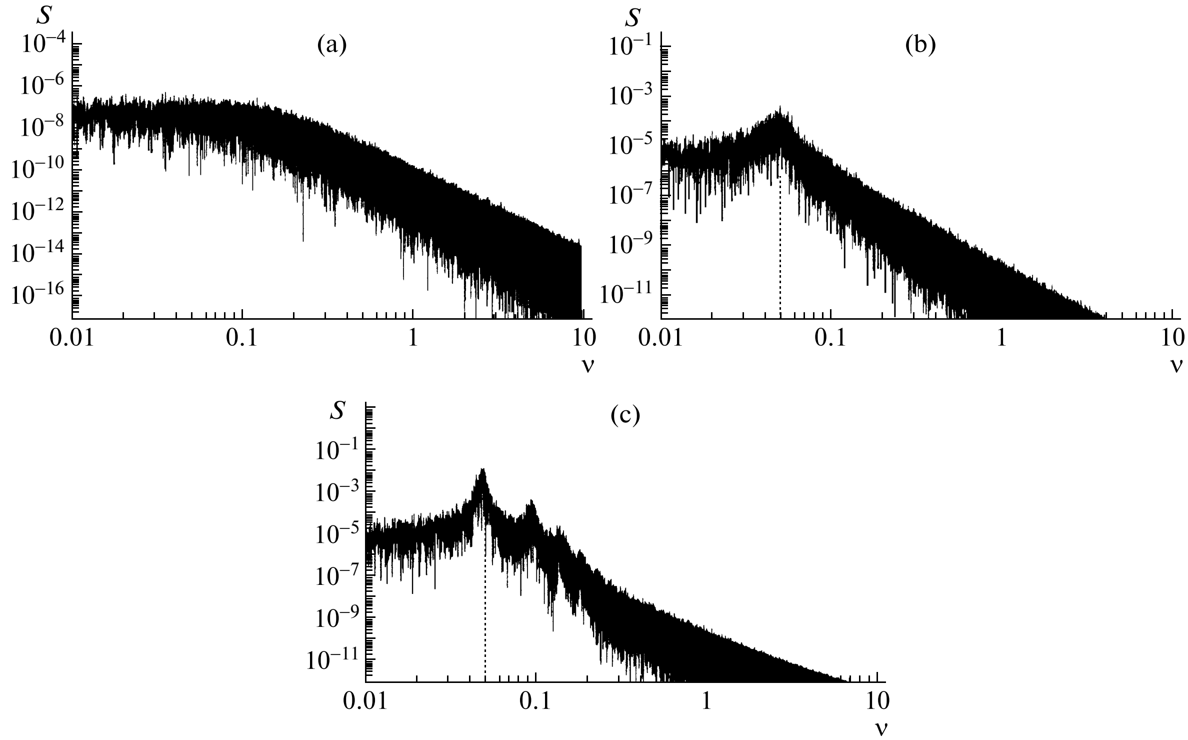

Fig. 4. Spectral power densities $S(\nu)$ corresponding to Fig. 3.

stresses also decrease (the descending segment of the curve). When the stresses acquire (as a result of relaxation) the values insufficient for maintaining the lubricant in the liquid-like state, the lubricant solidifies, and the process is repeated. Thus, melting occurs at large stresses just like before. It should be noted that the regime described here differs from that shown in Fig. 3a. In the case under investigation, periodic transitions occur between the solid-like and liquid-like structures of the lubricant, and random changes of stresses in it are fluctuations that do not lead to melting/solidification. Since the dependence in Fig. 3b is not strictly periodic, such a regime corresponds to experiments with chain molecules [1] that cannot easily form ordered structures; as a result, fluctuations are superimposed on oscillations. The amplitude of the stick–slip transitions is not constant in this case. Moreover, the effect of fluctuations may lead to instability of a focus, which corresponds to a continuous increase in the amplitude of stress oscillations resembling resonance in the system.

1.3. The Case of $\tau_\sigma \ll \tau_\varepsilon$, τ_T

In this case, setting $\tau_\sigma\dot{\sigma} \approx 0$ and measuring time in the units of τ_ε, we obtain

$$\ddot{\sigma} + \left[\frac{1}{\tau} - \frac{\dot{\sigma}}{\sigma}\right]\dot{\sigma} + \frac{\sigma}{\tau}[1 + \sigma^2 - g(T_e - 1 + \sigma^2)] = \xi(t), \tag{15}$$

where $\tau = \tau_T/\tau_\varepsilon$. The damping factor in this case is defined as

$$\beta = 0.5\tau^{-1}. \tag{16}$$

In contrast to the two previous cases, the value of β depends only on parameter τ and decreases upon an increase in this parameter, which implies that the duration of oscillations increases with τ. Consequently, a larger number of oscillations take place about the focus in Fig. 1c as compared to the previous case before the stationary state is established. Figure 2c, in which long-term oscillations take place without damping even for $t = 1000$, also illustrates this process. Figure 3c shows the time dependence of stresses under the action of noise ($D = \text{const}$ in all cases considered here), which is smoother and more regular than in Fig. 3b.

The corresponding spectrum (Fig. 4c) has a narrower peak at $\nu \approx 0.05$ than in Fig. 4b. Other peaks are also observed, but these peaks are less intense and more blurred; consequently, the fundamental frequency is $\nu \approx 0.05$. Figure 3c shows the dependence on time interval $\delta t = 400$, which corresponds (at the given frequency) to 20 full oscillations, which are observed on the dependence. It can be concluded that a more stable stick–slip regime with a larger amplitude sets in, in this case, on the time scale. Thus, the emergence of stick–slip friction should be expected in systems with $\tau_\sigma \ll \tau_\varepsilon \ll \tau_T$. It should be noted that for parameters corresponding to Fig. 2 ($g = 0.3$ and $T_e = 25$), the stationary value of stresses $\sigma_0 \approx 2.9761$ sets in with time in all cases considered here. It can be seen from Fig. 3 that noise may lead to small fluctuations near the steady-state value of σ_0 (see Fig. 3a) or may affect the system critically, changing the regime of flow (see Figs. 3b, 3c).

CONCLUSIONS

The above analysis shows that the experimentally observed periodic stick–slip regime of friction can be described using a rheological model parameterized by shear stresses, strain, and temperature of rubbing surfaces. The main feature of this regime is that transitions between the liquid-like and solid-like structures of the lubricant occur in this regime at different values of stresses, which is due to the effect of fluctuations leading to a periodic regime. It is shown that the most stable periodic stick–slip regime is realized when the stress relaxation time has the smallest value, followed by the strain relaxation time; the longest relaxation time corresponds to temperature. When the temperature relaxation time is the shortest, the stick–slip regime cannot be realized in the system. This is apparently due to rapid transfer of heat from the lubricant to the rubbing surfaces during dissipation due to heat conduction processes.

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