

## The Viscosity Temperature Dependence in the Boundary Friction Model under a Periodic External Action

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The effective viscosity temperature dependence of the lubricant is taken into consideration within the thermodynamic model of boundary friction. Transitions between the friction kinetic modes are described as the second-order phase transitions. For the system kinetic investigation the mechanical analogue of the simple tribological system has been considered. The time dependences of block coordinate, velocity, friction force and shear stress have been build. Therefore, the stick-slip motion has been described for a wide range of the system parameters. A comparison of simulation results with experimental data in sequence increasing shear rate has been made.

**Keywords:** Boundary friction, Phase transition, Elastic and viscous stress, Effective viscosity.

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### 1. INTRODUCTION

The dynamic properties of an ultrathin film which is confined between two atomically flat solid surfaces are significantly different from a bulk lubricant [1,2]. The boundary friction mode set in, when the lubricant thickness is less than several molecular layers. In this case, the lubricant film does not form stable thermodynamic solid or liquid phase. Instead, the regions with solid-like or liquid-like structures appear [1,3]. After a prolonged rubbing mechanisms operation at high pressures, the lubricant squeezes out of the surfaces and the boundary mode is established [3]. The periodical phase transitions of the first-order [3] and the second-order [4] between structural states lead to the interrupted motion. One of the most common experimental techniques is based on the use of surface force apparatus (SFA) [3]. It is used for investigation of quasi-spherical and chain molecules behavior which are confined between two atomically flat mica surfaces [3,5].

The detailed experimental investigation of the effective viscosity dependence on the temperature and the velocity gradient has been done in the work [6]. The main aim of this research is to investigate the tribological system behavior with help of the thermodynamic model, which takes into consideration the dependence of the effective viscosity on the temperature and the shear rate.

### 2. TRIBOLOGICAL SYSTEM

Let us consider the mechanical analogue of the tribological system [7] which shown in Fig. 1.

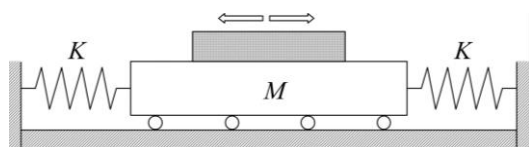


Fig. 1 – The scheme of the tribological system.

A block of mass  $M$  is located on rollers. The rolling

friction for the block can be neglected. The block is confined between two springs with stiffness  $K$  each. On the surface of the block another one is placed, which is forced to move by external forces. In the presence of interaction forces between the two blocks, the motion of the upper block causes the motion of the lower one. However, their trajectories do not coincide and depend on the friction mode established in the system.

Let us denote the upper block coordinate and velocity by  $X$  and  $V = \dot{X}$ , respectively. Moreover,  $x$ ,  $v = \dot{x}$  are a coordinate and a velocity of the lower block. The upper block moves according to the cyclic law:

$$X = X_m \cos \omega t, \quad (2.1)$$

$$V = -X_m \omega \sin \omega t, \quad (2.2)$$

where  $X_m$  is the amplitude and  $\omega$  is the cyclic frequency. In this case motion equation takes form [8]:

$$M\ddot{x} + 2Kx - F = 0. \quad (2.3)$$

### 3. VISCOSITY AND FRICTION FORCE DEPENDENCE ON TEMPERATURE

The stresses arising in the lubricant layer during the movement contain elastic  $\sigma_{el}$  and viscous  $\sigma_v$  components. The friction force  $F$  is determined as a multiplication of the total stresses and the rubbing surfaces contact area  $A$ :

$$F = \sigma_{el} + \sigma_v A. \quad (3.1)$$

The viscous stresses is found according to the equation [6]:

$$\sigma_v = \eta_{eff} \dot{\epsilon}, \quad (3.2)$$

where  $\eta_{eff}$  is the effective viscosity.

In the work [6] the effective viscosity  $\eta_{eff}$  dependence on the temperature  $T$  and on the deformation rate  $\dot{\epsilon}$  of the layers consisting of hydrocarbon polymeric

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chain molecules has been investigated by molecular dynamics methods. The resultant dependence has the form [6]:

$$\eta_{eff} = 10^C \dot{\varepsilon}^{-n}, \quad (3.3)$$

where  $C(n)$  and  $n$  are the temperature dependant parameters.

It is experimentally found, that the parameter  $n$  varies from 1 in the case of low temperatures (solid-like state) to  $n = 0$  (liquid-like state) in the case of high temperatures. In order to take this peculiarity into account, authors of article [6] introduce relationship between  $T$  and  $n$  in form:

$$n = \frac{1}{1 + (T/T_k)^\beta}, \quad (3.4)$$

where constants  $T_k$  and  $\beta$  depend on the polymer used. Particularly, for eicosan  $C_{20}H_{42}$  those parameters equal  $T_k = 353$  K,  $\beta = 4.09$  [6].

For all lubricants kinds, the dependence  $C(n)$  can be approximately expressed by equality [6]:

$$C = 10.9n - 3.8. \quad (3.5)$$

#### 4. FREE ENERGY

Let's write down the free energy density in the form [4,9]:

$$f = \alpha(T - T_c)\phi^2 + \frac{a}{2}\phi^2\varepsilon_{el}^2 + \frac{b}{4}\phi^4, \quad (4.1)$$

where  $T$  is the lubricant temperature,  $T_c$  is the critical temperature,  $\varepsilon_{el}$  is the shear component of elastic strain,  $a$ ,  $b$  are positive constants,  $\phi$  is the order parameter which in the solid-like state takes nonzero values and equals to zero, when lubricant is liquid-like [4,9].

The elastic stresses arising in the lubricant are defined as  $\sigma_{el} = \partial f / \partial \varepsilon_{el}$  [4]:

$$\sigma_{el} = \alpha\phi^2\varepsilon_{el} = \mu\varepsilon_{el}, \quad (4.2)$$

where  $\mu$  is the shear modulus which takes zero values in liquid-like state. Nonzero values of the shear modulus which correspond to solid-like state, can be found from condition  $\partial f / \partial \phi = 0$  [4]:

$$\mu = \frac{a}{b} 2\alpha(T_c - T) - a\varepsilon_{el}^2, \quad (4.3)$$

From the expression (4.3) let us find the elastic strain critical value [4]

$$\varepsilon_{el,c} = \sqrt{\frac{2\alpha(T_c - T)}{a}}. \quad (4.4)$$

The lubricant melts when it exceeds this value.

The elastic stresses (4.4) stationary value depends on the film thickness  $h$ , the internal stresses relaxation time  $\tau_\varepsilon$  and the surfaces relative velocity  $(V-v)$  [10,11]:

$$\varepsilon_{el}^0 = \frac{(V-v)\tau_\varepsilon}{h}. \quad (4.5)$$

From the Eq. (4.4) using expressions (4.5) we can evaluate the critical velocity  $V_{c0}$ :

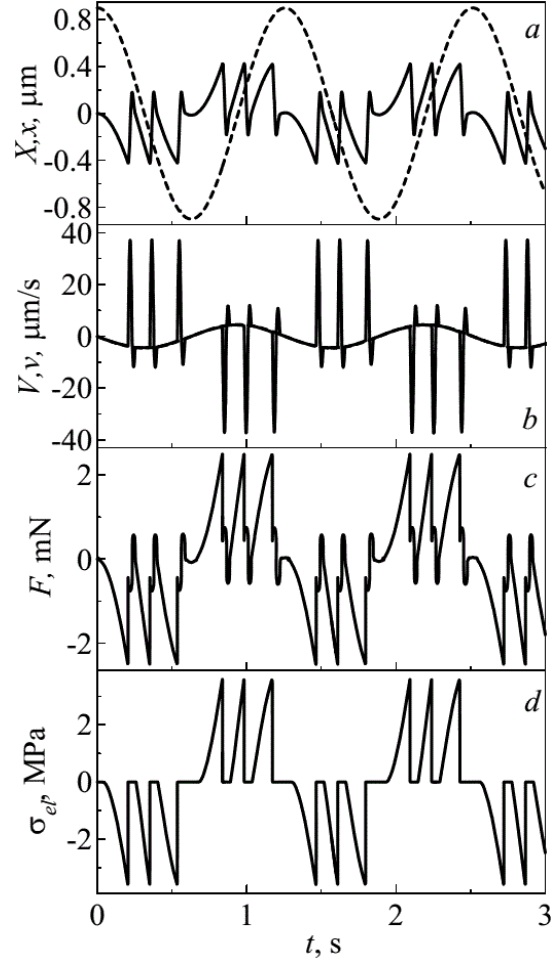
$$V_{c0} = \frac{h}{\tau_\varepsilon} \sqrt{\frac{2\alpha(T_c - T)}{a}}. \quad (4.6)$$

The friction force  $F$  can be written as combination of (3.2) – (3.5), (4.2) and (4.5):

$$F = \left[ \sigma_{el} + \text{sgn}(V-v) 10^{10.9n-3.8} \left( \frac{|V-v|}{h} \right)^{1-n} \right] A. \quad (4.7)$$

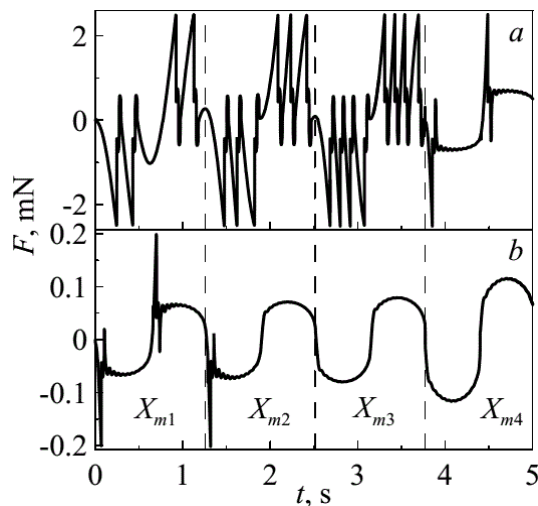
#### 5. STICK-SLIP FRICTION

The typical time dependences of the system parameters are shown in Fig. 2. Here the dashed curves correspond to the upper block coordinates  $X$  and velocities  $V$  and the solid curves to the bottom block coordinates  $x$  and velocities  $v$ .



**Fig. 2** – The dependences of coordinates  $X$  and  $x$ , velocities  $V$  and  $v$ , elastic stresses  $\sigma_{el}$  and friction force  $F$  on the time  $t$  at the parameters  $\alpha = 0.95$  J · K<sup>-1</sup>/m<sup>3</sup>,  $T_c = 290$  K,  $a = 4 \cdot 10^{12}$  Pa,  $b = 230$  J/m<sup>3</sup>,  $T_k = 353$  K,  $\beta = 4.09$ ,  $A = 0.6 \cdot 10^{-9}$  m<sup>2</sup>,  $\tau_\varepsilon = 10^{-8}$  s,  $h = 10^{-9}$  m,  $M = 0.4$  kg,  $\delta = 100$  J<sup>-1</sup>·m<sup>3</sup>/s,  $K = 3000$  N/m,  $T = 235$  K,  $X_m = 0.9 \cdot 10^{-6}$  m,  $\omega = 5$  rad/s.

At the initial time moment, the lubricant is solid-like. At time  $t > 0$  the upper block starts motion in accordance with Eq. (2.1). The bottom blocks moves together with the upper one, because of a friction force  $F$ , arising between the surfaces. However, the bottom block motion is prevented by stretching out and compressing of the springs. Therefore, in general case the velocities  $V$  and  $v$  do not coincide. Consequently, the relative shear velocity  $|V-v|$  increases. After exceeding critical value  $|V-v| > V_{c0}$  (4.6), the lubricant melts. The friction force  $|F|$  abruptly decreases (Fig. 2, c), and the block slips on a significant distance (Fig. 2, a). The bottom block velocity sharply grows and as a result the value  $|V-v|$  increases. At this moment in Fig. 2, b an increase in the friction force absolute value can be observed due to the stress viscous component increase according to Eq. (4.2), (4.5) and (4.7). Whereupon  $|F|$  decreases since the relative velocity reduces and  $F$  reverses sign. Here the block slips so much, so stretched spring compresses and vice versa. After fulfillment of the condition  $|V-v| < V_{c0}$  the lubricant solidifies.



**Fig. 3** – The friction force  $F$  dependences on the time  $t$  for the same parameters as in Fig. 2 and  $X_{m1} = 0.67 \cdot 10^{-6}$ ,  $X_{m2} = 0.9 \cdot 10^{-6}$ ,  $X_{m3} = 1.31 \cdot 10^{-6}$ ,  $X_{m4} = 5 \cdot 10^{-6}$  m. The panel  $a$  corresponds to the lubricant temperature  $T_1 = 235$  K, and the panel  $b$  to the  $T_2 = 280$  K one.

Lets consider the shear amplitude influence on the system behavior. Fig. 3,  $a$  demonstrates the friction force time dependence for 4 different shear amplitudes of the upper block. It is shown that the increase in amplitude  $X_m$  leads to the rise of phase transition amount over period (Fig. 3,  $a$ , amplitudes  $X_{m1}$ ,  $X_{m2}$ ,  $X_{m3}$ ). It is explained by the fact, that the block velocity

$X_m \omega \sin(\omega t)$  increases. Therefore the phase transition frequency grows [3,5,8,12]. As the amplitude is elevated to  $X_m = 5 \mu\text{m}$  the lubricant solidifies twice over a period. In this situation the lubricant solidifies when the block changes its direction. It is also worth noting that amplitudes were chosen so that their subsequent changing leads to stationary state at minimal time. For instance, for intermediate value  $X_m = 0.75 \mu\text{m}$ , which is between values  $X_{m1}$  and  $X_{m2}$ , the stick-slip motion is observed. In this case, the blocks twice “stick” and twice “slip” over a period. However, significant time is required to establish the stationary state.

Fig. 3,  $b$  was built for the higher lubricant temperature  $T_2 = 280$  K. The increase in temperature causes the decrease in the phase transition frequency. At the same time, the maximum friction force value reduces [3,5,7,8,11-13]. The figure first section determines the stick-slip motion, where lubricant melts twice over a period. This case is similar to that one described above for  $X_{m4}$ . For the subsequent amplitude value  $X_{m2}$ , the lubricant does not solidify after melting. Therefore, the further amplitude growing does not influence the friction mode and slide friction is established in the system [13, 14].

## 6. CONCLUSION

In the work the thermodynamic model of ultrathin lubricant melting has been further developed. The novelty of the investigation consists in consideration of the dependence of the polymeric lubricant viscosity on the temperature and velocity gradient within the second-order phase transition model. The dependence evidences, that the effective viscosity logarithm is proportional to the logarithm of the shear rate. The proportionality factor value varies from 1 in the solid-like state to 0 in the liquid-like state. It has been shown that stick-slip motion is a result of second-order phase transition between liquid-like and solid-like lubricant structures.

The time dependencies of coordinate, velocity, friction force and elastic stress have been built. The computer simulation results have been presented, which predicts stick-slip disappearance and further system functioning in the sliding friction mode.

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