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Development and Research of Probabilistic Models of Quality Assessment of Management Information Systems Operation

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Abstract - The purpose of the research is the development of stochastic models of management information systems (MIS) operation based on queueing systems. It has been shown that it is possible to use a single line queueing systems with the generalized erlang flow of random events. The study of such systems is carried out and the main characteristics of operation are obtained. This allows the planning of procedures for MIS operation.

***Key words* - time, criterion, *model,* service, process, *design, flow,* system, event.**

I. INTRODUCTION

MIS are difficult to implement, their work is often associated with numerous and mutually independent streams of input events, wearing casual character. In this regard, on the design phase of such systems the use of stochastic modeling is necessary [1, 2].

II. THE MAIN PART

Software of MIS provides the control of wide range MIS individual sub-system tasks solution in real time. These tasks are characterized by frequency of execution, duration of work, priority, volumes of output and input information. The launch on the execution of various tasks carried out by MIS supervisor system on the commands, which the planner of MIS periodically prepares [3]. The simple form of planning of the real time tasks execution is synchronous, when slot between sequential starts of the same task are equal to the selected slot. The flexibility and efficiency of such planning is greatly improved with the introduction of priorities, as well as with the ability to solve the background tasks when the computer system of MIS is free from solving real-time tasks [4, 5]. A variety of characteristics can serve as a criterion of the effectiveness of the dispatching operation of the system. For example the delay of start time of individual tasks on the planned time: the mean time, the probability that the time delay exceeds a certain fixed value and so on [6]. The knowledge of these characteristics can be used for synthesis of the system of planning of MIS software.

Let the whole time interval is split with the timer into separate intervals Δ. For each interval a number of high-priority and low priority tasks in accordance with the operation of MIS is planned to perform. Time left free in the interval from solving real-time tasks is used to solve background tasks (for example, test equipment monitoring).Interruption of low priority tasks is authorized.

Interrupted low priority task complete service after processing of high priority tasks. If a low priority task is interrupted in *i* - th interval, it continuation is added to the execution of tasks in (*i* + 1) - th time interval. Each task perform is planned with a certain period T_i. You can select the main computing cycle ***T***, which is the least common multiple of $T_i,\ i = 1,2,...,N$, where *N* - total number of individual tasks, that can be solved with the help of MIS software. In general, the time of execution of each task can be random; therefore it is important to know their solving time delay. This is especially true for low priority tasks, the beginning of the implementation of which can be shift by high priority tasks.

For considering execution of high priority tasks assume the next. Let ξ_{ij} is the time of execution of *i*-th priority task in the *j*-th interval of time. Suppose that $\Phi_j = \sum_{i \in I_j} \xi_{ij}, i = 1,2,...,n, n+1, n+2,$ is the time of execution of all high priority tasks in the *j*-th interval of time, where I_j are scheduled tasks on the *j*-th interval, and $n = T_h / \Delta$, where T_h is the main cycle of execution of high priority tasks. Let φ_j is the average time of execution of priority tasks in the *j*-th interval. The overall distribution function of service time can be written as a superposition of flows for each service interval -

$$B(t) = 1 - \frac{1}{m} \sum_{j=1}^{m} e^{-\mu_j t}$$

, so it is described as hypereksponentsial distribution. Input stream for such a system is regular, but it can be approximated by Erlang distribution in the event that the coefficient of variation approaches to zero. So, the entire system can be represented as a single line queueing system of type E/H/1. Let us investigate this system. Let

$\overline{\lambda} = \frac{\lambda}{k}$ and $\overline{\mu} = \left[\sum_{i=1}^{m} \frac{1}{m\mu_i} \right]^{-1}$ are average intensity of the flows described in accordance with the distribution of $A(t)$ and $B(t)$. $\alpha(s)$ and $\beta(s)$ are the Laplace-Stieltjes transformation of these distributions. So

$$\alpha(s) = \left(\frac{\lambda}{\lambda + s} \right)^k \text{ and } \beta(s) = \frac{1}{m} \sum_{i=1}^{m} \frac{\mu_i}{\mu_i + s}.$$

If $\overline{\mu} > \overline{\lambda}$, then for a stationary mode exists a function of the distribution of the waiting time $F(t)$, for which it is possible to compile the Lindley integral equation [7]. It corresponds to the factorization equationon

$$\gamma(s) = \frac{K_+(s)}{K_-(s)} = \alpha(-s)\beta(s) - 1 = = \frac{1}{m} \left(\frac{\lambda}{\lambda - s} \right)^k \sum_{i=1}^{m} \frac{\mu_i}{\mu_i + s} - 1 = 0 \quad (1)$$

Notice, that function

$$\beta(s) = \frac{\frac{1}{m} \sum_{i=1}^{m} \mu_i \frac{\prod_{j=1}^{m} (\mu_j + s)}{\mu_i + s}}{\prod_{i=1}^{m} (\mu_i + s)} = \frac{P_{m-1}(s)}{Q_m(s)} \quad (2)$$

is the ratio of two polynomials, and the degree of the numerator is less than the denominator. Then the Laplace-Stieltjes transformation from $F(t)$ will be

$$\varphi(s) = \frac{Q_m(s)}{Q_m(0) \prod_{i=1}^{m} \left(1 - \frac{s}{q_i} \right)}, \quad (3)$$

where $q_i \ (i = 1,2,...,m)$ are the roots of equation

$$\gamma(s) = 0 \Rightarrow \frac{1}{m} \left(\frac{\lambda}{\lambda - s} \right)^k \sum_{i=1}^{m} \frac{\mu_i}{\mu_i + s} - 1 = 0, \quad (4)$$

which are located in the left half-plane $\mathrm{Re}\, s < 0$. Using (2), we find in the left half-plane $\mathrm{Re}\, s < 0$

$$\varphi(s) = \frac{Q_m(s)}{\prod_{i=1}^{m} \mu_i \left(1 - \frac{s}{q_i} \right)}. \quad (5)$$

Let $s = \lambda(1 - z)$, then the equation (4) will have the form

$$\frac{1}{m} \sum_{i=1}^{m} \frac{\mu_i}{\mu_i + \lambda(1 - z)} - z^k = 0. \quad (6)$$

Solving this equation after the transformations, we will get

$$\varphi(s) = \lambda^m \prod_{i=1}^{m} \frac{(\mu_i + s)(1 - z_i)}{\mu_i + [\lambda(1 - z_i) - s]}, \quad (7)$$

where z_i are the roots of equation (5) and

$$1 < z_1 < 1 + \frac{\mu_1}{\lambda} < z_2 < ... < 1 + \frac{\mu_{k-1}}{\lambda} < z_k < < 1 + \frac{\mu_k}{\lambda} < ...... < z_m < 1 + \frac{\mu_m}{\lambda}. \quad (8)$$

Taking the inverse Laplace-Stieltjes transformation from (7), we can obtain an expression for the distribution function of delay time in serving high-priority tasks $F(t)$.

Let's find the average delay time in the service of high-priority tasks

$$\bar{t}_d = -\varphi'(s)\big|_{s=0} = \sum_{i=1}^{m} \left[\frac{1}{\lambda(z_i - 1)} - \frac{1}{\mu_i} \right]. \quad (9)$$

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Probability that time of solving of high-priority tasks ξ will exceed Δ is determined by the next expression

$$P\{\xi > \Delta\} = \sum_{j=1}^{m} e^{-\mu_j \Delta} \Big/ m. \quad (10)$$

Real-time conditions have a strong impact on the execution of lower priority tasks, for which the interruption of service is allowed [6]. Suppose that *i* is the priority level and $T_i = N_i / \Delta$ the period of performance of the low priority task, which are scrutinized. Let u_j is the time required to execute this task in the *j*-th interval, and ω_j is the total time of execution of all high priority task in this interval including the time for the end of execution all tasks from (*j* - 1)- th interval. Then the time that remains in the j-th period for the execution of low priority task is defined by the expression $v_j^i = \max\{0, T_i - w_j\}$, where v_j^i is a random variable. It can be either greater or less than the time u_j required to solve low priority task. Thus, the relationship between these random variables is similar to the process of service of requests in a single-server queueing system of type G/G/1; the sequence of values v_j^i describes the input stream of requests, and u_j - service time. To determine the relationship between the delay time Z when executing a low-priority task and the waiting time in an equivalent single-line queueing system, we will use the fact that if the solution of the problem starts in the planned time interval Δ , then $\eta = z$, otherwise $\eta = z - \Delta \cdot \mathrm{int}(z/\Delta)$, where $\mathrm{int}(x)$ is the integer part of the expression. So

$$P\{z > t\} = P^*\{\eta > t^*\} = 1 - F(t - \Delta \cdot \mathrm{int}(t/\Delta)) =$$
$$= 1 - W(t - \Delta \cdot \mathrm{int}(t/\Delta)),$$

Where $F(t)$ - is the law of the distribution of the waiting time for an equivalent single-line queueing system, and $W(t)$ a is the distribution function of the waiting time.

Let us approximate the input stream and the service flow with distributions that belong to the family of common Erlang distributions. The Laplace-Stieltjes transformation from their distribution functions have the form

$$f(s) = \prod_{i=1}^{k} \sum_{j=1}^{n_i} \frac{a_{ij}\lambda_{ij}}{s + \lambda_{ij}}, \quad \sum_{j=1}^{n_i} a_{ij} = 1, \qquad (11)$$

and is the ratio of two polynomials.

You can get comprehensive performance characteristics of low priority tasks in real time if you will approximates the input flow and service time by general Erlang distribution. Exponential, hyper-exponential and Erlang distribution are special cases of distribution (11). Any distribution of the duration of intervals between neighboring events can be approximated with any accuracy by the general Erlang distribution, it is only necessary to choose the parameters appropriately $a_{ij}, \lambda_{IJ}, n_i, k$. It is desirable that the values n_i and k will be small.

Let us consider a single-line queueing system with expectation that simulates the process of servicing low priority tasks, the input flow of requests and services for which are given by generalized Erlang distributions with Laplace-Stieltjes transforms from the distribution functions, respectively $\alpha(s)$ and $\beta(s)$.

The expression for the average intensity of requests for low-priority tasks is calculated as follows:

$$\overline{\lambda} = -\alpha'(0)^{-1} = \left[\sum_{i=1}^{k} \sum_{g=1}^{l_i} \frac{a_{ig}}{\lambda_{ig}} \right]^{-1}. \qquad (12)$$

The expression for the average intensity of the process of servicing low-priority tasks will have the form

$$\overline{\mu} = -\beta'(0)^{-1} = \left[\sum_{i=1}^{n} \sum_{q=1}^{c_i} \frac{b_{iq}}{\mu_{iq}} \right]^{-1}. \qquad (13)$$

If $\overline{\mu} > \overline{\lambda}$, then for the system, there is a steady-state regime, for the study of which the above approach can be used

III. CONCLUSIONS

MIS operate under random event flows. In this regard, on the design phase of such systems the use of stochastic modeling is necessary, in particular on the basis of queueing systems. In paper process of planning of real time application is analyzed. The planning cycle is composed of solutions of high-priority and low-priority tasks. It has been shown that it is possible to use a single line queueing systems with the Erlang input stream and hyperexponential service time for the simulation of high priority tasks. The integral equation Lindley for such a system is received. The solution of such equation is obtained by factorization of integral equation. Explicit expressions of the distribution function of the delay time of the real-time high-priority tasks is received, as well as its average value. The process of execution of low-priority tasks is analyzes. It is shown that the analysis of this process can be done by the use of queueing systems with the generalized Erlang flow of random events. On the basis of the method of the integral equation Lindley the Laplace-Stieltjes transform of the waiting time in the queue and the average value of this time is obtained.

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