

## Parametric Study of Stacked Microstrip Patch Antenna with Dissimilar Substrates

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(Received 20 January 2018; revised manuscript received 09 August 2018; published online 25 August 2018)

A complete parametric study of stacked rectangular microstrip patches printed on non-magnetic isotropic substrate is performed. The numerical results are obtained by applying the method of moments to the electric field integral equations. Detailed closed-form expressions of Green's functions are presented. The new results found indicate that, the lower resonant frequency is mainly determined by the patch etched on the thicker layer. The layer having the higher permittivity defines which resonance is mainly determined by the bottom patch, either the upper resonance if the upper layer has the higher permittivity, or the lower resonance in opposite case. All these results offer better understanding and thus a better control of the dual-band operating of the microstrip antenna. Therefore, a proper choice of the antenna parameters becomes possible in order to obtain the desired functional characteristics.

**Keywords:** Stacked Microstrip Patches, Resonance, Dissimilar Substrates, Ual Frequency, Mutual Coupling.

DOI: [10.21272/jnep.10\(4\).04004](https://doi.org/10.21272/jnep.10(4).04004)

PACS numbers: 77.22.Ch, 84.40. - x, 84.40.Ba

### 1. INTRODUCTION

Narrow bandwidth is one of the major deficiencies of microstrip antennas. Moreover, they suffer from the fact that they cannot operate efficiently over relatively large distinct bands, as required by some applications. To overcome these shortcomings, stacked- patches microstrip antennas have been proposed [1]. These structures, sometimes called dual-frequency antennas, can resonate at two different frequencies. Tunable stacked-patches configuration [2], [3] has been also investigated. Similar studies have been conducted for triangular [4], rectangular [5-8] and circular [9] patches.

Most of the published works on rectangular stacked-patches antennas have considered substrates with similar layers, consequently, when the effects of the substrate parameters are studied, the parameters of all layers must be varied simultaneously. This paper is devoted to study rectangular stacked-patches printed on dissimilar substrates; specifically, the effect of permittivity and thickness of each layer is investigated. The rest of the paper is organized as follows: the theoretical formulation of the analysis method is provided in section 2. Simulation results are presented and analyzed in section 3, and finally, some observations and results are reported in the conclusion.

### 2. THEORETICAL FORMULATION

The proposed antenna structure is depicted in Fig. 1. The substrate is a non-magnetic isotropic dielectric, with free space permittivity and permeability given by  $\epsilon_0$  and  $\mu_0$  respectively.

The process yielding the resonant frequencies of the stacked-patches microstrip antenna starts by applying the method of moment to the following integral equations

$$E_1 = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} F(k_s, r_s) \times \\ \times [Q_{11}(k_s) \cdot J_1(k_s) + Q_{12}(k_s) \cdot J_2(k_s)] dk_s, r_s \in patch_1 \quad (1)$$

$$E_2 = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} F(k_s, r_s) \times \\ \times [Q_{21}(k_s) \cdot J_1(k_s) + Q_{22}(k_s) \cdot J_2(k_s)] dk_s, r_s \in patch_2 \quad (2)$$

where  $E_1$  and  $E_2$  are the transverse electric fields associated to the lower and the upper patches respectively. At resonance, the field and the current can sustain themselves without an external source, thus the transverse fields of (1) and (2) become

$$E_1 = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} F(k_s, r_s) \times \\ \times [Q_{11}(k_s) \cdot J_1(k_s) + Q_{12}(k_s) \cdot J_2(k_s)] dk_s = 0, r_s \in patch_1 \quad (3)$$

$$E_2 = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} F(k_s, r_s) \times \\ \times [Q_{21}(k_s) \cdot J_1(k_s) + Q_{22}(k_s) \cdot J_2(k_s)] dk_s = 0, r_s \in patch_2 \quad (4)$$

where  $J_1(k_s)$  and  $J_2(k_s)$  are the vector Fourier transform of the electric current densities on the lower and the upper patches  $j_1(r_s)$ ,  $j_2(r_s)$ , respectively,

$$j_1(r_s) = \frac{1}{4\pi^2} \iint_{-\infty}^{+\infty} F(k_s, r_s) J_1(k_s) dk_s, \\ j_2(r_s) = \frac{1}{4\pi^2} \iint_{-\infty}^{+\infty} F(k_s, r_s) J_2(k_s) dk_s,$$

where  $F(k_s, r_s) = \frac{1}{k_s} \begin{bmatrix} k_x & k_y \\ k_y & -k_x \end{bmatrix} e^{ik_s r_s}$ ,  $Q_{11}$ ,  $Q_{12}$ ,  $Q_{12}$  and  $Q_{22}$  are diagonal matrices having the form  $Q = \begin{bmatrix} Q^e & 0 \\ 0 & Q^h \end{bmatrix}$ .

Each matrix is related to the corresponding spectral

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Green's function having the form  $\mathbf{G} = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix}$ .

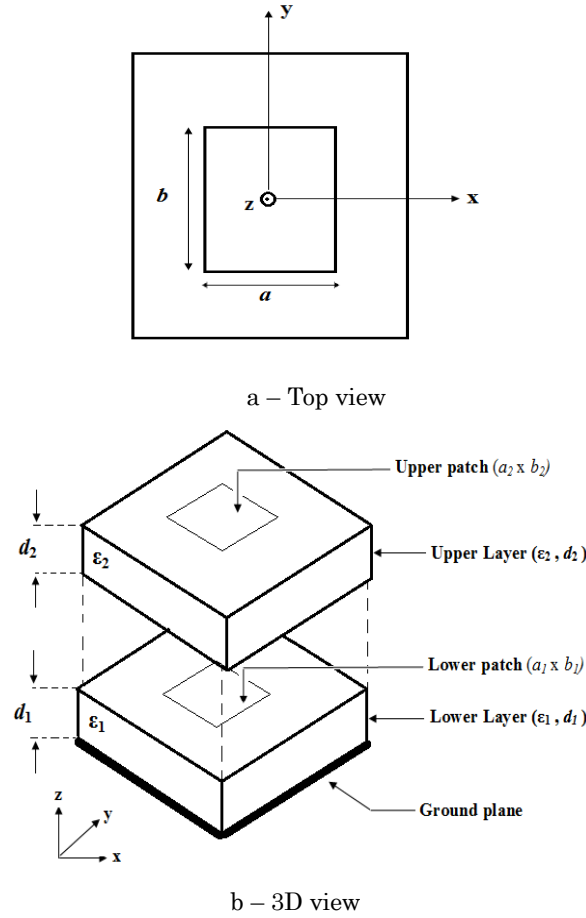


Fig. 1 – Stacked rectangular microstrip antenna structure

By the expressions

$$\begin{aligned} G_{11} &= \frac{1}{k_s^2} (k_x^2 Q^e + k_y^2 Q^h), \\ G_{12} = G_{21} &= \frac{k_x k_y}{k_s^2} (Q^e - Q^h), \\ G_{22} &= \frac{1}{k_s^2} (k_y^2 Q^e + k_x^2 Q^h), \end{aligned}$$

where  $k_0 = \sqrt{k_s^2 + k_z^2} = \omega \sqrt{\epsilon_0 \mu_0}$ ,  $k_s = \sqrt{k_x^2 + k_y^2}$ .

It is given that  $\mathbf{Q}_{11}$  and  $\mathbf{Q}_{22}$  are related to the bottom and top patches respectively,  $\mathbf{Q}_{12}$  and  $\mathbf{Q}_{21}$  represent the interaction between the two patches.

The basic idea of the method of moments is to approximate the unknown current densities on the patches using a limited number of known basis functions, and these current densities are expressed in space domain as:

$$\begin{aligned} \mathbf{j}_1(\mathbf{r}_s) &= \sum_{n=1}^{N_0} a_n \begin{bmatrix} J_{xn}^1(\mathbf{r}_s) \\ 0 \end{bmatrix} + \sum_{m=1}^{M_0} b_m \begin{bmatrix} 0 \\ J_{ym}^1(\mathbf{r}_s) \end{bmatrix}, \\ \mathbf{j}_2(\mathbf{r}_s) &= \sum_{p=1}^{P_0} c_p \begin{bmatrix} J_{xp}^2(\mathbf{r}_s) \\ 0 \end{bmatrix} + \sum_{q=1}^{Q_0} d_q \begin{bmatrix} 0 \\ J_{yq}^2(\mathbf{r}_s) \end{bmatrix}, \end{aligned}$$

where  $J_{xn}^1$ ,  $J_{ym}^1$ ,  $J_{xp}^2$ ,  $J_{yq}^2$  are the selected basis functions, and  $a_n$ ,  $b_m$ ,  $c_p$ ,  $d_q$  are the unknown weighting coefficients to be computed. In this paper, we used

entire domain sinusoidal basis functions given by the following expressions

$$\begin{aligned} J_{xn}^1 &= \sin \left[ \frac{n_1 \pi}{a_1} \left( x + \frac{a_1}{2} \right) \right] \cos \left[ \frac{n_2 \pi}{b_1} \left( y + \frac{b_1}{2} \right) \right], \\ J_{ym}^1 &= \sin \left[ \frac{m_2 \pi}{b_1} \left( y + \frac{b_1}{2} \right) \right] \cos \left[ \frac{m_1 \pi}{a_1} \left( x + \frac{a_1}{2} \right) \right], \\ J_{xp}^2 &= \sin \left[ \frac{p_1 \pi}{a_2} \left( x + \frac{a_2}{2} \right) \right] \cos \left[ \frac{p_2 \pi}{b_2} \left( y + \frac{b_2}{2} \right) \right], \\ J_{yq}^2 &= \sin \left[ \frac{q_2 \pi}{b_2} \left( y + \frac{b_2}{2} \right) \right] \cos \left[ \frac{q_1 \pi}{a_2} \left( x + \frac{a_2}{2} \right) \right]. \end{aligned}$$

Consequently, the integral equations in (3) and (4) are transformed into a matrix equation as

$$\begin{bmatrix} \mathbf{Z}_{11} & \mathbf{Z}_{12} \\ \mathbf{Z}_{21} & \mathbf{Z}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = 0$$

where  $\mathbf{I}_1$  is a  $((N_0 + M_0) - \text{by} - 1)$  vector representing the weighting coefficients related to  $\mathbf{j}_1$ ,  $\mathbf{I}_2$  is a  $((P_0 + Q_0) - \text{by} - 1)$  vector representing the weighting coefficients related to  $\mathbf{j}_2$ .

The complex resonant frequencies are the roots of the following characteristic equation

$$\det(\mathbf{Z})=0$$

where  $\mathbf{Z} = \begin{bmatrix} \mathbf{Z}_{11} & \mathbf{Z}_{12} \\ \mathbf{Z}_{21} & \mathbf{Z}_{22} \end{bmatrix}$ .

The detailed expressions of  $\mathbf{Z}_{11}$ ,  $\mathbf{Z}_{12}$ ,  $\mathbf{Z}_{21}$  and  $\mathbf{Z}_{22}$  can be found in [10].

### 3. RESULTS AND DISCUSSION

At the beginning of this section, we compare the experimental results reported in [11] to our numerical results, the considered structure has the following characteristics:  $a_1 \times b_1 = 2.1 \times 1.5$  cm,  $a_2 \times b_2 = 3.1 \times 1.5$  cm,  $d_1 = 0.9$  mm,  $d_2 = 1.8$  mm,  $\epsilon_{psr1} = \epsilon_{psr2} = 2.33$ .

Table 1 – The resonant frequencies for single and stacked patches structures

|             | One patch alone        |                | Two patches stacked    |                |
|-------------|------------------------|----------------|------------------------|----------------|
|             | Theo-ry(present paper) | Meas-ured [11] | Theo-ry(present paper) | Meas-ured [11] |
| lower-patch | 4.428                  | 4.33           | 4.732                  | 4.55           |
| upper patch | 3.087                  | 3.08           | 3.083                  | 3.09           |

Table 1 reveals that our results are in good agreement with measurements, which validates the subsequent results. In order to investigate the influence of the structure parameters on resonance characteristics of the antenna, this section is divided into two parts, first, we study the effect of the substrate's thickness then the effect of relative permittivity is investigated in the second part.

#### 3.1 Effect of Dielectric Thickness

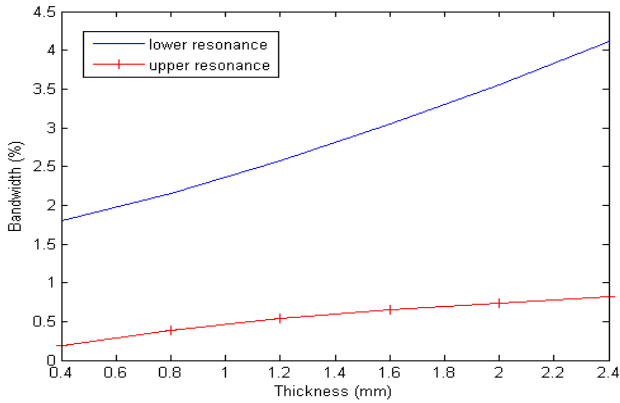
The thickness of the first layer is varied from 0.4 to 2.4 mm, and the second layer thickness is fixed to 1.2 mm. Both layers have the same relative permittivity  $\epsilon_r = 2.33$ , and the patches are identical and have di-

mensions  $a_1 \times b_1 = a_2 \times b_2 = 1.8 \text{ cm} \times 2.7 \text{ cm}$ . The patches' dimensions are chosen so that the length to the width ratio remains constant (equals to 1.5).

Table 1, represents the variation of resonant frequencies of stacked and isolated patches for different thicknesses of the lower layer of the substrate, whereas Fig. 2 depicts the variation of the upper and the lower resonance bandwidths (%) versus the lower layer thickness.

**Table 2** – The resonant frequencies of stacked and isolated patches,  $d_2 = 1.2 \text{ mm}$

| $d_1(\text{mm})$ | Resonant frequency (GHz) |                 |                 |                    |
|------------------|--------------------------|-----------------|-----------------|--------------------|
|                  | Stacked patches          |                 | Top patch alone | Bottom patch alone |
|                  | Upper resonance          | Lower resonance |                 |                    |
| 0.4              | 3.596                    | 3.531           | 3.538           | 3.561              |
| 0.8              | 3.585                    | 3.498           | 3.511           | 3.517              |
| 1.2              | 3.579                    | 3.465           | 3.485           | 3.479              |
| 1.6              | 3.575                    | 3.432           | 3.458           | 3.445              |
| 2                | 3.572                    | 3.401           | 3.431           | 3.412              |
| 2.4              | 3.57                     | 3.37            | 3.405           | 3.38               |



**Fig. 2** – Upper and lower resonance bandwidths as a function of  $d_1$ ,  $d_2 = 1.2 \text{ mm}$

From Table 2, we notice that as  $d_1$  increases the upper and lower resonant frequencies decrease, but the bandwidths increase, also the distance between the upper and lower resonant frequencies increases as  $d_1$  increases. Moreover, we observe that when  $d_1 < d_2$ , the lower resonant frequency is close to the top patch resonant frequency when the bottom patch is absent, which indicates that the lower resonant frequency is mainly determined by the top patch.

Similarly, the lower resonant frequency is mainly determined by the bottom patch when  $d_1 > d_2$ , because the lower resonant frequency is close to the bottom patch resonant frequency when the top patch is absent. From Fig. 2, it is clear that the lower resonance bandwidth is larger than the bandwidth of the upper resonance.

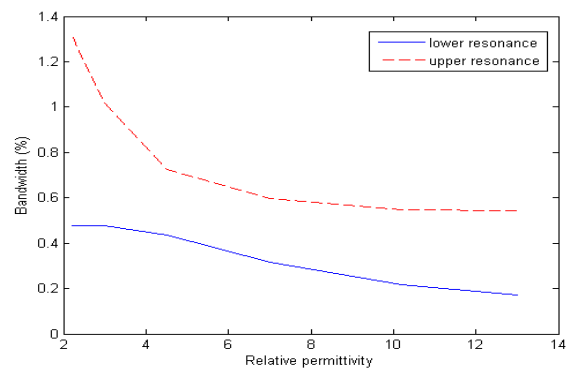
### 3.2 Effect of Relative Permittivity

To investigate the effect of the relative permittivity on the characteristics of the antenna, the relative permittivity of the first layer is varied from 2.2 to 13, and the relative permittivity of the second layer is fixed to 6. The two layers have the same thickness  $d$ , where

$d = 1 \text{ mm}$ , and the two patches have the same dimensions  $a_1 \times b_1 = a_2 \times b_2 = 1.8 \text{ cm} \times 2.7 \text{ cm}$ . The corresponding resonant frequencies of stacked and isolated patches are presented in Table 3. The data presented in Table 3 and Fig. 3 show that the resonant frequencies and the bandwidths of both resonances are inversely proportional to the relative permittivity. Also it is found that when  $\epsilon_{r1} < \epsilon_{r2}$  the upper resonant frequency is primarily determined by the bottom patch, but the bottom patch is the main factor in determining the lower resonant frequency when  $\epsilon_{r1} > \epsilon_{r2}$ . Moreover, it is also noted that when  $\epsilon_{r1}$  increases, the difference between the upper and lower resonant frequencies decreases until it reaches its minimum for  $\epsilon_{r1} = \epsilon_{r2}$ , and after that it starts increasing steadily. When the results of Tables 2, 3 are compared, we can see clearly that the ratio of maximum to minimum resonant frequencies for the case of relative permittivity is greater than the ratio of the ratio of maximum to minimum resonant frequencies for the case of thickness. This means that the impact of the relative permittivity on the resonant frequencies is larger than that of the thickness. Finally, it is important to report that, we have conducted a similar study for  $d_2$  and  $\epsilon_{r2}$ , and the results are similar to those reported above.

**Table 3** – Resonant frequencies of stacked and isolated patches,  $\epsilon_{r2} = 6$ .

| $\epsilon_{r1}$ | Resonant frequency (GHz) |                 |                 |                    |
|-----------------|--------------------------|-----------------|-----------------|--------------------|
|                 | Stacked patches          |                 | Top patch alone | Bottom patch alone |
|                 | Upper resonance          | Lower resonance |                 |                    |
| 2.2             | 3.464                    | 2.281           | 2.932           | 3.462              |
| 2.33            | 3.38                     | 2.281           | 2.883           | 3.379              |
| 3               | 3.03                     | 2.278           | 2.685           | 3.029              |
| 4.5             | 2.529                    | 2.274           | 2.418           | 2.527              |
| 6               | 2.278                    | 2.21            | 2.265           | 2.215              |
| 7               | 2.274                    | 2.061           | 2.193           | 2.062              |
| 10.2            | 2.27                     | 1.726           | 2.049           | 1.726              |
| 13              | 2.269                    | 1.536           | 1.976           | 1.536              |



**Fig. 3** – Upper and lower resonance bandwidths as a function of  $\epsilon_{r1}$ ,  $\epsilon_{r2} = 6$

The influence of the dimensions of the stacked patches has been investigated by [1], where it was found that the dual frequency behavior depends on the relative size of one patch with respect to the other.

Furthermore, it is well known that the stacked patches interact with each other by the mutual cou-

pling, in which the field of each patch affects the current distribution in the other patch, and hence its resonant frequency. The results presented in [1] show that the resonant frequency of the patch having the larger size is basically the less affected by the mutual coupling. This could be noticed when comparing the resonant frequencies of the stacked and the isolated patches.

#### 4. CONCLUSION

In this work, we conducted a detailed parametric study on the effect of the substrate's parameters on the resonance characteristics of the stacked patches microstrip antenna. Numerical results are based on the use of the method of moments in spectral domain. Close

examination of the simulation results yields the following conclusions; first, increasing the thickness of the layers or decreasing the permittivity increases the bandwidths of both resonances. Second, if the patches have the same size, then the lower resonant frequency is mainly determined by the size of the patch etched on the thicker layer. Third, the permittivity has a larger effect on the resonant frequencies than does the thickness. Finally, it is important to mention that the stacked configuration in which the dielectric layers and the patches have the same characteristics, possesses minimum separation between the upper and lower resonant frequencies, and this leads generally to overlapping bands, and consequently larger bandwidth. However, this situation is preferred only when distinct bands are not required.

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