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# Natural Light Diffraction on Endless Grating of Metal Strips

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In this paper, the problem of diffraction of natural light on a grating of infinite periodic sequence of infinitely thin metal strips is solved. A quantum approach was applied to its solution. The solution of the diffraction problem of a natural, unpolarized wave is based on the results obtained separately for the cases of diffraction of *H*- and *E*-polarized photons, solved by the strict method of the Riemann-Hilbert boundary value problem. The work is devoted to the calculation of the density of the probability of finding an *H*- or *E*-polarized photon at a given point in space for the case when the photon flow falling on the grating is a mixture of equal densities of *H*- and *E*-polarized photons. Since the diffraction pattern repeats with the grating period *l*, the paper presents the results of calculations of the probability density $|\Psi|^2$ depending on the *y* coordinate for one period within the limits of the change of *y*/*l* from zero to one. From the comparison of graphs $|\Psi|^2$ for *H*- and *E*- of polarized photons and their superposition, the relationship between the number of maxima in the diffraction pattern of natural light and the ratio of the grating period to the photon wavelength is established. As it turned out, the number of maxima increases proportionally to the specified ratio. At the same time, the main masses are located against the gaps. For unpolarized, natural light, the diffraction pattern is qualitatively similar to the corresponding pattern for *H*-polarized photons, but differs in the height of the maxima

**Keywords**: Diffraction, Grating, Quantum, psi-function, Diffraction pattern, Phonon.

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## 1. INTRODUCTION

Photon diffraction of *H*- and *E*-polarized photons on a grating formed by an infinite periodic sequence of infinitely thin metal strips was solved in works [1-2]. A quantum approach to solving problems was applied. Photon Ψ-function presented in the form of a Fourier series, which is a superposition of functions corresponding to one of the possible states of a photon scattered by a grating. The problems are solved by the rigorous mathematical method of the Riemann-Hilbert boundary value problem [3].

But in optics, as a rule, we deal with unpolarized or partially polarized light. Therefore, in our opinion, consideration of such a problem in an exact setting has a certain practical interest.

Based on the results of the above works, this paper proposes a solution to the problem for natural, unpolarized light.

## 2. FORMULATION AND SOLUTION OF THE PROBLEM

A uniform flow of photons falls at a normal angle on a grating, formed by an endless sequence (in the direction of the *Y* axis), boundless in the direction of the *X* axis, of infinitely thin metal strips. The width of the strips is equal to *d*, the period of the grating is *l*, the width of the strip is $l - d$. The problem consists in determining the square of the module of the psi-function – the probability of finding a photon at one or another point in space, for example, a screen for observing a diffraction pattern.

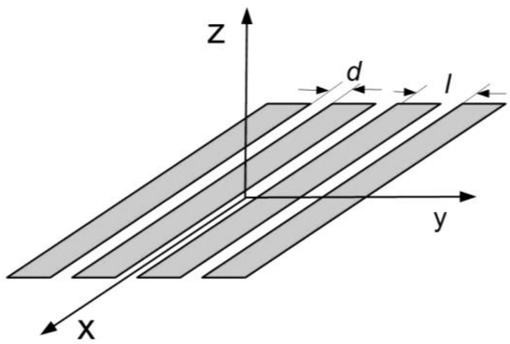


**Fig. 1** – Diffraction grating

Experiments on the polarization of natural light show that unpolarized light behaves as such, which can be considered as a superposition of *H*- and *E*-polarized light – a flow of photons, which is a mixture in equal proportions of *H*- and *E*-polarized photons [4]. The only difference is that in the case of natural light, the intensity of these waves is the same, and in the case of partially polarized light, it is different. According to this representation, oscillations of amplitude A (probability amplitude) are carried out in a plane that forms an angle with the plane of the polarizer can be divided into two oscillations with amplitudes $A_1 = A\cos\alpha$, and $A_2 = A\sin\alpha$ (the direction of propagation of the photon flow is perpendicular to the plane of the polarizer). Photon corresponding to the amplitude $\Psi_1$ will pass through the device, a photon, which corresponds to the amplitude $\Psi_2$ will be delayed. For the amplitudes of the probabilities we have, respectively

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$$\left|\Psi_1\right| = A\cos\alpha\,, \quad \left|\Psi_2\right| = A\sin\alpha$$

In quantum mechanics, to determine the probability of one of two mutually exclusive events (the first is that an *H*-polarized photon hits a given point of the screen, the second – an *E*-polarized photon), the law of adding probability amplitudes is introduced [5]:

$$\Psi = \Psi_1 + \Psi_2\,, \tag{1}$$

where $\Psi_1 = \left|\Psi_1\right| e^{i\varphi_1}$, $\Psi_2 = \left|\Psi_2\right| e^{i\varphi_2}$ probability amplitudes, respectively, for *E*- and *H*-polarized photons. Probability amplitudes are often called state vectors, because the square of the modulus of the resulting amplitude is calculated by the rule of adding two vectors

$$\left|\Psi\right|^2 = \left|\Psi_1\right|^2 + \left|\Psi_2\right|^2 + 2\left|\Psi_1\right|\left|\Psi_2\right|\cos\varphi\,, \tag{2}$$

where $\varphi = \varphi_1 - \varphi_2$ – is the phase difference of the amplitudes $\Psi_1$, $\Psi_2$. But since the state vectors, correspond to two mutually perpendicular polarizations, the phase difference $\varphi = \pi/2$, we get

$$\left|\Psi\right|^2 = \left|\Psi_1\right|^2 + \left|\Psi_2\right|^2. \tag{3}$$

But here it should also be taken into account that when solving each of the diffraction problems of *E*- and *H*-polarized photons, we assume that all 100 % of the photons of the flow have *E*- or *H*-polarization, respectively, and for natural light, or partially polarized light the composition of the flow will depend from the degree of polarization. In each of the partial cases of solving problems of diffraction of *E*- or *H*-polarized photons, the following relations are fulfilled:

$$\begin{aligned} \left|\Psi^E\right|^2 &= \left|\Psi_R^E\right|^2 + \left|\Psi_T^E\right|^2 \\ \left|\Psi^H\right|^2 &= \left|\Psi_R^H\right|^2 + \left|\Psi_T^H\right|^2 \end{aligned} \tag{4}$$

where $\left|\Psi_R^E\right|^2$ is the probability that an *E*-polarized photon will be reflected from the lattice; $\left|\Psi_T^E\right|^2$ is the probability that an *E*-polarized photon will pass through the grating, $\left|\Psi_R^H\right|^2$, $\left|\Psi_T^H\right|^2$ are the corresponding probabilities for an *H*-polarized photon; $\left|\Psi^E\right|^2$, $\left|\Psi^H\right|^2$ are the corresponding probabilities of detecting *E*- (*H*-) polarized photons at any point in space.

According to (3), in the case of natural or partially polarized light

$$\left|\Psi^E\right|^2 + \left|\Psi^H\right|^2 = \left|\Psi_R^E\right|^2 + \left|\Psi_T^E\right|^2 + \left|\Psi_R^H\right|^2 + \left|\Psi_T^H\right|^2 \tag{5}$$

For natural light $\left|\Psi^E\right|^2 = \left|\Psi^H\right|^2 = \left|\Psi\right|^2$. So, we have

$$2\left|\Psi\right|^2 = \left|\Psi_R^E\right|^2 + \left|\Psi_T^E\right|^2 + \left|\Psi_R^H\right|^2 + \left|\Psi_T^H\right|^2$$

or $\left|\Psi\right|^2 = \dfrac{1}{2}\left[\left|\Psi_R^E\right|^2 + \left|\Psi_T^E\right|^2 + \left|\Psi_R^H\right|^2 + \left|\Psi_T^H\right|^2\right].$

For the photon any polarization that passed through the grating as a result, we get

$$\left|\Psi\right|^2 = \frac{1}{2}\left[\left|\Psi_T^E\right|^2 + \left|\Psi_T^H\right|^2\right]. \tag{6}$$

Calculation results for density of the probability for *E*-, *H*-polarized photons and for the case for unpolarized light are represented on the figures (Fig. 2) – (Fig.4). Diffraction pattern repeats with the period $l$, so Figures 2-4 present the results of calculations density of probability $|\Psi|^2$ depending on the $y$ coordinate for one period within the limits of the change of $y/l$ from zero to one. As noted in works [1], [2], the value $l/\lambda = 1$ is a threshold value.

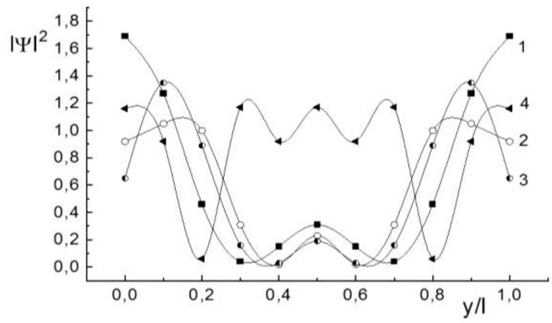


**Fig. 2** – Diffraction patterns of distribution $|\Psi|^2$ for *H*- at the different values of $l/\lambda$: 1 – ($l/\lambda = 1.1$); 2 – ($l/\lambda = 2.1$); 3 – ($l/\lambda = 3.1$); 4 – ($l/\lambda = 4.1$), at $d/l = 0{,}5$

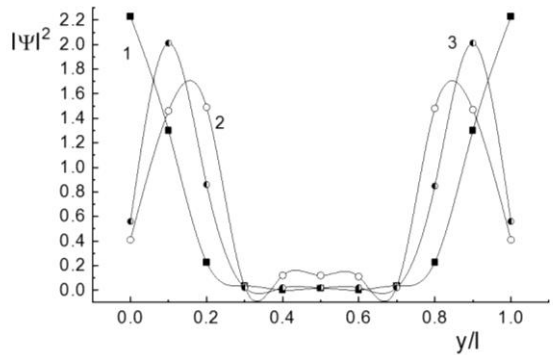


**Fig. 3** – Diffraction patterns of distribution $|\Psi|^2$ for *E*- at the different values of $l/\lambda$: 1 – ($l/\lambda = 2.1$); 2 – ($l/\lambda = 3.1$); 3 - ($l/\lambda = 4.1$), at $d/l = 0{,}5$

At $l/\lambda < 1$, the phenomenon of diffraction is not observed. Figure 2 shows the dependence $|\Psi|^2$ for the case of *H*-polarization from the coordinate $y$, more precisely from $y/l$. From the comparison of the graphs, it can be seen that with an increase in the ratio of the grating period to the wave length one additional maximum appears per unit of the $l/\lambda$.

Therefore, when $l/\lambda > n$, $n$ maxima are observed in the diffraction pattern. Moreover, initially, when $l/\lambda > 1$, $l/\lambda > 2$, $l/\lambda > 3$ maxima with the largest value $|\Psi|^2$ are located opposite the slits but already at $l/\lambda > 4$ the maxima align and shift from the center of the slit towards the strip. Following the alternating slits and strips can be seen in Fig. 1. For *E*-polarized photons, the diffraction pattern looks somewhat different. When $l/\lambda > 2$ we have one maximum, at $l/\lambda > 3$, $l/\lambda > 4$ – two maxima in the interval $y/l$ from 0 to 1.

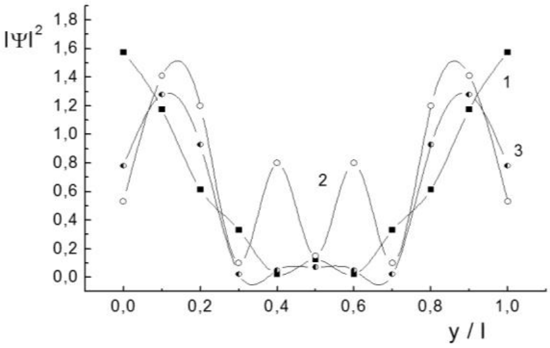


**Fig. 4** – Diffraction patterns of distribution $|\Psi|^2$ for natural light at the different values of $l/\lambda$: 1 – ($l/\lambda = 2.1$); 2 – ($l/\lambda = 3.1$); 3 – ($l/\lambda = 4.1$), at $d/l = 0{,}5$

For unpolarized, natural light, the diffraction pattern is shown in Fig. 4 and, as can be seen from the comparison, it is qualitatively more similar to the corresponding pattern for *H*-polarized photons. But it differs quantitatively in the height of the maxima.

## 3. CONCLUSIONS

First of all, it should be noted that this work, as far as we know, is the first attempt to consider the problem of diffraction of natural light on a periodic structure in an exact formulation.

A quantum approach to solving the problem is applied. Numerical calculations of the square of the modulus of the psi function - the probability of an *E*- or *H*-polarized photon hitting one or another point of the screen for a wide range of changes in the parameters characterizing the photon and the grating structure - were carried out.

The relationship between the number of maxima in the diffraction pattern of natural light and the ratio of the grating period to the photon wavelength has been established. The diffraction pattern is periodically repeated with a period equal to the period of the grating.

## REFERENCES

1. A.V. Bezugly, O.M. Petchenko, G.O. Petchenko, *J. Nano- Electron. Phys.* **13** No 1, 01002 (2021).
2. A.V. Bezougly, O.M. Petchenko, G.O. Petchenko, G.Y. Dulfan, *J. Nano- Electron. Phys.* **14**, No 3, 01002 (2022).
3. V.P. Shestopalov, *The method of the Riemann-Hilbert problem in the theory of diffraction and propagation of electromagnetic waves* (Kharkiv: Publishing house of Kharkiv University: 1971).
4. G.S. Landsberg, *Optics* (Moscow: Physmatlit: 2010) [In Russian].
5. I.O. Vakarchuk, *Quantum Mechanics* (Lviv: LNU: 2012) [In Ukrainian].

### Дифракція природного світла на безмежній ґратці металевих стрічок

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В даній роботі розв'язана задача дифракції природного, неполяризованого світла на ґратці, що утворена нескінченною періодичною послідовністю нескінченно тонких металевих стрічок. Застосовано квантовий підхід до її розв'язання. Розв'язок задачі дифракції природного, неполяризованого світла базується на результатах отриманих окремо для випадків дифракції *H*- та *E*-поляризованих фотонів, розв'язаних строгим методом крайової задачі Рімана-Гільберта. Робота присвячена розрахунку густини ймовірності знаходження *H*- чи *E*- поляризованого фотона в даній точці простору для того випадку коли потік фотонів, що падає на ґратку являє собою суміш рівної густини фотонів *H*- та *E*-поляризації. Оскільки дифракційна картина повторюється з періодом ґратки *l*, в роботі представлені результати розрахунків густини ймовірності $|\Psi|^2$ в залежності від координати *y* для одного періоду в межах зміни $y/l$ від нуля до одиниці. Із порівняння графіків $|\Psi|^2$ для *H*- та *E*-поляризованих фотонів і їх суперпозиції встановлена залежність між числом максимумів в дифракційній картині природного світла та відношенням періоду ґратки до довжини хвилі фотона. Як виявилось число максимумів зростає пропорційно вказаному відношенню. Основні максимуми при цьому розташовуються проти щілин. Для неполяризованого, природного світла дифракційна картина, якісно більше схожа на відповідну картину для *H*-поляризованих фотонів, але відрізняється висотою максимумів. При зростанні відношення періоду ґратки до довжини хвилі фотона спостерігається тенденція до вирівнювання їх висоти.

**Ключові слова:** Дифракція, Ґратка, Квант, псі-функція, Амплітуда вірогідності, Дифракційна картина, Фотон.