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SIMULATION MODEL OF THE MONO-PHASED INVERTER WITH PULSES WIDTH MODULATION

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INTRODUCTION

The performances of the adjustable electrical drives with asynchronous motors are dependent on the capability of frequency static converters to ensure a power supply close to a sinusoidal form.

The converters yield non-sinusoidal currents or voltages, which determine a deformed regime in the motor and in the supply network. The high harmonics have negative effects on the functioning of the ensemble converter – motor through: the increase of currents in the chain winding of the motor, the increase of the power loss, the apparition of oscillating couples and the worsen of commutation phenomena in the power semiconductor devices. The unfavourable effect of the oscillating couples shows up especially at low frequencies and consists in a jerky movement of the rotor, even a resonance phenomenon in the mechanical transmission of the drive being possible [1].

In order to eliminate these effects, the commutation program of the thyristors should be adjustable depending on the frequency of the fundamental of the voltage or current in the motor. This could be achieved by modulating the pulses in duration or width (PWM) after a sinusoidal function. The modulation of the pulse width is applied at frequencies $f < 50$ Hz and has a double role: the variation of amplitude of the fundamental correlated with its frequency and the nullifying of low frequency harmonics. In the literature [1, 4], several modulation methods are known: the comparison of a sinusoidal modulator signal with a high frequency triangular modulated signal, the sampling of the angular position of the spatial phasor of the phase voltages, the equality between the area sampled from the proposed sinusoidal voltage and the area of the pulse voltage.

In this paper, one analyses a method for width modulation, which could be software implemented in the command system of the inverter. On this simulation model, the commutation moments of the thyristors, the width of the pulses, the waveforms and the spectral analysis of the inverter's voltage are determined.

MATHEMATICAL MODEL OF THE PULSES WIDTH MODULATION

At a frequency of 50 Hz, the inverter functions in non-modulated regime and the output voltage $u(t)$ has a rectangular shape. The duration of a pulse is 10 ms, and the amplitude of the pulse is equal to the direct voltage of the intermediate circuit. The magnitude of that voltage can be computed from the equality between the effective values of the nominal voltage on the charge and of the fundamental:

$$E = \frac{\sqrt{2}\pi}{4} U_{nom} = \frac{\sqrt{2}\,\pi}{4} U_{ef1}\ , \qquad (1)$$

where: E is the voltage of the intermediate voltage; $U_{ef\,1}$ – the effective value of the fundamental of the inverter voltage; U_{nom} – the effective value of the nominal voltage of the charge.

At frequencies $f < 50$ Hz, the inverter functions in modulated regime, and the voltage has the shape of pulses with amplitude E and duration modulated after a sinusoidal function. In this case, the sinusoidal voltage proposed at the inverter terminals is:

$$u_s = A \sin n\varpi t \ , \qquad (2)$$

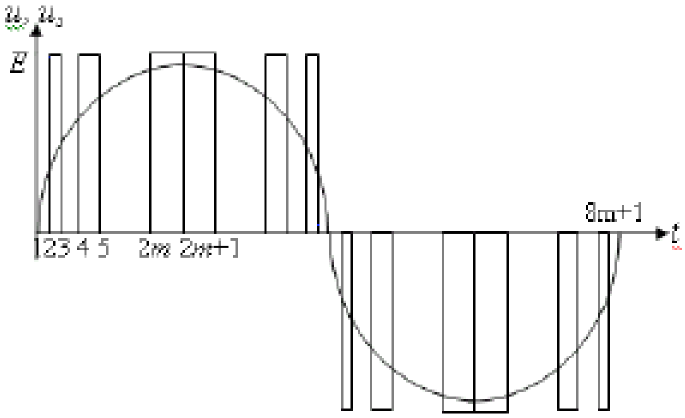

Figure 1. The waveforms of the synthetic voltage and proposed voltage

where A is the amplitude.

The proposed voltage can be approximated with a synthetic voltage (synthesized from pulses) defined in a period T like this:

during the first quart of period:

$$u(t) = U_{2k-1} = 0 \ , \quad \text{for} \ \ t_{2k-1} < t < t_{2k} \ , \qquad (3)$$

$$u(t) = U_{2k} = E \ , \quad \text{for} \ \ t_{2k} < t < t_{2k+1} \ , \qquad (4)$$

where: $k = 1, 2, \ldots m$ represents the number of pulses in the interval $0 - T/4$; $t_1 = 0$ and $t_{2m+1} = T/4$, being the limits of the interval;

in the second quart of period the pulses are symmetrical with respect to the moment $T/4$:

$$u(t) = U_j = U_i \ , \quad \text{for} \quad t_j \le t \le t_{j+1} \ , \qquad (5)$$

$$t_{j+1} = T/2 - t_i \ , \qquad (6)$$

where: $j = 2m + k$, $i = 2m - k + 1$, $k = 1, 2, \ldots 2m$; $t_{2m+1} = T/4$ and $t_{4m+1} = T/2$, being the limits of the interval:

– in the second semi period the pulses are negative and symmetrical with respect to the moment $T/2$:

$$u(t) = U_j = -U_k \ , \quad \text{for} \ \ t_j \le t \le t_{j+1} \ , \qquad (7)$$

$$t_{j+1} = T/2 + t_{k+1} \ , \qquad (8)$$

where: $j = 4m + k$, $k = 1, 2, \ \ldots 4m$; $t_{4m+1} = T/2$ and $t_{8m+1} = T$, being the limits of the interval.

In figure 1 the voltages $u_s(t)$ and $u(t)$ are graphically presented on the interval $0 - T$; the notations on the time axis being the indexes of the commutation moments.

The Fourier series expansion of the synthetic voltage $u(t)$ contains only odd harmonics in sinus [1]:

$$u(t)=\sum_{n=1,3,5}^{\infty} b_n \sin n\varpi t=\sum_{n=1,3,5}^{\infty} \sqrt{2} U_{efn} \sin n\varpi t, \tag{9}$$

where the series coefficients are:

$$b_n=\frac{2}{T}\int_0^T u(t)\sin n\varpi t\,\mathrm{d}t=\frac{4E}{n\pi}\sum_{k=1}^{m} U_k\left(\cos n\varpi t_{2k}-\cos n\varpi t_{2k+1}\right). \tag{10}$$

From the conditions that the fundamental equals the proposed voltage and that the first 2(m – 1) high harmonics nullify:

$$\begin{cases} b_1=A \\ b_3=b_5=\ldots=b_{4m-3}.=0 \end{cases} \tag{11}$$

From the relations (10) and (11) the following system of equations results:

$$\begin{cases} \sum_{k=1}^{m}\left(\cos \varpi t_{2k}-\cos \varpi t_{2k+1}\right)=\frac{\pi A}{4E} \\ \sum_{k=1}^{m}\left(\cos 3\varpi t_{2k}-\cos 3\varpi t_{2k+1}\right)=0. \\ . \\ \sum_{k=1}^{m}\left(\cos (4m-3)\varpi t_{2k}-\cos (4m-3)\varpi t_{2k+1}\right)=0 \end{cases} \tag{12}$$

with the commutations moments $t_2,\ t_3,\ \ldots t_{2m}$ as unknowns.

The non linear system of equations is solved numerically and the commutation moments from the interval 0 – T/4 are computed; the other commutation moments are found with relations (6) and (8).

For M an imposed number of harmonics (odd and even) nullified, the synthetic voltage contains $m=\left[\frac{M+2}{4}\right]+1$ pulses in the interval 0 – T/4, where [ ] represents the integral part of a number.

SPECTRAL ANALYSIS OF THE SYNTHETIC VOLTAGE

The effective values of the harmonics and the distortion coefficients are computed with [1]:

– the effective value of the n^{th} harmonic:

$$U_{efn}=\frac{b_n}{\sqrt{2}},\ n=1,3,5,\ldots; \tag{13}$$

the total effective value:

$$U_{ef.t}=\left[\frac{1}{T}\int_0^T u^2(t)\,\mathrm{d}t\right]^{\frac{1}{2}}=2E\left[\frac{1}{T}\sum_{k=1}^{m}\left(t_{2k+1}-t_{2k}\right)\right]^{\frac{1}{2}}; \tag{14}$$

the total effective value of the harmonics:

$$U_{ef.t.a} = \left[\frac{1}{T} \int_0^T \left(\sum_{n=3,5,..}^{\infty} b_n \sin n\varpi t \right)^2 \mathrm{d}t \right]^{\frac{1}{2}} = \left[\frac{1}{2} \sum_{n=3,5,..}^{\infty} b_n^2 \right]^{\frac{1}{2}} = \left[U_{ef.t}^2 - U_{ef.1}^2 \right]^{\frac{1}{2}} ; \qquad (15)$$

the distortion coefficients:

$$k_{d1} = \frac{U_{ef.t.a}}{U_{ef.1}} \quad , \quad k_{d2} = \frac{U_{ef.t.a}}{U_{ef.t}} \quad . \qquad (16)$$

RESULTS OF THE NUMERICAL SIMULATION

For the numerical simulation of the PWM, the Matlab toolbox is used [2]. Having as brick element the matrix, this package offers facilities for the time – voltage vectors construction, for the spectral analysis of the synthetic voltage, for the graphical representation of the synthetic voltage and of the frequency spectrum.

The input data are: *A*, *E*, *f*, *m*, *itm*, *era*, $t^{(0)} = \left[t_2^{(0)}, t_3^{(0)}, .. t_{2m}^{(0)} \right]$, where: *itm* is the maximum number of iterations; *era* – the maximum allowable error,; $t^{(0)}$ – the initial approximation of the time vector. The elements of the vector $t^{(0)}$ are generated with a step of 1/(8 *f m*). Taking into the account the practical possibilities to realize the commutation moments in static frequency converters, the precision was limited to 0,2 ms (*era* = 2 e –4).

For the numerical resolution of the non-linear system (12), the iterative method Newton – Raphson was used. The approximation at iteration $j + 1$ has the following vectorial form:

$$\mathrm{t}^{(j+1)} = t^{(j)} + dt^{(j)} , \qquad (17)$$

where: $t^{(j)}$ is the approximation at the j^{th} iteration; $dt^{(j)}$ – the solution of the linear system of equations deducted with the method Newton – Raphson; $j =$ 0, 1, 2, 3, …being the iteration counter.

The system of equations (12) could be written:

$$\begin{cases} f_1 = \sum_{k=1}^{m} \left(\cos \varpi t_{2k} - \cos \varpi t_{2k+1} \right) - \frac{\pi A}{4E} \\ f_i = \sum_{k=1}^{m} \left(\cos (2i-1)\varpi t_{2k} - \cos (2i-1)\varpi t_{2k+1} \right) = 0 \end{cases} \qquad (18)$$

The jacobian matrix of the system is:

$$g(t) = \begin{bmatrix} \frac{\partial f_1}{\partial t_2} & \frac{\partial f_1}{\partial t_3} \; . \; . \; . \; . & \frac{\partial f_1}{\partial t_{2m}} \\ \frac{\partial f_2}{\partial t_2} & \frac{\partial f_2}{\partial t_3} \; . \; . \; . \; . & \frac{\partial f_2}{\partial t_{2m}} \\ \frac{\partial f_{2m-1}}{\partial t_2} & \frac{\partial f_{2m-1}}{\partial t_3} \; . \; . \; . \; . & \frac{\partial f_{2m-1}}{\partial t_{2m}} \end{bmatrix} = \left[g_{ik} \right] \quad , \qquad (19)$$

where the elements of the matrix are:

$$g_{ik} = \frac{\partial f_i}{\partial t_{k+1}} = (-1)^k (2i-1)\,\omega \sin(2i-1)\,\omega\, t_{k+1}, \qquad (20)$$

The linear system deducted with the Newton – Raphson at j^{th} iteration, in vectorial form is:

$$g\left(t^{(j)}\right) dt^{(j)} = -f\left(t^{(j)}\right), \qquad (21)$$

and on components:

$$\sum_{k=1}^{2m-1} g_{ik}\left(t^{(j)}\right) dt_{k+1}^{(j)} = -f_i\left(t^{(j)}\right), \qquad (22)$$

with the unknowns $dt_{k+1}^{(j)}$, $k = 1, 2, \ldots 2m - 1$.

The iteration process is finished when the estimated precision of the solution is attained, evaluated at the n^{th} iteration through the Euclidian norm of the vector $dt^{(n)}$:

$$\left\| dt^{(n)} \right\|_2 = \left[\sum_{k=1}^{2m-1} \left| dt_{k+1}^{(n)} \right|^2 \right]^{\frac{1}{2}} \leq era\ , \qquad (23)$$

or when the maximum number of iterations is attained, $n \leq itm$,where *era* and *itm* are given.

The final solution of the non-linear system (12) is:

$$t^{(n+1)} = t^{(n)} + dt^{(n)}. \qquad (24)$$

Table 1 - The results of the numerical simulation

Input data	*A* [V]	44						
	f [Hz]	10						
	E	55 √2 π						
	Itm	10						
	Era	2.e – 4						
Output data	*m*	1	2	3	4	5	6	7
Pulses commutation moments in 0 – T/4 [ms]	*T*1	0	0	0	0	0	0	0
	*T*2	22,7	11,7	7,9	6,0	4,9	4,1	3,5
	*T*3	25,0	13,3	8,7	6,5	5,1	4,3	3,7
	*T*4		23,9	16,0	12,1	9,7	8,1	7,1
	*t*5		25,0	17,3	12,9	10,2	8,5	7,3
	t6			24,2	18,2	14,6	12,2	10,6
	*t*7			25,0	19,3	15,3	12.8	11,1
	*t*8				24,4	19,6	16,3	14,3
	*t*9				25,0	20,4	17,0	14,9
	t_{10}					24,5	20,5	18,4
	*t*11					25,0	21,2	19,2
	*t*12						24,6	22,5
	*t*13						25,0	22,8
	*t*14							24,7
	*t*15							25,0
	*b*1	44,00	44,00	44,00	44,00	44,00	44,00	44,00
	*U*ef 1	31,11	31,11	31,11	31,11	31,11	31,11	31,11
	*b*3	42,83	0	0	0	0	0	0
	*U*ef 3	30,28	0	0	0	0	0	0

The amplitude/ effective value of the first 25 harmonics [V]	*b*5	40,54	0	0	0	0	0	0
	*U*ef 5	28,66	0	0	0	0	0	0
	*b*7	37,24	42,83	0	0	0	0	0
	*U*ef 7	26,33	30,29	0	0	0	0	0
	*b*9	33,09	41,68	0	0	0	0	0
	*U*ef 9	23,40	29,47	0	0	0	0	0
	*b*11	28,28	1,15	42,83	0	0	0	0
	*U*ef 11	20,00	0,81	30,29	0	0	0	0
	*b*13	23,04	1,13	41,68	0	0	0	0
	*U*ef 13	16,29	0,80	29,47	0	0	0	0
	*b*15	17,60	38,36	1,14	42,82	0	9	0
	*U*ef 15	12,44	27,13	0,81	30,28	0	0	0
	*b*17	12,20	36,24	0,01	41,69	0	0	0
	*U*ef 17	8,62	25,62	0,01	29,48	0	0	0
	*b*19	7,06	3,14	0	1,14	42,83	0	0
	*U*ef 19	5,00	2,22	0	0,80	30,29	0	0
	*b*21	2,39	3,05	1,14	0,01	41,68	0	0
	*U*ef 21	1,69	2,16	0,81	0,01	29,47	0	0
	*b*23	1,65	31,28	38,36	0	1,14	42,83	0
	*U*ef 23	1,16	22,12	27,13	0	0,80	30,29	0
	*b*25	4,91	28,51	36,23	0,02	0,01	41,68	0
	*U*ef 25	3,47	20,16	25,62	0,01	0,01	29,47	0
Tot. Effectiv value [V]	*U*ef.t	73,44	80,58	81,78	82,20	82,39	82,50	82,55
Tot.eff.val. harm. [V]	*U*ef.t.a	66,53	74,33	75,64	76,09	76,29	76,41	76,47
The distortion coefficients	*k*d1	2,138	2,390	2,431	2,445	2,452	2,456	2,458
	*k*d2	0,906	0,922	0,925	0,925	0,926	0,926	0,926

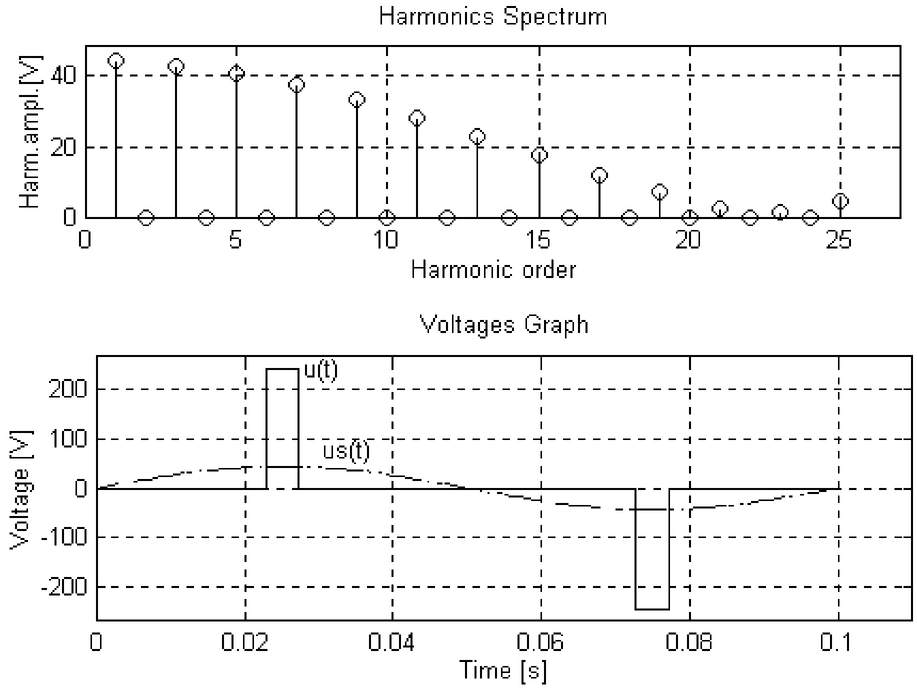

Figure 2. The graphical results of the simulation for m = 1

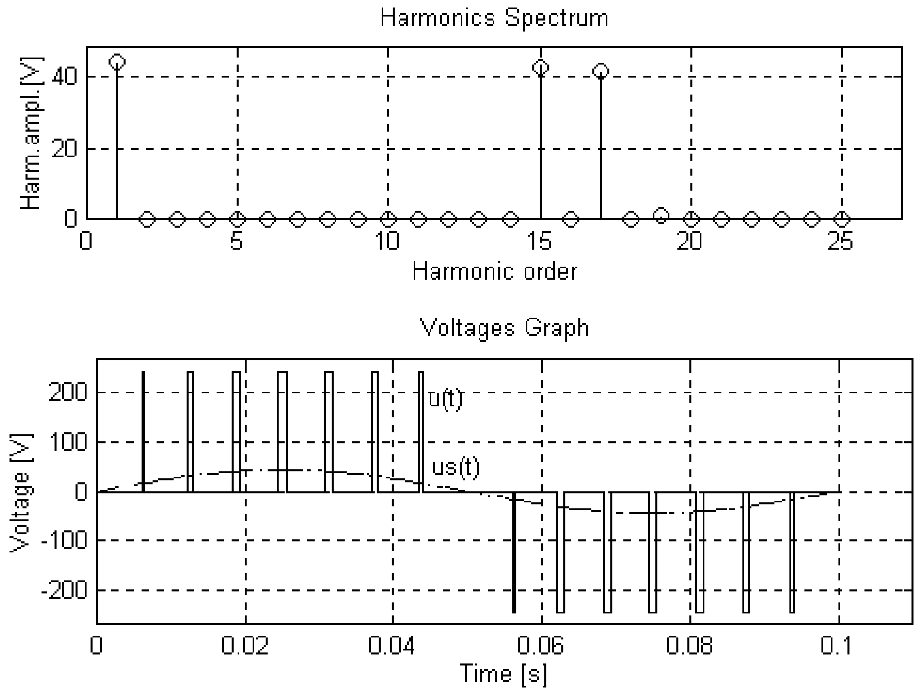

Figure 3. The graphical results of the simulation for m =4

The output data of the simulation program are: the commutation moments, the amplitude and the effective value of the 1st, 2nd, ..25th harmonics, the distortion coefficients, the graphical representation of the synthetic voltage and of the frequency spectrum.

For a sinusoidal voltage with the amplitude A = 44V and frequency f = 50Hz the numerical results of the simulation for m = 1 – 7 pulses in the interval 0 – T/4 are presented in table 1. In figures 2 and 3 the synthetic voltage and the spectrum of harmonics up to order 25 are presented for m= 1 and m = 4.

At frequency f = 10 Hz and voltage $U_{ef\ 1}$ = 31.11V, the inverter functions in modulated regime with m = 1 – 7 pulses in the interval 0 – T/4. The model is conceived such that the fundamental of the synthetic voltage is identical to the proposed sinusoidal voltage, and the odd and even harmonics up to order $4m$ – 2 are null. Through simulation, one can see that the harmonics of order $4km \pm 1$, k = 1, 2, 3, …, have a high amplitude (similar to the fundamental), the others being practically negligible, which means that a high number of harmonics are eliminated. By increasing the number of pulses, a better synthesis of the sinusoidal voltage is achieved, because the high amplitude harmonics are shifted towards high frequencies, which do not disturb the functioning of the asynchronous motor.

The voltage synthesis after the PWM principle nullifies the harmonics in a domain of the frequency spectrum, but increases the harmonics from another domain of the spectrum, such that the distortion factors do not change, even if the number of pulses increases. This is a disadvantage of the PWM voltage synthesis [1].

CONCLUSIONS

The presented mathematical model of the pulse width modulation is based on the assessment of a harmonics content on the synthetic voltage, conforming to relations (11); this method is different from those listed in the literature. The results obtained through numerical simulation on the model with the Matlab toolbox are better than those obtained with PWM methods.

The command program of the thyristors obtained through simulation of the presented PWM model could be implemented on a microprocessor.

ABSTRACT

The article presents a model of pulse width modulation based on the assessment of a certain harmonic content of the inverter synthetic voltage. The commutation moments of the pulses (with the same amplitude but with different widths) are computed in the conditions when the fundamental of the synthetic voltage is equal to the proposed sinusoidal voltage, and the high harmonics up to order 4m – 2 are null

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