STRUCTURE OF QUANTUM LEVELS FOR TWO-DIMENTIONAL ELECTRON IN THE HOMOGENEOUS MAGNETIC FIELD AND THE POTENTIAL CONFINING NEAR TO THE RING

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ABSTRACT

Studying of properties of quantum rings (nano-scaling and mesoscoping ring structures) in a magnetic field is one of directions on which there are interesting results (see, for example, [1]). Quantum transitions in such structures are accompanied by radiation on border of infra-red light, and interest is caused by periodic structures in which quantum rings are cooperate with neigh calls in pseudo-crystal. In the given work results of calculations of such structure in two-dimension statement (periodicity is provided with decomposition of the solution in Fourier series on both spatial coordinates) are presented.

METHODS OF COMPUTATION AND ANALYSIS

The solution of stationary equation Schrödinger in two-dimensional linear statement is considered. Following designations are accepted: m_e, B, U - mass of electron, intensity magnetic field, the confining potential, $c, e, \hbar$ - speed of light, an elementary charge, a constant of Planck, $\Delta, \psi(x, y), E$ - two-dimensional harmonic operator, wave function from the Cartesian coordinates of electron, energy of stationary quantum state.

$$\left(-\frac{\hbar^2}{2m_e}\Delta + i\frac{\hbar eB}{m_e c}\left(y\frac{\partial}{\partial x} - x\frac{\partial}{\partial y} \right) + \frac{e^2B^2}{2m_e c^2}\left(x^2 + y^2\right) + U(x, y) - E \right)\psi = 0 \quad (1)$$

After introduction of magnetic scale of length $\alpha = \sqrt{\frac{\hbar c}{eB}}$, scaling-less confining potential $V = \frac{2m_e c}{\hbar eB}U$ and energies of quantum states $\Lambda = \frac{2m_e c}{\hbar eB}E$ the equation (1):

$$\left(-\alpha^2\Delta + 2i\left(y\frac{\partial}{\partial x} - x\frac{\partial}{\partial y} \right) + \frac{x^2 + y^2}{\alpha^2} + V - \Lambda \right)\psi = 0 \quad (2)$$

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The solution of a problem (2) was searched in the form of a trigonometrically polynomial (final of series on Fourier - harmonics). Let R, L is self-length of areas of periodicity and number of harmonics: $\psi = \frac{1}{2R}\sum_{n,k=-L}^{L} a_{n,k} \exp\frac{\pi i}{R}(nx+ky)$ After substitutions of it in the equation (2), multiplication on $\exp\frac{\pi i}{R}(-qx-sy)$ and integration on area of periodicity $\Omega = \{-R \le x, y \le R\}$ the algebraic problem on eigen values and vectors turns out.

$$2\pi^2\alpha^2(q^2+s^2)\frac{a_{q,s}}{R} + 4iR\sum_{n,k=-L}^{L} a_{n,k}\left(\delta_{n,q}\left.\frac{(-1)^{k-s}n}{k-s}\right|_{k\neq s} - \delta_{k,s}\left.\frac{(-1)^{n-q}k}{n-q}\right|_{n\neq q}\right) +$$

$$+2R\sum_{n,k=-L}^{L}\left(B_{n-q,k-s} + B^0_{n-q,k-s}\right)a_{n,q} - 2R\Lambda a_{q,s} = 0\,;\; q,s = -L,...,L$$

Here $\delta_{n,k}$ - symbol of unit matrix, $B_{n,k} = \frac{1}{4R^2}\int_\Omega V(x,y)\exp\frac{\pi i}{R}(-nx-ky)dxdy$ - Fourier-harmonics of decomposition of the confining potential, and a harmonic of decomposition of symmetric component of a magnetic field whereas

$$\frac{1}{4R^2}\int_\Omega \frac{x^2+y^2}{\alpha^2}\exp\frac{\pi i}{R}(-nx-ky)dxdy = \left[\begin{array}{l} \frac{2}{3}\left(\frac{R}{\alpha}\right)^2 : n=k=0 \\ \frac{2(-1)^n}{\pi^2 n^2}\left(\frac{R}{\alpha}\right)^2 : n\neq 0, k=0 \\ 0 : nk\neq 0 \end{array}\right.$$ and

designations $\rho = \left(\frac{R}{\alpha}\right)^2$, are represented so:

$$B^0_{0,0} = \frac{2}{3}\rho;\; B^0_{n,0} = B^0_{0,n} = \frac{2(-1)^n}{\pi^2 n^2}\rho : n\neq 0;\; B^0_{n,k} = 0 : nk\neq 0$$

Having designated following

$$A_{n,k,q,s} = \frac{\pi^2}{\rho}(q^2 + s^2)a_{q,s}\delta_{nq}\delta_{ks} +$$

$$+ 2i\sum_{n,k=-L}^{L} a_{n,k}\left(\delta_{n,q}\left.\frac{(-1)^{k-s}n}{k-s}\right|_{k\neq s} - \delta_{k,s}\left.\frac{(-1)^{n-q}k}{n-q}\right|_{n\neq q}\right) + \tag{3}$$

$$+ B_{n-q,k-s} + B^0_{n-q,k-s} \;; k, n, q, s = -L,...,L$$

we receive, that the solution of an algebraic problem (2) is reduced to a finding of eigen numbers and vectors of Hermetic matrix $C = (C_{j,p})$ with physically scaling-less elements,

$$\left(C_{jp} = A_{n,k,q,s} : j = k + L + 1 + (n + L)(2L + 1), p = s + L + 1 + (q + L)(2L + 1)\right)$$

, $n, k, q, s = -L,...,L$.

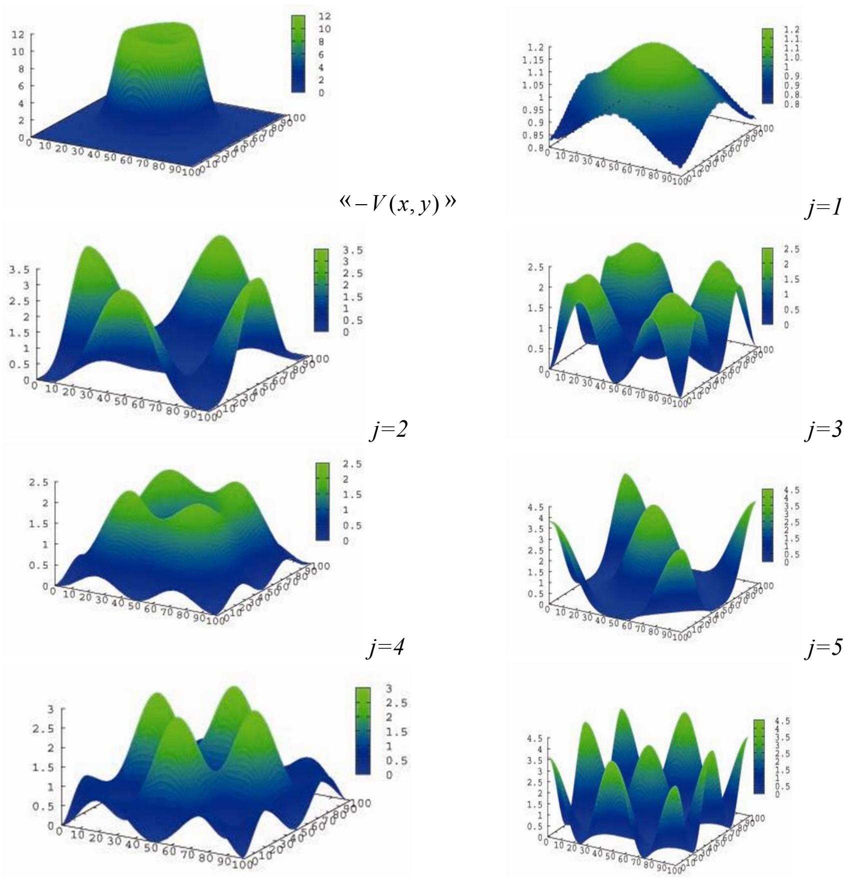
For a finding of eigen values and vectors for this matrix it is used QL - algorithm with shift [2]. Eigen-vectors consist of Fourier - decomposition terms of eigen-functions $D^j = \left(v^j_p : p = 1,...,(2L+1)^2\right)$.

On presented below graphs (fig.1) are presented the confining potential with a sign "minus" and squares of modules of first seven eigen-functions (are signed by their numbers ***j***). Horizontal coordinates «x, y» are specified in % from the spatial period «2R», vertical the coordinate is scaled according to a norm-condition.

It is calculated own 37 values, since the least, they are located by way of increases, according to numbering of own functions):

0.6497803; 9.086677; 9.597447; 9.700025; 10.52466; 18.94184; 19.91620; 21.60274; 22.66856; 37.10139; 37.11093; 37.34475; 37.61111; 47.99970; 48.18777; 48.34706; 49.18681; 52.59499; 52.60853; 52.97558; 53.71998; 78.64438; 78.96074; 81.34107; 81.78598; 83.75708; 83.81923; 83.87377; 84.10400; 96.76934; 96.79221; 96.90021; 97.14402; 103.6655; 103.8549; 103.9458; 103.9949 (it is A_j for ***j***=1, …, 37).

Infringement of symmetry of the confining potential (its deviation from a figure of rotation) less than 1% in relation to the maximal absolute value, and it leads to infringement of frequency rate eigen-numbers with a relative error of the same order (apparently from resulted above values). Value of parameter $\rho = 1$, that is $R = \alpha$, number of harmonics L=7.



«$-V(x,y)$» *j=1*

j=2 *j=3*

j=4 *j=5*

j=6 *j=7*

Fig. 1 – Confining potential (with sign “minus”) and square of modules of eigen-functions with number “***j***”

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