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# QUANTISETS AND THEIR QUANTIRELATIONS

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The sets with either unit or zero quantities of their possible elements, the fuzzy sets with intermediate quantities in the indeterminate case only, and the multisets whose element quantities are any cardinal numbers cannot express many typical collections, e.g., that of half an apple and a quarter of a pear. For concrete (mixed) quantities, e.g., "5 *l* fuel", there is no suitable mathematical model and no known operation, say between "5 *l*" and "fuel" (neither "5 *l*" × "fuel" nor "fuel" × "5 *l*"). Set operations with absorption are only restrictedly reversible and hinder constructing universal quantity degrees. Elastic mathematics by the author and its fundamental quantianalysis introduce corresponding adequate concepts.

**Notation 1.** A set quantioperation (quantirelation) is a set operation (relation) such that the actual quantity of each element of its operands (objects) is exactly taken into account, and can be denoted by the sign of a similar usual set operation (relation, respectively) with a little circle on the right above by noncoinciding results of the operation and the quantioperation.

**Definition 2.** A *quantiset* is a non-positional quantiunion of *quantielements* of the form ${}_q a$, each of them consisting of its *element* (*basis*), say $a$, with its *own quantity* (named: *uniquantity*), say $q$, inside in the quantiset, the elements and element quantities being any objects (possibly fuzzy etc.):

$A =^\circ \{\ldots, {}_q a, \ldots, {}_r b, \ldots, {}_s c, \ldots\} ^\circ=^\circ \ldots \cup^\circ {}_q a \cup^\circ \ldots \cup^\circ {}_r b \cup^\circ \ldots \cup^\circ {}_s c \cup^\circ \ldots$ .

*Quantifying* is a set quantioperation of the form ${}_q$: $a \to {}_q a$.

The *empty quantielement* ${}_0 a =^\circ {}_q\#$ (# the *empty element*, $\# \in \varnothing$) is the empty set $\varnothing$ and has to be *reduced* to *canonical form* ${}_0\#$.

In a quantiset, all the quantielements with the same basis have to be *reduced* (*collected*) by adding their own quantities:

$$\ldots \cup^\circ {}_q a \cup^\circ \ldots \cup^\circ {}_r a \cup^\circ \ldots \cup^\circ {}_s a \cup^\circ \ldots =^\circ {}_{\ldots + q + \ldots + r + \ldots + s + \ldots} a.$$

*Quantisets* are *quantiequal* if, after the reduction, they contain all quantielements in common.

*Outside quantifying* a quantiset means multiplying the inside quantities by the outside one:

$${}_t A =^\circ {}_t\{\ldots , {}_q a, \ldots , {}_r b, \ldots, {}_s c, \ldots\}^\circ =^\circ \{\ldots , {}_{tq} a, \ldots , {}_{tr} b, \ldots , {}_{ts} c, \ldots\}^\circ.$$

**Example 3.** $\{{}_{2\ \text{loaves}}$bread, ${}_{1.5\ kg}$meat, ${}_{1/2}$water-melon, ${}_{\$\ -25}$money, ${}_{-2\ h}$time, ${}_{-3\ l}$petrol$\}$ is a possible result of shopping.

**Definition 4.** An *ordinary set* is a reduced quantiset with unit element quantities only.

**Definition 5.** An *algebraic quantiunion of quantisets* is a quantiset quantiunifying all quantielements of the quantisets, each quantity in the subtrahends changing sign:

$$\ldots\cup^\circ \{\ldots , {}_q a, \ldots\}^\circ \setminus^\circ \ldots \setminus^\circ \{\ldots , {}_r b, \ldots\}^\circ \cup^\circ \ldots \cup^\circ \{\ldots , {}_s c, \ldots\}^\circ \setminus^\circ$$
$$\ldots \setminus^\circ \{\ldots , {}_t d, \ldots\}^\circ \cup^\circ \ldots =^\circ \{\ldots , {}_q a , \ldots , {}_{-r} b, \ldots , {}_s c, \ldots , {}_{-t} d, \ldots\}^\circ.$$

**Definition 6.** An *algebraic addition* or *unification* is called *algebraically commutative* and/or *associative* if it becomes commutative and/or associative provided that each negative operation sign is avoided by changing the own signs of the corresponding operands, say:

$$- 3 - 5 + 2 - (- 4) + (-1) = (- 3) + (- 5) + 2 + 4 + (-1),$$
$${}_q a \setminus^\circ {}_r b \cup^\circ {}_s c \setminus^\circ {}_t d =^\circ {}_q a \cup^\circ {}_{-r} b \cup^\circ {}_s c \cup^\circ {}_{-t} d.$$

**Corollary 7.** *If the quantities of each basis in quantisets form a commutative additive group, then the algebraic quantiunification of the quantisets is an algebraically commutative and associative set quantioperation, and the quantisets form a commutative additive group with zero* ${}_0\#$.

**Notation 8.** For a one-variable basis function (a completely algebraically additive one-variable quantity function), quantifying the preimage means quantifying (multiplying) the image, say $f({}_q x) = {}_q f(x)$ ($Q({}_x a) = xQ(a)$, respectively).

The introduced concepts apply to information problems etc.