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Effective Landau-Lifshitz-Gilbert Equation for a Conducting Nanoparticle

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We study the role of conductivity in the magnetization dynamics of single-domain ferromagnetic particles. Our approach is based on the coupled system of Maxwell's and Landau-Lifshitz-Gilbert (LLG) equations that describes both the induced electromagnetic field and the magnetization dynamics. We show that the effective LLG equation for a conducting particle contains two additional terms compared to the ordinary LLG equation. One of these terms accounts for the magnetic field of eddy currents induced by an external magnetic field, and the other is magnetization dependent and is responsible for the conductivity contribution to the damping parameter. By analytically solving Maxwell's equations, we determine this contribution and demonstrate the importance of conduction effects for large nanoparticles.

Keywords: Ferromagnetic nanoparticles, Eddy currents, Magnetization dynamics, Conduction effects.

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1. INTRODUCTION

The Landau-Lifshitz (LL) equation [1] and its modification, the Landau-Lifshitz-Gilbert (LLG) equation [2,3], are the basic equations for studying the magnetization dynamics in ferromagnetic materials. Though these equations are equivalent from the mathematical point of view [4] (specifically, the LL equation reduces to the LLG one by a simple rescaling of the gyromagnetic ratio and damping parameter), the latter is more preferable with the physical point of view. In general, the magnetic state in finite samples, e.g., in ferromagnetic particles, is multi-domain, and so the magnetization direction is space-dependent. However, if the particle size is small enough (usually of the order of a few tens of nanometers) then the formation of domain walls becomes energetically unfavorable (see, e.g., Ref. [5]). As a consequence, in this case the single-domain state with a uniform distribution of the magnetization is realized, and the LLG equation simplifies to that describing the coherent rotation of the magnetization. This equation is widely used for studying the nonlinear effects in the magnetization dynamics, regimes of forced precession, magnetization switching, etc. [6]. Moreover, if the effective magnetic field acting on the magnetization contains the noise term then the LLG equation becomes stochastic and it can be used to investigate the effects of thermal fluctuations, including the phenomenon of superparamagnetism [7]. In particular, within this approach we have studied a number of thermal effects in the magnetization dynamics driven by the rotating magnetic field [8−11].

Because the effective field in conducting ferromagnets contains the magnetic field of eddy currents, the magnetization dynamics in these materials differs from that in non-conducting ones. In the multi-domain case, this difference arises from the conductivity contribution to the effective mass and damping coefficient of domain walls. It was shown in particular that, due to the acceleration dependence of the dissipation, the eddy mass is negative [12,13]. Recently, this prediction has been confirmed by analyzing the leftward asymmetry experimentally observed in the Barkhausen effect [14,15].

In the case of conducting single-domain particles, the LLG equation should be supplemented by Maxwell's equations, which determine the eddy-current contribution to the effective magnetic field [16]. It is usually assumed that this contribution is negligible for nano-sized particles. However, in this paper we show that if the particle size is close to the critical one (which determines the appearance of the single-domain state of the particle) then the eddy-current contribution to the Gilbert damping parameter can be comparable with that of non-conducting samples.

2. DESCRIPTION OF THE MODEL

We consider a spherical particle of electrically conductive and ferromagnetic material. It is assumed that the particle radius R is so small that the magnetic state of the particle is single-domain and the magnetization $\mathbf{M}$ depends only on time and has a constant value, i.e., $\mathbf{M} = \mathbf{M}(t)$ and $M = |\mathbf{M}| = \text{const}$. In this case, the dynamics of $\mathbf{M}$ can be described by the LLG equation

$$\frac{d\mathbf{M}}{dt} = -\gamma \mathbf{M} \times (\mathbf{H}_{\text{eff}} + \boldsymbol{\mathcal{H}}) + \frac{\alpha}{M}\mathbf{M} \times \frac{d\mathbf{M}}{dt}, \qquad (2.1)$$

where $\gamma(>0)$ is the gyromagnetic ratio, the cross denotes the vector product, $\alpha(>0)$ is the damping parameter, $\mathbf{H}_{\text{eff}}$ is the effective magnetic field acting on $\mathbf{M}$, and $\boldsymbol{\mathcal{H}}$ is the averaged magnetic field of eddy currents. Note that since α and $\mathbf{H}_{\text{eff}}$ are assumed to be the same as in the case of non-conducting particles, the LLG equation (2.1) differs from the ordinary one only by the presence of the current-induced field $\boldsymbol{\mathcal{H}}$. In general, $\boldsymbol{\mathcal{H}}$ contains two contributions: one from the magnetic field $\mathbf{H} = \mathbf{H}(\mathbf{r}, t)$ induced by changing the magnetization and the other from the magnetic field $\mathbf{H}_1 = \mathbf{H}_1(\mathbf{r}, t)$ induced by a time-varying external magnetic field. Thus, taking into account the linearity of the Maxwell equations, we obtain $\boldsymbol{\mathcal{H}} = \overline{\mathbf{H}} + \overline{\mathbf{H}}_1$, where the overbar denotes an average over the particle volume V, i.e.,

$$\overline{\mathbf{H}} = \frac{1}{V}\int_V d\mathbf{r}\, \mathbf{H}(\mathbf{r}, t) \qquad (2.2)$$

and similarly for $\overline{\mathbf{H}}_1$. Since the term $\overline{\mathbf{H}}_1$ does not depend on the magnetization, it can be considered as an additional external magnetic field. It should be noted that the source magnetic field $\mathbf{H}_1$ satisfies the quasi-stationary Maxwell equations, which in some cases (e.g., when the external magnetic field is linearly polarized) can be solved analytically [17−19].

Because we are interested here in deriving the effective LLG equation for conducting particles, it is necessary to find the magnetic field $\mathbf{H}$ generated by the magnetization $\mathbf{M}$. To this end, we should solve Maxwell's equations for an arbitrary dependence of $\mathbf{M}$ on time. In the quasi-stationary approximation, i.e., when the condition $\omega \ll \pi c/R$ holds, these equations can be written in the form

$$\operatorname{rot}\mathbf{E} = -\frac{4\pi}{c}\Theta(r)\frac{d\mathbf{M}}{dt}, \qquad \operatorname{div}\mathbf{E} = 0, \tag{2.3}$$

$$\operatorname{rot}\mathbf{H} = \frac{4\pi}{c}\Theta(r)\mathbf{j}, \qquad \operatorname{div}\mathbf{H} = 0. \tag{2.4}$$

Here, $\mathbf{E} = \mathbf{E}(\mathbf{r},t)$ is the electric field induced by changing the magnetization direction, $\mathbf{j} = \mathbf{j}(\mathbf{r},t)$ is the current density satisfying Ohm's law $\mathbf{j} = \sigma\mathbf{E}$, σ is the conductivity, c is the light velocity, $r = |\mathbf{r}|$, and the function $\Theta(r)$ is defined as $\Theta(r) = 1$ at $r \le R$ and $\Theta(r) = 0$ at $r > R$ (we assume here that the origin of the coordinate system is located at the center of the particle). It should also be noted that, besides the condition of quasi-stationarity, in Eq. (2.4) we have used the condition $\omega \ll 4\pi\sigma$, which permits us to neglect the displacement current density.

3. SOLUTION OF MAXWELL'S EQUATIONS

An important feature of the vector equation in Eq. (2.3), which represents Faraday's law for the time-dependent magnetization, is that it does not depend on $\mathbf{H}$. This fact gives us an opportunity to solve Eqs. (2.3) and (2.4) exactly, i.e., determine both the magnetization-induced electric and magnetic fields.

3.1 Induced Electric Field

Since, according to Eq. (2.3), the induced electric field is solenoidal, it can be written as $\mathbf{E} = \operatorname{rot}\mathbf{F}$. Assuming that the vector potential $\mathbf{F} = \mathbf{F}(\mathbf{r},t)$ satisfies the Coulomb gauge condition $\operatorname{div}\mathbf{F} = 0$ and so $\operatorname{rot}\operatorname{rot}\mathbf{F} = -\Delta\mathbf{F}$ (Δ is the Laplace operator), from Eq. (2.3) we obtain the vector Poisson equation

$$\Delta\mathbf{F} = \frac{4\pi}{c}\Theta(r)\frac{d\mathbf{M}}{dt} \tag{3.1}$$

The solution of this equation, which vanishes at $r \to \infty$, is given by

$$\mathbf{F} = -\frac{1}{c}\frac{d\mathbf{M}}{dt}\int_V \frac{d\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|}. \tag{3.2}$$

The integral in the right-hand side of Eq. (3.2) can easily be calculated yielding

$$\int_V \frac{d\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|} = \frac{2\pi}{3}(3R^2 - r^2) \tag{3.3}$$

($r \le R$). Therefore, using the identity

$$\operatorname{rot}[f(r)\mathbf{a}(t)] = -\frac{1}{r}\frac{df(r)}{dr}\mathbf{a}(t)\times\mathbf{r}, \tag{3.4}$$

for the induced electric field inside the particle (i.e., when $r \le R$) we get

$$\mathbf{E} = -\frac{4\pi}{3c}\frac{d\mathbf{M}}{dt}\times\mathbf{r}. \tag{3.5}$$

A simple analysis shows that the lines of electric field (3.5) and so the lines of current density $\mathbf{j}$ lie in the planes perpendicular to the vector $d\mathbf{M}/dt$ and have the form of concentric circles. Because $\mathbf{E}$ linearly depends on $\mathbf{r}$, the concentration of these lines increases with increasing the circles radius.

3.2 Induced Magnetic Field

Similarly to the electric field, we represent the induced magnetic field as $\mathbf{H} = \operatorname{rot}\mathbf{G}$ and choose the Coulomb gauge ($\operatorname{div}\mathbf{G} = 0$) for the vector potential $\mathbf{G} = \mathbf{G}(\mathbf{r},t)$. In this case, from Eq. (2.4) we again obtain the vector Poisson equation

$$\Delta\mathbf{G} = -\frac{4\pi}{c}\Theta(r)\mathbf{j}. \tag{3.6}$$

Using Ohm's law and Eq. (3.5), the physically relevant solution of this equation can be written in the form

$$\mathbf{G} = -\frac{4\pi\sigma}{3c^2}\frac{d\mathbf{M}}{dt}\times\int_V \frac{\mathbf{r}'d\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|}. \tag{3.7}$$

Finally, calculating the integral in Eq. (3.7),

$$\int_V \frac{\mathbf{r}'d\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|} = \frac{2\pi}{15}(5R^2 - 3r^2)\mathbf{r} \tag{3.8}$$

($r \le R$), and using the identity

$$\operatorname{rot}[f(r)\mathbf{a}(t)\times\mathbf{r}] = \mathbf{a}(t)\left(2f(r) + r\frac{df(r)}{dr}\right) - \frac{1}{r}\frac{df(r)}{dr}(\mathbf{a}(t)\cdot\mathbf{r})\mathbf{r} \tag{3.9}$$

(the dot denotes the scalar product), we arrive to the following expression for the magnetic field induced inside the particle:

$$\mathbf{H} = -\frac{16\pi^2\sigma}{45c^2}\left[(5R^2 - 6r^2)\frac{d\mathbf{M}}{dt} + 3\left(\frac{d\mathbf{M}}{dt}\cdot\mathbf{r}\right)\mathbf{r}\right]. \tag{3.10}$$

It should be mentioned that though the magnetization-induced magnetic field can also be calculated outside the particle (when $r > R$), we will use only the above result because it is this magnetic field which determines $\bar{\mathbf{H}}$ and influences the magnetization dynamics. According to Eq. (3.10), the magnetic field $\mathbf{H}$ is non-uniform and possesses axial symmetry about the axis which passes through the particle center and is parallel to the vector $d\mathbf{M}/dt$.

4. EFFECTIVE LLG EQUATION

Next we use Eq. (3.10) and the definition (2.2) to calculate $\bar{\mathbf{H}}$. Taking into account that

$$\frac{1}{V}\int_V d\mathbf{r} = 1, \quad \frac{1}{V}\int_V d\mathbf{r}\, r^2 = \frac{3}{5}R^2,$$
$$\frac{1}{V}\int_V d\mathbf{r}\left(\frac{d\mathbf{M}}{dt}\cdot\mathbf{r}\right)\mathbf{r} = \frac{1}{5}R^2\frac{d\mathbf{M}}{dt}, \tag{4.1}$$

for the magnetization-induced magnetic field averaged over the particle volume we find

$$\bar{\mathbf{H}} = -\frac{32\pi^2\sigma R^2}{45c^2}\frac{d\mathbf{M}}{dt}. \tag{4.2}$$

As was expected from axial symmetry of $\mathbf{H}$, the average field $\bar{\mathbf{H}}$ is parallel to the vector $d\mathbf{M}/dt$ and, in accordance with Lenz's law, its direction is opposite to $d\mathbf{M}/dt$.

Now we are in a position to write the effective LLG equation. Substituting the current-induced magnetic field $\boldsymbol{\mathcal{H}} = \bar{\mathbf{H}} + \bar{\mathbf{H}}_1$ [with $\bar{\mathbf{H}}$ given by Eq. (4.2)] into Eq. (2.1), we obtain the desired equation

$$\frac{d\mathbf{M}}{dt} = -\gamma\mathbf{M} \times (\mathbf{H}_{\mathrm{eff}} + \bar{\mathbf{H}}_1) + \frac{\alpha + \alpha'}{M}\mathbf{M} \times \frac{d\mathbf{M}}{dt}, \qquad (4.3)$$

where

$$\alpha' = \frac{32\pi^2\sigma\gamma M R^2}{45c^2}. \qquad (4.4)$$

According to this equation, the influence of conductivity on the magnetization dynamics is accounted for by both the magnetic field $\mathbf{H}_1$, which modifies the external time-dependent magnetic field, and the additional contribution α' to the damping parameter.

In order to assess the importance of conduction effects, let us compare α' with the damping parameter α, which is related to non-conducting materials. Usually, α ranges from about 10^{-4} to 10^{-1} (for example, in garnets $\alpha \sim 10^{-4} - 10^{-3}$). Considering iron particles with $\sigma = 10^{18}\mathrm{s}^{-1}$, $M = 1.7 \cdot 10^3\mathrm{G}$ and $\gamma = 1.85 \cdot 10^7 \mathrm{s}^{-1}\mathrm{Oe}^{-1}$, from Eq. (4.4) one gets $\alpha' \approx 2.5 \cdot 10^{-6} R^2$, where R is measured in nanometers. Because particles are considered to be single-domain, their radius must not exceed some critical value R_{cr}, i.e., $R < R_{\mathrm{cr}}$. For iron particles $R_{\mathrm{cr}} \approx 10$ nm, therefore $\max\{\alpha'\} = \alpha'|_{R=R_{\mathrm{cr}}} \approx 2.5 \cdot 10^{-4}$. These estimations show that for rather large nanoparticles, α' can be of the order of α. Clearly, in these cases the conduction effects cannot be neglected, and the effective LLG equation (3.4) should be used for studying the magnetization dynamics.

5. CONCLUSIONS

We have derived the effective Landau-Lifshitz-Gilbert equation that describes the magnetization dynamics in conducting ferromagnetic nanoparticles. The influence of conductivity is accounted in this equation by two terms. The first accounts for the magnetic field of eddy currents that are induced by the external time-dependent magnetic field. Because this induced field does not depend on the magnetization, it can be considered as an addition external magnetic field.

The second term describes the influence of the magnetic field of eddy currents that are induced by the time-dependent magnetization. By solving the corresponding Maxwell's equations, we have shown that this influence is completely accounted by an addition contribution to the damping parameter. It has been established that for large nanoparticles a given contribution is essential and cannot be neglected.

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