Diffusion: from normal to anomalous

Yu.Bystrik, *postgraduate student*

The chaotic motion of small particles was first observed and investigated by the botanist Robert Brown in 1827. Brown was studying pollen grains of the plant suspended in water under a microscope when he observed minute particles, ejected by the pollen grains, which were executing a jittery motion. By repeating the experiment with particles of inorganic matter he was able to rule out that the motion was life-related. So it was suggested that Brownian particle is displaced by bombardments of smaller particles.

In 1905 Albert Einstein brought the solution of the problem of Brownian motion in one of his most famous paper. He did the assumption that "the random walker" forgets the direction of motion after a certain time, and that the mean-squared displacement during this time is finite. As a result from microscopic principles he derived the equation for diffusion of particles, from which it follows that the particles are placed according to the normal Gaussian distribution and the variance of the walker position is proportional to the time. Einstein’s predictions were experimentally verified and confirmed by Jean Baptiste Perrin. These were the first serious steps to understand the wide class of the stochastics processes in the nature. In addition, they were indirectly claimed the existence of atoms and molecules.

But the further researches of different phenomena showed that systems demonstrate abnormal behaviour very often. While in the case of typical Brownian dynamics the corresponding mean-square displacement scales linearly in time, many complex systems manifest features of anomalous diffusion, characterized by a non-linear relationship to time. It occurs because the dynamics of anomalous processes is dominated by long time or/and space correlations , so these types of systems have a memory. Usually, the variance follows the power law, i.e., $\sigma^2(t) \propto t^\alpha$, and two types of anomalous diffusion, subdiffusion (when $0 < \alpha < 1$) and superdiffusion (when $\alpha > 1$), are distinguished. The class of systems with such behavior of the variance is vast and growing. Subdiffusion has been observed in amorphous solids, percolation clusters, living cells, and superdiffusion has been observed in turbulent flows, optical lattices and bacterial motion. In general, the time-dependence of the variance is not restricted by a power function. For example, very interesting alternative is superslow diffusion, i.e., the long-time diffusion of particles whose variance grows slower than any power of time. Some examples of superslow diffusion have been found in resistor networks, charged polymers, aperiodic environments, iterated maps, Langevin dynamics, fractional kinetics and others. Also there are classes of processes which are characterized by an infinite mean-square displacement of particle's position for all values of time. The most known example of these processes is Levy flights, which stem from mathematics related to chaos theory and play useful role in simulation and understanding random phenomena. Examples include earthquake data analysis, financial mathematics, end even patterns of movements of hunting animals.

All normal systems are very similar to each other, whereas any anomalous system exhibits its very own type of behavior. Thus, anomalous transport refers to nonequilibrium processes that cannot be described by using standard methods of statistical physics. It asks for a synergy of many different disciplines, from the mathematical theory of dynamical systems over the theory of stochastic processes to the statistical physics of disordered systems. Moreover, the study of natural phenomena is impossible without deep collaboration between of mathematics, physics, computer science, chemistry and biology. Stochastic systems play a dominant role in exact and life sciences, embracing a richness of systems such as glasses, liquid crystals, proteins, biopolymers, organisms or even ecosystems.

Frankly, a lot of important stochastic problems have not been solved yet. But the longer scientists try to understand random and diffusive processes, the most clear picture emerges, that all these phenomena, which are called anomalous, are in our lives at each step. Nature never tires to tell us that anomalous is completely normal.

A. Diadechko, ELA

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