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Constant Electric Field Influence on the Rectification of the Transversal Current Induced by the Sinusoidal and Cnoidal Electromagnetic Waves in Graphene Superlattice

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In graphene superlattice the direct current induced in perpendicular direction with relation to the superlattice axis by cnoidal and sinusoidal electromagnetic waves under the presence of longitudinal constant electric field was calculated. Direct current dependences on the cnoidal wave amplitude and on the longitudinal electric field intensity were investigated. Direct current dependence on the cnoidal wave amplitude was shown to have resonant character.

Keywords: Graphene, Superlattice, Conoidal wave.

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1. INTRODUCTION

There are intensive investigations of features of graphene [1-4] and of superlattices (SL) based on the graphene [5-9] last time. Dispersion low of SL based on the graphene sheet deposited on the periodic subsrate is studied in [10] where the energy of electron motion along the SL axis was shown to depend on the quasimomentum directed along its axis periodically near the Dirac point.

The increased attention to the carbon system with the additional SL potential is due to the possibility of using of the superlattice as a working medium of generators and amplifiers of terahertz (THz) electromagnetic (EM) waves. The ability to create so-called Bloch oscillator based on the semiconductor SL was discussed [11].

In [12,13] the nonlinear effects in optical fibers – namely, the so-called effects of the coherent wave mixing [14] – were studied. The important problem arising in the study of THz wave mixing is the investigation of the mutual influence of powerful radiation with different frequencies propagating in the nonlinear medium [15]. One manifestation of the mutual influence of radiation – the so-called effect of mutual rectification of EM waves – the appearance of a constant component of the electric current under the number of EM waves with frequencies are treated as integer numbers [14,16,17].

The interest to the effect of THz radiation rectification in the SL is due to its direct relation with the diagnosis of kinetics features of nonlinear mediums, with detection, amplification and generation of THz radiation [12,13,18]. In [19-27] application of a strong polychromatic pump field is proposed to amplify THz radiation in SL. Amplification of even harmonics of the pump field in this case is accompanied by the appearance of a constant component of the field. The possibility of rectification of current induced by the mixing of the sinusoidal EM wave and its second harmonic in the semiconductor SL was shown in [16,17,28]. Moreover, in [16,17] the situation when the waves polarization planes coincide with the axis of the bulk SL was considered.

The mutual rectification of cnoidal and weak sinusoidal waves in the bulk SL was studied in [29] where the case of polarization planes of both waves contain the SL axis was considered. In [29] the presence or absence of direct current was shown to be regulated by change of cnoidal wave intensity. The rectification of the transversal current induced in graphene SL by two sinusoidal EM waves with orthogonal polarization planes was studied in [30,31]. In [30] the effect of the mutual rectification of sinusoidal EM waves frequency of one of which is in two times more than that of the other in graphene SL was investigated. In [31] the electric field influence on the direct current induced by two sinusoidal waves of equal frequencies perpendicularly to the graphene SL axis was studied.

The nonlinearity of current constant component dependence on the EM wave intensity is most notably appeared at high power of wave. However the intensive EM wave with electric field intensity directed along the SL axis is not of sinusoidal form as was assumed in [16,17,30,31]. In collisionless approximation in one-dimensional SL the EM wave equation is a solution of d'Alembert equation taking the form of sine-Gordon (SG) equation which has the Jacobi elliptic functions as a general periodic solution.

Below, the longitudinal electric field influence on the transverse current rectification induced in a graphene SL by two EM waves, one of which is polarized along the SL axis and has cnoidal form is considered.

2. CALCULATIONS

Let graphene lies in the plane xz. Numerical analysis of the dispersion law [10] carried out in [31] showed that the electronic spectrum of the superlattice based on graphene can be approximately written in the following explicit form:

$$\varepsilon(\mathbf{p}) = \sqrt{\varepsilon_0^2 + p_x^2\upsilon_F^2} + \frac{\varepsilon_1^2}{\sqrt{\varepsilon_0^2 + p_x^2\upsilon_F^2}}\left(1 - \cos\frac{p_z d}{\hbar}\right), \quad (1)$$

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where $\varepsilon_0 = 0.059$ eV, $\varepsilon_1 = 0.029$ eV, $d = 2\cdot 10^{-6}$ cm is the SL period, $\upsilon_F = 10^8$ cm/s is the Fermi surface velosity, Oz is the SL axis. In the new notations: $q_x = p_x d/\hbar$, $q_z = p_z d/\hbar$, $\gamma = \hbar\upsilon_F/d\varepsilon_0$, $\gamma_1 = \varepsilon_1/\varepsilon_0$, the expression (1) takes the form:

$$\varepsilon(\mathbf{p}) = \varepsilon_0\sqrt{1+\gamma^2 q_x^2} + \frac{\varepsilon_0\gamma_1^2(1-\cos q_z)}{\sqrt{1+\gamma^2 q_x^2}}. \tag{2}$$

The constant electric field with intensity is equal to $\mathbf{E} = (0,0,E)$ is suggested to be applied along the SL axis. In addition the electric field of cnoidal EM wave is applied. Another alternate electric field does oscillations in graphene plane perpendicular to the SL axis. Under these conditions the electric current density arising through the axis Ox in the constant relaxation time approximation is calculated with the following formula:

$$j_x(t) = -\frac{e}{\tau}\int_{-\infty}^{t} \mathrm{d}t' e^{-\frac{t-t'}{\tau}} \sum_{\mathbf{p}} f_0(\mathbf{p}) \times V_x\left(\mathbf{p} + \frac{e}{c}\big(\mathbf{A}(t') - \mathbf{A}(t) + \mathbf{A}_1(t') - \mathbf{A}_1(t)\big) + e\mathbf{E}(t'-t)\right) \tag{3}$$

where $\mathbf{A} = (0,0,A_z)$ is the vector potential of cnoidal EM wave field satisfying to the SG equation, $\mathbf{A}_1 = (A_{1x},0,0)$ is the vector potential of sinusoidal EM wave field:

$$A_{1x} = \frac{cE_1}{\omega}\sin(\omega t + \phi), \tag{4}$$

where E_1, ω, ϕ are amplitude, frequency and initial phase of the transversal field oscillations, $f_0(\mathbf{p})$ is equilibrium state function, $V_x(\mathbf{p})$ is the electron velocity along the SL axis calculated by the formula: $V_x = \partial\varepsilon/\partial p_x$. Rewrite (3) in the following view:

$$j_x(t) = -\frac{e\upsilon_F\gamma}{\tau}\int_{-\infty}^{t} \mathrm{d}t' e^{-\frac{t-t'}{\tau}} \sum_{\mathbf{p}} f_0(\mathbf{p})$$
$$\left(\frac{q_x + \beta' - \beta + \gamma^2(q_x+\beta'-\beta)^3}{\left(1+\gamma^2(q_x+\beta'-\beta)^2\right)^{3/2}} + \frac{\gamma_1^2(q_x+\beta'-\beta)\cos\left(q_z + \omega_B(t'-t) + \alpha' - \alpha\right)}{\left(1+\gamma^2(q_x+\beta'-\beta)^2\right)^{3/2}}\right) \tag{5}$$

where $\alpha(t) = edA_z(t)/c\hbar$, $\beta(t) = edA_{1x}(t)/c\hbar$, $\alpha' = \alpha(t')$, $\beta' = \beta(t')$, $\omega_B = edE/\hbar$ is the Bloch oscillations frequency. Further the sinusoidal EM wave is supposed to be weak: $b = edE_1/\hbar\omega << 1$. In the linear approximation in the parameter b formula (5) has the view:

$$j_x(t) = -\frac{e\upsilon_F\gamma^3}{\tau}\int_{-\infty}^{t}\mathrm{d}t' e^{-\frac{t-t'}{\tau}}(\beta'-\beta)\sum_{\mathbf{p}} f_0(\mathbf{p})\frac{1}{\left(1+\gamma^2 q_x^2\right)^{3/2}} - \frac{e\upsilon_F\gamma_1^2\gamma}{\tau}\int_{-\infty}^{t}\mathrm{d}t' e^{-\frac{t-t'}{\tau}}(\beta'-\beta)\cos\left(\omega_B(t'-t)+\alpha'-\alpha\right)$$
$$\times\sum_{\mathbf{p}} f_0(\mathbf{p})\frac{\left(1-2\gamma^2 q_x^2\right)\cos q_z}{\left(1+\gamma^2 q_x^2\right)^{5/2}} \tag{6}$$

After sum of the momenta was calculated we obtain:

$$j_x(t) = -\frac{j_0\gamma^4}{\tau\gamma_1^2}\frac{F_1(\gamma,\gamma_1,\varepsilon_0/\theta)}{F_0(\gamma,\gamma_1,\varepsilon_0/\theta)}\int_{-\infty}^{t}\mathrm{d}t' e^{-\frac{t-t'}{\tau}}(\beta'-\beta) - \frac{j_0\gamma^2}{\tau}\frac{F_2(\gamma,\gamma_1,\varepsilon_0/\theta)}{F_0(\gamma,\gamma_1,\varepsilon_0/\theta)}\times$$
$$\times\int_{-\infty}^{t}\mathrm{d}t' e^{-\frac{t-t'}{\tau}}(\beta'-\beta)\cos\left(\omega_B(t'-t)+\alpha'-\alpha\right) \tag{7}$$

where we define:

$$j_0 = \frac{n_0 e\upsilon_F\gamma_1^2}{\gamma},$$

$$F_0(\gamma,\gamma_1,\varepsilon_0/\theta) = \int_0^{\infty} I_0\left(\frac{\varepsilon_0\gamma_1^2}{\theta\sqrt{1+\gamma^2 q_x^2}}\right)\exp\left(-\frac{\varepsilon_0}{\theta}\sqrt{1+\gamma^2 q_x^2}\right)\mathrm{d}q_x,$$

$$F_1(\gamma,\gamma_1,\varepsilon_0/\theta) = \int_0^{\infty}\frac{1}{\left(1+\gamma^2 q_x^2\right)^{3/2}}\times I_0\left(\frac{\varepsilon_0\gamma_1^2}{\theta\sqrt{1+\gamma^2 q_x^2}}\right)\exp\left(-\frac{\varepsilon_0}{\theta}\sqrt{1+\gamma^2 q_x^2}\right)\mathrm{d}q_x$$

,

$$F_2(\gamma,\gamma_1,\varepsilon_0/\theta) = \int_0^{\infty}\frac{1-2\gamma^2 q_x^2}{\left(1+\gamma^2 q_x^2\right)^{5/2}}\times I_1\left(\frac{\varepsilon_0\gamma_1^2}{\theta\sqrt{1+\gamma^2 q_x^2}}\right)\exp\left(-\frac{\varepsilon_0}{\theta}\sqrt{1+\gamma^2 q_x^2}\right)\mathrm{d}q_x$$

At low temperatures $\theta << \varepsilon_0$:

$$j_x(t) = -\frac{j_0\gamma^4}{\tau\gamma_1^2}\int_{-\infty}^{t} \mathrm{d}t' e^{-\frac{t-t'}{\tau}}(\beta'-\beta) - \frac{j_0\gamma^2}{\tau}\int_{-\infty}^{t} \mathrm{d}t' e^{-\frac{t-t'}{\tau}}(\beta'-\beta)\cos\left(\omega_{\mathrm{B}}(t'-t)+\alpha'-\alpha\right) \tag{8}$$

Further to calculate the second integral in (8) we use the following Fourier series expansion:

$$e^{i\alpha(t)} = \sum_{n=-\infty}^{\infty} c_n(a) e^{in\Omega(a)t}, \tag{9}$$

where

$$\Omega(a) = \frac{\pi\Omega_0}{2\mathrm{K}(a)},\ c_0 = \frac{2\mathrm{E}(a)}{\mathrm{K}(a)} - 1,$$

$$c_n = \frac{n\pi^2}{\mathrm{K}^2(a)}\frac{1}{q_a^{-n/2} + (-1)^{n+1} q_a^{n/2}}$$

при $0 < a \le 1$,

$$\Omega(a) = \frac{\pi\Omega_0 a}{\mathrm{K}(a^{-1})},\ c_0 = 1 - 2a^2 + \frac{2a^2\mathrm{E}(a^{-1})}{\mathrm{K}(a^{-1})},$$

$$c_n = \frac{4n\pi^2 a^2}{\mathrm{K}^2(a^{-1})}\frac{1}{q_{1/a}^{-n} - q_{1/a}^{3n}},\ \text{при } a > 1,$$

$$a = \frac{eE_0 d}{2\hbar\Omega_0},\ q_a = \exp\left(-\frac{\pi\mathrm{K}\left(\sqrt{1-a^2}\right)}{\mathrm{K}(a)}\right),\ \Omega_0 = \frac{\omega_{\mathrm{pl}} u}{c}\sqrt{\left|1 - \frac{u^2}{c^2}\right|},$$

$$\omega_{\mathrm{pl}} = \frac{2ed\varepsilon_1}{\hbar}\sqrt{\frac{\pi n_0}{\varepsilon_0}\frac{F_3(\gamma,\gamma_1,\varepsilon_0/\theta)}{F_0(\gamma,\gamma_1,\varepsilon_0/\theta)}},$$

$$F_3(\gamma,\gamma_1,\varepsilon_0/\theta) = \int_0^{\infty}\frac{1}{\sqrt{1+\gamma^2 q_x^2}} \times I_1\left(\frac{\varepsilon_0\gamma_1^2}{\theta\sqrt{1+\gamma^2 q_x^2}}\right)\exp\left(-\frac{\varepsilon_0}{\theta}\sqrt{1+\gamma^2 q_x^2}\right)\mathrm{d}q_x,$$

E_0 is the amplitude of cnoidal wave, u is the wave velocity, c is the wave velocity in the absence of electrons, $\mathrm{K}(x)$ и $\mathrm{E}(x)$ are complete elliptic integrals of the first and second kind correspondingly. After substituting (9) in (8), calculation of integrals, and averaging over the wave period, we obtain the following formula for the constant component of the transverse current density:

$$\langle j_x \rangle = -\sigma_0 E_1 R(a)\cos\phi, \tag{10}$$

where $\sigma_0 = n_0 e^2 \upsilon_F^2 \varepsilon_1^2 / \varepsilon_0^3 \omega$,

$$R(a) = \sum_{k,n=-\infty}^{\infty} c_k c_n \frac{\left(\delta_{n\Omega,k\Omega-\omega} - \delta_{n\Omega,k\Omega+\omega}\right)(\omega_{\mathrm{B}} + n\Omega)\tau}{1 + (\omega_{\mathrm{B}} + n\Omega)^2\tau^2} \tag{11}$$

3. DISCUSSIONS

The plot of the dependence of the function (11) on the cnoidal wave amplitude under the different values of the longitudinal electric field intensity is shown on the figure 1 $(0 < a \le 1)$.

This function can be seen to have the resonant character. The constant component of the transversal current occurs whenever the sinusoidal wave frequency is the integer number of cnoidal wave frequency. The possibility of the direct current in the transverse direction with relation to the SL axis is explained by the nonadditivity of the graphene SL spectrum. In the bulk SL with additive spectrum such effect is impossible.

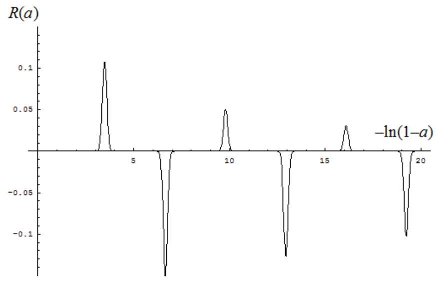

Fig. 1 – $\omega_{\mathrm{B}} = \omega$, $\omega\tau = 10$, $\Omega_0\tau = 10$

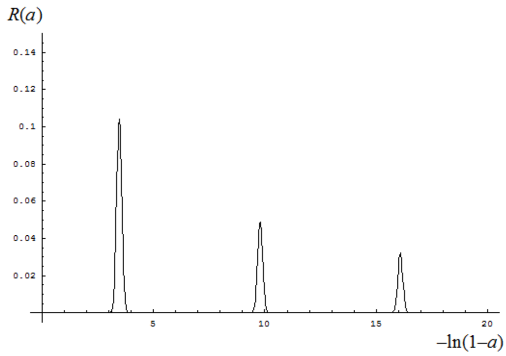

Fig. 2 – $E = 0$, $\omega\tau = 10$, $\Omega_0\tau = 10$

On the Fig. 2 the direct current dependence on the cnoidal wave amplitude when constant electric field is absent $(E = 0)$ is shown. In this case (as in [29]) direct current resonance occurs when ω is in even number more than Ω .

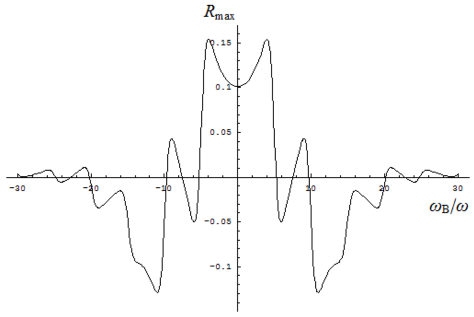

Fig. 3 – a = 0.97, $\omega\tau$ = 10, $\Omega_0\tau$ = 10

However when longitudinal electric field is present $(E \neq 0)$ the new resonances of transversal direct current appears (Fig. 1).

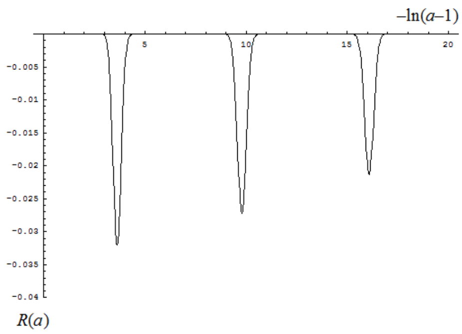

Fig. 4 – $\omega_B = \omega$, $\omega\tau$ = 10, $\Omega_0\tau$ = 10

Moreover, the resonances values and resonances directions can be regulated by changing of longitudinal electric field. This effect is also possible due to nonadditivity of the electron spectrum (1). The direct current dependence on the longitudinal electric field intensity is shown on the Fig. 3.

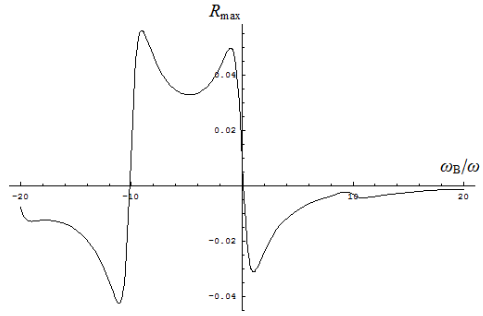

Fig. 5 – a = 1.028, $\omega\tau$ = 10, $\Omega_0\tau$ = 10

The dependences of the function (11) on the cnoidal wave amplitude and on the intensity of the longitudinal electric field in the case $a > 1$ are shown on the Fig. 4 and Fig. 5, correspondingly. The constant component of the transversal current occurs whenever the sinusoidal wave frequency is in integer number more than cnoidal wave frequency. In the case $a > 1$ the function $R_{\max}(\omega_B)$ can be seen from Fig. 5 to be not symmetric, unlike the case of $0 < a \leq 1$ (Fig. 3). This fact is due to the constant component of electric field of cnoidal wave in the case when $a > 1$.

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