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Theory of the Scope Change Under Magnetic Field, Temperature and Stress

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Mathematical model that describes connections between elastic and electromagnetic properties is presented. These connections are based on possibility to change a volume of solid by temperature, pressure or magnetic field. New parameter of solid "scope change", that depends on elastic modulus, heat and thermal conductivity, thermal expansion, is proposed. The comparison of theoretical and experimental results was made for conductivity data of $LaMnO_3$.

Keywords: Volume Changes; Pressure; Magnetic Field; Stress; Temperature.

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1. INTRODUCTION

In recent years, an essential progress in study of properties of magneto-containing materials has been achieved due to new experimental technologies. Numerous effects, anomalies, special features have been discovered ([1-3] for example). However, the work in this direction is still far from being complete. Experimental conditions were never taking into account in full. For example, volume changes and dependence of material density [4] on temperature, magnetic field and pressure were not considered.

There are many theoretical explanations of these effects but they are mostly based on special experimental conditions. However, the general principles, which would explain the influence of acting parameters on phase transitions and states, have not been formulated untill now. The pioneering work on understanding of the role of volume changes in the formation of resistance properties was done by P. Kapitsa [5], where he revealed the linear dependence of conductivity on temperature and magnetic field for many pure metals. Further investigations of dependence of temperature, magnetic field on high pressure in the study of new materials (magnetic semiconductors and dielectrics) accumulated the large volume of data [6,7,8]. It follows from the results published in [9] that it is necessary to consider volume changes in the materials to understand the nature of critical regularities and phase states.

The main idea of the present paper is to show a good agreement between a new proposed mathematical model and previously obtained experimental data. The volume changes dependence on electromagnetic field, temperature and solid parameters was obtain. An explicit solution of the model system in the stationary case, namely, when behaviour of the solid is stabilized are presented. Analytical results with experimental data for LaMnO3 was compared.

2. NEW MODEL

To describe a solid influenced by electromagnetic field, temperature and mechanical stress we propose the following system of equations (in SI):

$$\tilde{\varepsilon}\vec{E}_t + \vec{J} = \operatorname{curl}\vec{H}, \quad \tilde{\varepsilon}\operatorname{div}\vec{E} = \tilde{\rho}, \tag{1}$$

$$\tilde{\mu}\vec{H}_t + \operatorname{curl}\vec{E} = 0, \quad \operatorname{div}\vec{H} = 0, \tag{2}$$

$$c_0 T_t + \operatorname{div}\vec{q} = -T_0\alpha_T(3\lambda + 2\mu)\,\theta_t + (\vec{J}\cdot\vec{E}), \tag{3}$$

$$\rho\vec{u}_{tt} = \mu\,\Delta\vec{u} + (\lambda+\mu)\nabla\theta - \alpha_T(3\lambda+2\mu)\nabla T + \tilde{\mu}[\vec{J}\times\vec{H}] \tag{4}$$

Using the basic dynamics law of linear deformed solid (cf. [10]), we introduce the term $\tilde{\mu}[\vec{J}\times\vec{H}]$ related to the electromagnetic field, and the thermal expansion term $-\alpha_T(3\lambda+2\mu)\nabla T$ in (4). The term $-T_0\alpha_T(3\lambda+2\mu)\,\theta_t$ in (3) follows from the thermo-elasticity theory for a solid (cf. [11]), and the term $(\vec{J}\cdot\vec{E})$ in (3) is a consequence of heat generation into the solid (cf. [12]), i.e. the Joule–Lenz law. Here

$$\vec{J} = \sigma\vec{E} + \beta\nabla T, \quad \vec{q} = \gamma\vec{E} - \lambda_T\nabla T, \quad \theta = \operatorname{div}\vec{u}, \tag{5}$$

where $\vec{E}$ and $\vec{H}$ are electric and magnetic fields; $\tilde{\rho}$ is charge density; $\vec{J}$ is current density, σ is electric conductivity; $\tilde{\varepsilon}$ is permittivity and $\tilde{\mu}$ is magnetic conductivity; T is temperature, T_0 is temperature without resiliencies, $\vec{q}$ is heat flux, c_0 is heat capacity without resiliencies, λ_T is heat conductivity, α_T is coefficient of linear heat expansion; $\vec{u}$ is displacement, θ is scope change; ρ is density; $\lambda = \nu E/[(1+\nu)(1-2\nu)]$ and $\mu = E/[2(1+\nu)]$ are the Lamé parameters, $E$is Young's modulus, ν is Poisson's ratio.

It is necessary proceed with rigorous analysis of the scope change, θ, depending on the other parameters, and consider stationary states of the system (1)-(4):

$$\vec{J} = \operatorname{curl}\vec{H}, \operatorname{curl}\vec{E} = 0, \tilde{\varepsilon}\operatorname{div}\vec{E} = \tilde{\rho}, \operatorname{div}\vec{H} = 0, \tag{6}$$

$$\operatorname{div}\vec{q} = (\vec{J}\cdot\vec{E}), \tag{7}$$

$$\mu\,\Delta\vec{u} + (\lambda+\mu)\nabla\theta - \alpha_T(3\lambda+2\mu)\nabla T + \tilde{\mu}[\vec{J}\times\vec{H}] = 0. \tag{8}$$

For simplicity sake, we assume that $\beta=\gamma=0$, $\tilde{\rho}$ and σ are constant. Taking the div-operator for (8), due to (5)-(7), it is obtain the main equations:

$$\Delta\vec{H} = 0, \quad \Delta\vec{E} = 0, \tag{9}$$

$$-\Delta T = \frac{\sigma}{\lambda_T}\vec{E}^2, \qquad (10)$$

$$-\Delta\theta = \frac{\sigma}{\lambda + 2\mu}\left(\frac{\alpha_T(3\lambda + 2\mu)}{\lambda_T} - \sigma\tilde{\mu}\right)\vec{E}^2. \qquad (11)$$

Here was use the equality: $\mathrm{div}(\vec{E}\times\vec{H}) = \vec{H}\,\mathrm{curl}\,\vec{E} - \vec{E}\,\mathrm{curl}\,\vec{H}$. Now consider electrostatic field (i.e. $\vec{E} = \vec{E}_0$ is a constant vector), and a spherical solid of radius R at the origin 0 (i.e. $\Omega = B_R(\mathbf{0})$ with the boundary $\partial\Omega = S_R(\mathbf{0})$). In this case, was obtain the following boundary value problem for the equation

$$-\Delta\theta = \frac{\sigma}{\lambda + 2\mu}\left(\frac{\alpha_T(3\lambda + 2\mu)}{\lambda_T} - \sigma\tilde{\mu}\right)E_0^2 \quad \text{in} \quad \Omega, \qquad (12)$$

$$\theta = \mathbf{0} \quad \text{on} \quad \partial\Omega \qquad (13)$$

The well-known solution of the problem (12), (13) is

$$\theta = \frac{\sigma E_0^2}{4\pi(\lambda + 2\mu)}\left(\frac{\alpha_T(3\lambda + 2\mu)}{\lambda_T} - \sigma\tilde{\mu}\right)\cdot \cdot\iiint_\Omega \frac{dx'dy'dz'}{\sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2}} \qquad (14)$$

3. DISCUSSION AND COMPARISON WITH EXPERIMENTAL RESULTS

To obtain experimental data, which describe change of volume of a solid under *P, H, T*, one needs to extract a specific (or molar) volume V_0 by analogy with the gases. A change of the form and the size of a solid depends on external fields as

$$d\varepsilon_i = \alpha_{ij}\, dT - s_{ij}\, dP + k_{ij}H_j, \qquad (15)$$

where α_{ij} are coefficients of anisotropic thermal expansion, s_{ij} are anisotropic modules of compressibility, k_{ij} are coefficients of anisotropic magnetostriction. Obviously, the value of macro-parameter V_0 depends on solid's atom sizes due to the fact that they determine the forces of interaction between the atoms, which deform an electron cloud. Any external field disturbs the balance inside the solid and deforms the electron cloud that changes all material properties. Therefore the volume change depends on multiple material parameters as it is reflected in the proposed model. This change can be evaluated by (14).

In order to verify the proposed model, we substitute the known parameters into (15) and compare the analytical results with the experimental ones. However, the absence of the complete set of the experimental parameters for the material with the identical structure does not allow us the direct comparison. It is still possible to attempt the indirect comparison for the relatively complete set of experimental data of resistivity measurements from [8].

To compare theoretical and experimental data the derivative $\partial\sigma/\partial\theta$ from (14), which shows a correspondence between the conductivity and the specific volume was find. Approximating *σ(θ)* from (14) by the first term of the Taylor series about $\theta = 0$ and taking into account the smallness of volume strains, it is obtain

$$\sigma \approx \frac{B}{\tilde{\mu}} - \frac{1}{ABS}\theta \qquad (16)$$

hence

$$\frac{\partial\sigma}{\partial\theta} = \frac{1}{ABS} \qquad (17)$$

where $A = \frac{E_0^2}{4\pi(\lambda + 2\mu)}$, $B = \left(\frac{\alpha_T(3\lambda + 2\mu)}{\lambda_T}\right)$, S is the surface of sample. The experimental parameter values in (17) was substitute: the elastic modules λ = 224 GPa, μ = 82 GPa, the coefficient of the thermal expansion α_T = 10^{-5} K^{-1} (cf. [13]), the thermo-conductivity λ_T = 2 W/mK [14], the surface of sample as tablet with 5 mm diameter of and a 1mm height is $S = 5{,}5\cdot 10^{-5}$ м2, E_0= 10^2 V/m. The value $\partial\sigma/\partial\theta = \mathbf{2{,}2}\cdot\mathbf{10^6}\,\Omega^{-1}$ m^{-1} was computed. The experimental value $\partial\sigma/\partial\theta = \mathbf{7{,}3}\cdot\mathbf{10^5}\Omega^{-1}m^{-1}$ is taken from [9] for the temperature interval 50-150 K, $P = 0$, $H = 0$. It can be seen that the experimental and analytical (obtained according to the proposed model) data are in a good agreement that testifies in favor of the proposed model.

The proportionality of the electric conductivity to the specific volume attests to the fact that these two macro-parameters of a solid are determined approximately by common factor $\partial\sigma/\partial\theta$. This factor reflects the change of the electron cloud density in comparison with a separate atom. A change of the mass density of orbitals is related to the internal stresses value in a solid, whereas a change in the charge density of cloud is related to the conductivity value. For example, if the mass density changes are small, i.e. conductivity measurements are done in the short intervals *P-T-H*, then these change in the mass density can be come significant under high pressures, strong magnetic field and extremely high or low temperatures. This leads to the change of internal stresses and the other material properties.

4. CONCLUSION

a) For the correct experimental investigations of solid properties it is necessary to include a new parameter (the scope change θ). The role of this factor grows strongly when an external impact tends to its outer limit.
b) The parameter θ determines the degree of change of the internal stress within material and shows its relation to the other parameters.
c) The coefficient $\partial\sigma/\partial\theta$ reflects close connection between the conductivity and the mechanical properties as they are determined by electron and mass density of an electron cloud correspondingly.
d) The proposed model allows studying conductivity properties of materials more deeply. Moreover, this model gives better description of the relations between solid properties.

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