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Spin excitations in a Ferromagnetic Nanorice-type Cluster

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In the paper, dipole-exchange spin excitations in a composite nanorice-type nanoparticle (prolate rotation ellipsoid) are investigated theoretically. A nanorice with a non-magnetic core and a shell composed of an uniaxial ferromagnet is considered. Spin dynamics in the above-described nanosystem is described using the linearized Landau-Lifshitz equation (in the magnetostatic approximation) with the addends that consider the magnetic dipole-dipole interaction, the exchange interaction and the anisotropy effects. An equation for the magnetic potential of the above-described spin excitations is obtained. For the case of a thin shell, a solution for the above-mentioned equation in the form of a combination of the generalized spheroidal functions is proposed. For the above-described case, a dispersion relation for such spin excitations is also found.

Keywords: composite nanostructure, nanoshell, nanorice, spin excitation, dipole-exchange theory.

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1. INTRODUCTION

Spin waves in different kinds of nanosystems are an actual and popular topic of research. Magnonics – a subfield of modern solid state physics that studies spin waves in nanosystems [1] – and spintronics (spin electronics) – a sub-field of modern solid state physics that studies the properties of the electron spin and ways of its manipulations in solid-state devices [2] – are promising, in particular, for creating new information storage, transmission and processing devices [3–5].

Magnetic properties of a nanosystem are known to depend essentially on its shape and size. Therefore, spin waves are studied in different types of nanosystems – thin films [6], micron-sized magnetic quantum dots [7], magnetic nanowires [8] and so on – individually. Magnetic propert $R_{||}^{(2)}$ ies of $R_{\perp}^{(1)}$ shell-type magnetic nanosystems [9,10] remain poorly researched at rhe moment. In particular, synthesized recently shell-type ellipsoid magnetic nanoparticles – magnetic nanorice [11-13] – remain practically unresearched. However, they exhibit unique properties that are not observed in a nanosystem with higher degree of symmetry. This makes study of a magnetic nanorice (in particular, of spin excitations in a magnetic nanorice) actual.

In the paper, dipole-exchange spin excitations in a nanorice cluster with the shape of a prolate rotation ellipsoid are investigated. The investigated nanorice consists of a non-magnetic core and a shell comprised of an uniaxial ferromagnet. For the above-described spin excitations, equation for the magnetic potential in the magnetostatic approximation (considering the dipole-dipole interaction, the exchange interaction and the anisotropy effects) is obtained. The equation is solved approximately for the case of a thin shell. A dispersion relation for this case is obtained.

2. SETTING OF THE PROBLEM

Let us consider a nanoshell with the shape of a prolate rotation ellipsoid (nanorice particle) composed of a non-magnetic core and a ferromagnetic shell. Therefore, the core of the nanorice is bounded by a rotation ellipsoid with semiaxes $R_{||}^{(1)}$, , and the external boundary of the particle is a rotation ellipsoid with semiaxes , $R_{\perp}^{(2)}$. Let us introduce prolate spheroidal coordinates (ξ,η,φ) with the length parameter a (so the internal and the external boundary of the shell is described by the equations $\xi = \xi_1$ and $\xi = \xi_2$, correspondingly, where the constants $\xi_1 = R_{||}^{(1)}/a$, $\xi_2 = R_{||}^{(2)}/a$) and assume that the ferromagnet is of an local "easy axis" type (so the ground state magnetization $\vec{M}_0$ is directed along the axis ξ, curvature of which is considered negligible inside the shell). Let us denote the ferromagnet parameters as follows: the exchange constant α, the uniaxial anisotropy parameter β and the gyromagnetic ratio γ. We neglect the dissipation – and, therefore, the spin excitations damping in the shell, discarding relaxation terms in the Landau-Lifshitz equation.

Let us consider linear spin excitations in the above-described shell (so the magnetization $\vec{m}$ and the magnetic field $\vec{h}$ of the excitation can be considered as a small perturbation of the overall magnetization density and the overall internal magnetic field, correspondingly). Therefore, the magnetization density $\vec{M} = \vec{M}_0 + \vec{m}$, $|\vec{m}| << |\vec{M}_0|$, and the magnetic field inside the shell $\vec{H}^{(i)} = \vec{H}_0^{(i)} + \vec{h}$, $|\vec{h}| << |\vec{H}_0^{(i)}|$ (here $\vec{M}_0$ and $\vec{H}_0^{(i)}$ are the ground state magnetization and

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the internal magnetic field, correspondingly).

The task of the paper is to obtain an equation for the magnetic potential of the above-described excitations (in a magnetostatic approximation) and, after solving this equation, to obtain an dispersion relation of these excitations.

3. SYSTEM OF EQUATIONS FOR A SPIN EXCITATION IN A NANORICE SHELL

Let us write down the Landau-Lifshitz equation for linear spin excitations in the above-described nanorice shell. The Landau-Lifshitz equation without dissipation term can be written as follows (see, e.g., [14]):

$$\frac{\partial \vec{m}}{\partial t} = \gamma\left(\vec{M}_0 \times \left(\vec{h} + \alpha\sum_i \frac{\partial^2 \vec{m}}{\partial x_i^2} + \beta\vec{n}\left(\vec{m}\vec{n}\right) - \frac{1}{M_0^2}\left(\vec{M}_0\vec{H}_0^{(i)} + \beta\left(\vec{M}_0\vec{n}\right)^2\right)\vec{m}\right)\right), \tag{1}$$

here $\vec{n}$ is a unit vector along the anisotropy axis of the system (for our system, it coincides with the unit vector $\vec{e}_\xi$). After substituting the magnetization perturbation in a periodic by time form

$$\vec{m}\left(\vec{r},t\right) = \vec{m}_0\left(\vec{r}\right)\exp\left(i\omega t\right), \vec{h}\left(\vec{r},t\right) = \vec{h}_0\left(\vec{r}\right)\exp\left(i\omega t\right) \tag{2}$$

into the Landau-Lifshitz equation (1) and considering $\vec{M}_0 \,||\, \vec{H}_0^{(i)} \,||\, \vec{n} \,||\, \vec{e}_\xi$, $\vec{m}_0 \perp \vec{e}_\xi$ as well as the system symmetry, we obtain the first necessary relation between the magnetization and the magnetic field. In order to obtain the second relation, let us use the magnetostatic approximation (see, e.g., [14]; possibility of using this approximation infers from the fact that the above-described configuration of the current does not create a magnetic field). Considering $\vec{h}$ as a potential field, so that $\vec{h} = -\nabla\Phi$, $\vec{h}_0 = -\nabla\Phi_0$, $\Phi = \Phi_0\left(\vec{r}\right)\exp\left(i\omega t\right)$ (here Φ is a magnetic potential), we can obtain the sought relation from the Maxwell equation $div\vec{h} = -4\pi \cdot div\vec{m}$. Therefore, the sought system of equations can be written in the following form:

$$\begin{cases} i\omega\vec{m}_0 = \gamma\left(M_0\vec{e}_z \times \left(\vec{h}_0 + \alpha\Delta\vec{m}_0 - \left(\beta + \dfrac{H_0^{(i)}}{M_0}\right)\vec{m}_0\right)\right) \\ \Delta\Phi_0 - 4\pi div\vec{m}_0 = 0 \end{cases} \tag{3}$$

Using this system of equations, we can find the sought equation for the magnetic potential.

4. EQUATION FOR THE MAGNETIC POTENTIAL

In order to obtain the equation for the magnetic potential of a spin wave in the system, let us eliminate the magnetization perturbation in the system of equations (3).

Let us substitute the second equation of (3) into the first one. After certain transformations we obtain

$$\begin{aligned} &-\frac{i\omega}{\gamma M_0} div\left[\vec{e}_\xi \times \vec{m}_0\right] = -\Delta\Phi_0 + \\ &+\frac{4}{a^2\left(\xi^2 - \eta^2\right)}\frac{\partial}{\partial\xi}\left(\left(\xi^2 - 1\right)\frac{\partial\Phi_0}{\partial\xi}\right) + \frac{1}{4\pi}\left(\alpha\Delta - \tilde{\beta}\right)\Delta\Phi_0, \end{aligned} \tag{4}$$

here $\tilde{\beta} = \beta + H_0^{(i)} / M_0$. After further transformations one can obtain the sought relation for the magnetic potential in the following form:

$$\begin{aligned} &\left(\frac{\omega^2}{\gamma^2 M_0^2} - \left(\tilde{\beta} - \alpha\Delta\right)\left(\tilde{\beta} + 4\pi - \alpha\Delta\right)\right)\Delta\Phi_0 + \\ &+4\pi\left(\tilde{\beta} - \alpha\Delta\right)\frac{4}{a^2\left(\xi^2 - \eta^2\right)}\frac{\partial}{\partial\xi}\left(\left(\xi^2 - 1\right)\frac{\partial\Phi_0}{\partial\xi}\right) = 0 \end{aligned} \tag{5}$$

Thus, we have obtained an equation for the magnetic potential of the spin excitations described in the previous chapter. Let us find a dispersion relation for these excitations using the above-written equation.

5. DISPERSION RELATION

In order to find the sought dispersion relation, let us substitute a solution of (5) in the form of the combination of the spheroidal functions:

$$\Phi_0(\xi,\eta,\phi) = R(\xi)S(\eta)\exp\left(\pm im\phi\right), \tag{6}$$

here the functions R and S satisfy the following equation:

$$\begin{cases} \dfrac{d}{d\xi}\left(\left(\xi^2 - 1\right)\dfrac{dR}{d\xi}\right) + \left(-\lambda + (ka)^2\left(\xi^2 - 1\right) - \dfrac{m^2}{\xi^2 - 1}\right)R = 0 \\ \dfrac{d}{d\eta}\left(\left(1 - \eta^2\right)\dfrac{dS}{d\eta}\right) + \left(\lambda + (ka)^2\left(1 - \eta^2\right) - \dfrac{m^2}{1 - \eta^2}\right)S = 0 \end{cases}, \tag{7}$$

λ is the variables separation constant and m is an arbitrary integer. The solution (6,7) satisfies the Helmholtz equation $\Delta\Phi_0 = -k^2\Phi_0$ for the function Φ_0. After replacement of the functions R and S with renormalized functions $R(\xi) = R_1(ka\xi)$, $S(\eta) = S_1(ka\eta)$ – where k is a generalized wavenumber – and substituting them into the equation (5) one can obtain

$$\begin{aligned} &-\left(\frac{\omega^2}{\gamma^2 M_0^2} - \left(\tilde{\beta} + \alpha k^2\right)\left(\tilde{\beta} + 4\pi + \alpha k^2\right)\right)k^2\Phi_0 + \\ &+4\pi\left(\tilde{\beta} - \alpha\Delta\right)\frac{4}{a^2\left(\xi^2 - \eta^2\right)} \cdot \\ &\cdot\left(\lambda - (ka)^2\left(\xi^2 - 1\right) + \frac{m^2}{\xi^2 - 1}\right)\Phi_0 = 0 \end{aligned} \tag{8}$$

As it can be seen, the equation (8) contains a variable component $\dfrac{\lambda - (ka)^2\left(\xi^2 - 1\right) + \dfrac{m^2}{\xi^2 - 1}}{a^2\left(\xi^2 - \eta^2\right)}$, so the function (6) is not, generally speaking, a solution of (5). However, it can be considered as an approximate solution in the case when this variable component can be considered approximately constant. In particular, this can be done when the shell is thin, so condition $(\xi_2 - \xi_1)/\xi_1 << 1$ (which is equivalent to the condition $\left(R_{||}^{(2)} - R_{||}^{(1)}\right)/R_{||}^{(1)} << 1$) fulfils and, at the same time, the particle is not strongly prolate, so the condition $\xi_1^2 >> 1$ (or $\left(\left(R_{||}^{(1)}\right)^2 - \left(R_{\perp}^{(1)}\right)^2\right)/\left(R_{||}^{(1)}\right)^2 << 1$, which is equivalent) does not fulfill. In this case,

$$\frac{\lambda - (ka)^2\left(\xi^2 - 1\right) + \dfrac{m^2}{\xi^2 - 1}}{a^2\left(\xi^2 - \eta^2\right)} \approx \frac{\lambda\xi_0^2 - (ka)^2\xi_0^4 + m^2}{a^2\xi_0^4} = const \quad (9)$$

where $\xi_0 = \sqrt{\left(\xi_1^2 + \xi_2^2\right)/2}$. Therefore, after considering $k \sim 1/d$ (the fact that implies from the physical sense of k) and expanding the constant λ into series

$$\lambda = \lambda_{lm} \approx -(ka)^2 + ka\left(2(l-m)+1\right) \approx -(ka)^2 \quad (10)$$

where l, m are the corresponding quantum numbers (this form of expansion is obtained, e.g., in [15]) one can finally obtain an approximate dispersion equation in the form

$$-\left(\frac{\omega^2}{\gamma^2 M_0^2} - \left(\tilde{\beta} + \alpha k^2\right)\left(\tilde{\beta} + 4\pi + \alpha k^2\right)\right)k^2 + 4\pi\frac{ka\left(2(l-m)+1\right)\xi_0^2 - (ka)^2\xi_0^4 + m^2}{a^2\xi_0^4}\left(\tilde{\beta} + \alpha k^2\right) = 0 \quad (11)$$

Therefore, the dispersion relation for this case can be written as follows:

$$\omega = \frac{\gamma M_0}{k}\sqrt{\alpha^2 k^6 + 2\alpha\tilde{\beta}k^4 + 4\pi\frac{\alpha\left(2(l-m)+1\right)}{a\xi_0^2}k^3 + \left(\tilde{\beta}^2 + \frac{4\pi\alpha m^2}{a^2\xi_0^4}\right)k^2 + \frac{4\pi}{a^2\xi_0^4}\left(\tilde{\beta}a\left(2(l-m)+1\right)\xi_0^2 k + \tilde{\beta}m^2\right)} \quad (12)$$

Note that the length of spin waves should be of the same order of magnitude or more with the exchange interaction length (that has an order of several nanometers for typical ferromagnets). At the same time, mean thickness of typical nanoshells has approximately the same order with the exchange length or slightly exceed it. Therefore, for such typical nanoshells only one non-zero "radial" – with $k \approx 2\pi/\left(a\left(\xi_2 - \xi_1\right)\right)$ – can be excited.

6. CONCLUSIONS

Thus, we have investigated dipole-exchange spin excitations (standing spin waves) in a nanorice particle with the shape of a prolate rotation ellipsoid. The particle consists of a non-magnetic core and a shell composed of an "easy axis" uniaxial ferromagnet. For such excitations, a differential equation for the magnetic potential in the magnetostatic approximation with account for the dipole-dipole magnetic interaction, the exchange interaction and the anisotropy effects has been obtained. The equation has been solved for the case of a thin (compared to its size) shell with the semiaxes relation that is close to unity ($\left(\left(R_{||}^{(1)}\right)^2 - \left(R_{\perp}^{(1)}\right)^2\right)/\left(R_{||}^{(1)}\right)^2 << 1$); for this case, a dispersion relation for the above-described spin excitations has been obtained.

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