JOURNAL OF NANO- AND ELECTRONIC PHYSICS *ЖУРНАЛ НАНО- ТА ЕЛЕКТРОННОЇ ФІЗИКИ*
Vol. **7** No 4, 04001(7pp) (2015) Том **7** № 4, 04001(7cc) (2015)

Phase Transition of Polariton and Magnetopolariton in Semiconductor Microcavity

S.C. Kenfack*, A.J. Fotue, N. Issofa, J-R.D. Djomou, A.B. Tchapda, M. Tiotsop, L.C. Fai

Mesoscopic and Multilayers Structures Laboratory, University of Dschang, P.O. Box 479 Dschang, Cameroon

(Received 02 September 2015; published online 10 December 2015)

The real interaction between matter and electromagnetic radiation is too complicated for a complete theoretical investigation. In this paper, we study phase transition of polariton and magnetopolariton in semiconductor microcavity. We have analyzed the polariton and magnetopolariton phase transition via the path integral approach in Dicke model. Numerical results showed that the system exhibits phase transition from normal phase to super-radiant phase. The transition is affected by the coupling term, Matsubara frequencies and temperature. We observed that the sudden transition is closed to absolute temperature, which means that these three parameters have considerable effect on the polariton formation and stability. Additionally, the introduction of the magnetic field in the semiconductor microcavity shows that the system still undergoes a phase transition. It is shown that weak magnetic field does not alter the phase transition significantly, apart from a small shift of the transition point. Compared to other parameters strong magnetic field drastically change sign at the critical temperature.

Keywords: Polariton, Magnetopolariton, Path integral approach, Dicke model.

PACS numbers: 71.36. + c, 71.35.Ji, 31.15.xk

1. INTRODUCTION

The observation of the strong coupling of light with exciton in semiconductor microcavities [1] has been generated much speculation regarding the possibility for low-threshold optical devices [2-3]. Realization of such devices based on the bosonic character of the optical eigenmode [4] of these structures would be a revolutionary step in semiconductor optics. However in 1992, Weisbush et al. [5] renewed the subject by showing that the strong coupling regime between excitons and light can also be reached when a quantum well is inserted in a plane microcavity. When the quantum well is placed in a high finesse microcavity, the strong coupling regime between excitons and light is easily reached, giving rise to exciton-photon mixed quasi particles called Polaritons. Then polariton is a mixed state which has half-light and half-matter. Polaritons were first considered theoretically by Kirill Borisovich Tolpygo [6], and were initially termed light-excitons in Ukrainian and Russian scientific literature. Being mixed exciton-photon quasi particles, the Polaritons have integral spins and can reveal bosonic properties [7] responsible for a number of interesting effects both predicted and observed, namely, stimulated scattering [8], Polariton lasing [9], Bose-Einstein condensation [10], Super fluidity [11] etc. Furthermore, the introduction of a magnetic field in a semiconductor microcavity shows that the polariton is magnetized. This is called Magnetopolariton.

Intense research was carried out in order to determine the life time of polaritons, their dispersion and the different processes which enable their relieving [12-13]. In 1998, Dang et al. measured a non-linear behavior of the emission in a microcavity, while preserving the mode of coupling even under strong excitation [14]. Parametric amplification and oscillation of polaritons were observed in 2000 [15], showing that polaritons behave like bosons with low density. The stake was then to show a phase transition of the polariton, which is the main objective of this work. The Dicke Hamiltonian was shown to exhibit a phase transition between a super-radiant phase with macroscopic occupations in the field of the atoms and a normal phase without excitations at zero temperature. In the context of phase transitions, a collection of two level systems coupled linearly to one scalar bosonic mode undergoes a phase transition from a normal to a super-radiant phase at certain critical coupling strength. This phase transition has been investigated theoretically a long time ago by Hepp and Lieb [16] and also by Wang and Hioe [17]. The super-radiant transition at zero temperature was shown [18-20].

In fact, experimental progress in superconducting circuits allows integrating multiple artificial atoms into micro-cavities. To maintain sufficiently long coherence times, the number of two-level systems in superconducting material must be kept moderate and far below the number of atoms in a cold atomic cloud. Furthermore, environmental degrees of freedom in the form of phonon like photon are always present in these structures. Nevertheless, one may assume that signatures of Dicke physics are observable, at least in certain ranges temperature. Another line of research combines polariton and magnetic field. The effect of the coupling parameter, Matsubara frequencies, temperature and magnetic effects parameters on the polariton have not been fully understood yet, despite their important role in semiconductors materials. The purpose of this work is to contribute towards closing this effect by deriving the energy of the system. Considering the ground state of the Dicke Hamiltonian, it is immediately obvious, that there is a competition between the parameter cited above respectively. This competition depends on the strength of those parameters.

In this work, we study the phase transition of polariton and magnetopolariton lasing in semiconductor microcavities. The problem of phase transitions in polaritonic systems has not yet been discussed in a number of

* kevinsadem@yahoo.fr

papers. Such a phase transition means the sudden change in the properties of a quantum many-body system as a control parameter is varied. We have used the Dicke model to describe the coupling term, Matsubara frequencies, temperature and magnetic effects on the polariton and magnetopolariton.

This paper is organized as follows. In the first section, we present the method which is the Path integral approach via the Dicke model. In the second section, this method is applied to polariton lasing in semiconductor microcavities. In the third section, the same method is also applied to magnetopolariton lasing in semiconductor microcavities. The last part is the conclusion. Throughout the paper we use units with $k_B = \hbar = 1$.

2. PATH INTEGRAL APPROACH TO DICKE MODEL

2.1 The Dicke Hamiltonian

The Dicke model, originally developed to describe a large number N of two-level atoms interacting with a single-mode radiation field [21], has regained substantial interest in the last decade [22]. The phenomenon of phase transition is usually studied in connection with the Dicke Hamiltonian which describes an array of N two-level atoms interacting with a photonic field. It can be written as the sum of three terms, where the first is the atom Hamiltonian, second is the cavity Hamiltonian and the third is the interaction Hamiltonian:

$$\mathrm{H}=\frac{\Omega}{2}\sum_{j=1}^{N}\sigma^{z}{}_{(j)}+\omega_0 b^{+}b+\frac{g}{\sqrt{N}}\sum_{j=1}^{N}(b\sigma^{+}_{(j)}+b^{+}\sigma^{-}_{(j)}) \quad (1.1)$$

In above equation, we defined the operators $\sigma_j^{\pm} = 0.5\left(\sigma_j^x \pm \sigma_j^y\right)$, where the operators σ_x^j, σ_y^j and σ_z^j satisfy the commutation relations $\left[\sigma_j^p, \sigma_j^q\right] = 2i \in^{pqr} \sigma_j^r$ with $p,q,r = x,y,z$ and there are Pauli spin matrices for the j[th] atom. Therefore, $\left[\sigma_+^j, \sigma_-^j\right] = \sigma_z^j$ and $\left[\sigma_j^z, \sigma_j^{\pm}\right] = \pm 2\sigma_j^{\pm}$. The b^+ and b are the creation and annihilation operators for the field that satisfy the usual commutation relation rules and g is the coupling term between the atom and the field measured in units of the field energy . Where ω_0 is the frequency of the mode of the field and is the energy gap between the energy eigenstates of the electron.

2.2 Path Integral Approach

This method has been broadly used in a large series of papers (Sami et al., 2011 [23]; Fai et al., 2010a [24], 2010b [25]), Emanuele et al. (2011) [26]). Full details of the calculation of the energy of the system described in equation (1.1) are given in [Aparicio et al. (2007) [27], Aparicio et al., 2011 [28]. Then, obtaining the Hamiltonian we used the path integral approach to derive the energy of the system. The method used here is identical to that used by Aparicio et al. (2013) [29]. In order to apply the path integral approach with the functional method, first it is necessary to change the atomic pseudo-spin operators of the model by a linear combination of Fermi operators to define the generalized Dicke model. Second, the thermodynamic limit $(N \to \infty)$ where N the number of two is-level atoms must be taken. In the following line, we consider the problem of defining the partition function of the Dicke model. First let's define the Euclidean action S of this model which describes a single quantized mode of the field and the ensemble of N identical two-level atoms. It is given by:

$$\mathrm{S}=\int_0^\beta d\tau \begin{pmatrix} b^*(\tau)\partial_\tau b(\tau)+ \\ +\sum_{i=1}^{N}\left(\alpha_i^*(\tau)\partial_\tau\alpha_i(\tau)+\beta_i^*(\tau)\partial_\tau\beta_i(\tau)\right)\end{pmatrix} - \int_0^\beta d\tau H_F(\tau) \quad (1.2)$$

Where α_i, α_i^+, β_i and β_i^+ satisfy the anti-commutator relations $\alpha_i\alpha_j^+ + \alpha_j^+\alpha_i = \delta_{ij}$ and $\beta_i\beta_j^+ + \beta_j^+\beta_i = \delta_{ij}$. H_F is the Hamiltonian of the fermions Dicke model, this is given by

$$H_F(\tau) = \omega_0 b^*(\tau)b(\tau) + \frac{\Omega}{2}\sum_{i=1}^{N}\left(\alpha_i^*(\tau)\alpha_i(\tau) - \beta_i^*(\tau)\beta_i(\tau)\right) +$$
$$\frac{g}{\sqrt{N}}\sum_{i=1}^{N}\left(\alpha_i^*(\tau)\beta_i(\tau)b(\tau) + \alpha_i(\tau)\beta_i^*(\tau)b^*(\tau)\right) \quad (1.3)$$

Let's define the formal quotient of two functional integrals i.e. the partition function of generalized Dicke model and the partition function of the free Dicke model. Therefore, we are interested in calculating the following quantity:

$$\frac{Z}{Z_0} = \frac{\int[d\eta]e^{S}}{\int[d\eta]e^{S_0}} \quad (1.4)$$

Where S is the Euclidean action of the Dicke mode given by Eq.1.2, S_0 is the free Euclidean action for the free single bosonic mode and the free atoms i.e. the expression of complete action S taking $g = 0$ and $[d\eta]$ is the functional measure. To calculate that quotient, we will use the free action for the single mode bosonic field $S_{BO}(b)$ given by

$$S_O(b) = \int_0^\beta d\tau b^*(\tau)\left(\partial_\tau - \omega_0\right)b(\tau). \quad (1.5)$$

By substituting this equation into the Euclidean action of the Dicke mode and making few developments, we obtained

$$\mathrm{S}=S_0 + \int_0^\beta d\tau \sum_{i=1}^{N}\rho_i^+(\tau)M\left(b^*,b\right)\rho_i(\tau) \quad (1.6)$$

Where

$$\rho_i^+(\tau) = \begin{pmatrix}\beta_i^*(\tau) & \alpha_i^*(\tau)\end{pmatrix}, \ \rho_i(\tau) = \begin{pmatrix}\beta_i(\tau) \\ \alpha_i(\tau)\end{pmatrix}$$

and the matrix

$$M\left(b^*,b\right)=\begin{pmatrix} \partial_\tau+\frac{\Omega}{2} & -N^{-1/2}gb^*(\tau) \\ -N^{-1/2}gb(\tau) & \partial_\tau-\frac{\Omega}{2}\end{pmatrix}$$

Let us find the functional integral form of the partition function of the Dicke model Z.

We know that $Z=\int\left[d\eta\right]e^{S}$. Substituting the action S given by Eq. (1.6) and use the determinant properties we obtain:

$$Z=\int\left[d\eta(b)\right]e^{S_0}\left(\det M\left(b^* \quad b\right)\right)^N \qquad (1.7)$$

this case, $\left[d\eta(b)\right]$ is the functional measure only for the bosonic field. In order to calculate the partition function of the Dicke model, let us use the determinant properties, the development limit in Taylor series for 2 variables in order 2, the functional derivative and its properties, Fourier representation for bosonic field, making certain transformations and using the fact that

$$\left[d\eta\left(b\right)\right]=\prod_{\omega}db(\omega)db^*\left(\omega\right)$$

we obtain:

$$Z=e^{N\Phi\left(b_0^*,b_0\right)}\frac{2\pi i}{\left(S^2\left(0\right)-R^2\left(0\right)\right)}\prod_{\omega\geq1}\frac{\left(2\pi i\right)^2}{S\left(\omega\right)S\left(-\omega\right)-R^2\left(\omega\right)}.\qquad (1.8)$$

We shall calculate the function $S\left(\omega\right)$ and $R\left(\omega\right)$ following the procedure of Aparicio et al. (2007) [30] as follow

$$R\left(\omega\right)=-\frac{\left(\Omega_\Delta^2-\Omega^2\right)\alpha^{-2}}{2\Omega_\Delta\left(\omega^2+\Omega_\Delta^2\right)}\tanh\left(\frac{\beta\Omega_\Delta}{2}\right)\qquad (1.9)$$

$$\begin{aligned}S\left(\omega\right)=i\omega&\left(1-\frac{\Omega\alpha^{-2}}{\Omega_\Delta\left(\omega^2+\Omega_\Delta^2\right)}\tanh\left(\frac{\beta\Omega_\Delta}{2}\right)\right)-\\ &-\omega_0g^2\alpha^2+\frac{\left(\Omega_\Delta^2+\Omega^2\right)\alpha^{-2}}{2\Omega_\Delta\left(\omega^2+\Omega_\Delta^2\right)}\tanh\left(\frac{\beta\Omega_\Delta}{2}\right)\end{aligned}\qquad (1.10)$$

From (Aparicio et al. (2007) we deduced $H\left(\omega\right)$:

$$H\left(\omega\right)=\frac{S\left(\omega\right)S\left(-\omega\right)-R^2\left(\omega\right)}{\omega_0^2+\omega^2}\qquad (1.11)$$

By substituting the expression of the function $S\left(\omega\right)$ and $R\left(\omega\right)$ in the last expression we get

$$\begin{aligned}H\left(\omega\right)=1&+\frac{2\Omega\omega^2g^2-\omega_0g^2\left(\Omega_\Delta^2+\Omega^2\right)}{\Omega_\Delta\left(\omega^2+\Omega_\Delta^2\right)\left(\omega^2+\omega_0^2\right)}\tanh\left(\frac{\beta\Omega_\Delta}{2}\right)+\\ &+\frac{g^4\Omega^2}{\Omega_\Delta^2\left(\omega^2+\Omega_\Delta^2\right)\left(\omega^2+\omega_0^2\right)}\tanh^2\left(\frac{\beta\Omega_\Delta}{2}\right)\end{aligned}\qquad (1.12)$$

Where the parameter ω takes the value $2\pi n/\beta$, with n being all the integers and β is the inverse of temperature. These values correspond to the Matsubara frequencies for bosonic fields, and $\Omega_\Delta=\sqrt{\Omega^2+4g^2\left|b_0\right|^2}$, where is a critical value of the bosonic fields.

3. POLARITON LASING IN SEMICONDUCTOR MICROCAVITIES

3.1 Polariton Energy

Polaritons are half-light, half-matter quasi particles resulting from the strong coupling of the photon mode of a microcavity with an exciton resonance of the embedded semiconductor structure [31]. We consider a system consisting of N identical exciton interacting with the photons in the cavity. The Hamiltonian of a bosonic system, coupled with the reservoir of exciton, in thermal equilibrium at temperature β^{-1} can be written in the following form:

$$H=H_{ph}+H_{ex}+H_{ex-ph}\qquad (2.1)$$

In the above relation, the first term describes a photonic Hamiltonian:

$$H_{ph}=\omega_0a^+a\qquad (2.2)$$

The second term represents an excitonic Hamiltonian:

$$H_{ex}=\frac{\Omega}{2}\sum_{i=1}^{N}\sigma^z{}_{(i)}\qquad (2.3)$$

and the third term describes a Hamiltonian of exciton-photon interaction which is written as:

$$H_{ex-ph}=\frac{g}{\sqrt{N}}\sum_{j=1}^{N}\left(a\sigma_{(j)}^++a^+\sigma_{(j)}^-\right)\qquad (2.4)$$

In these equations, a^+ and a are the photon creation and annihilation operators for the cavity mode frequency ω_0, σ_i^+ and σ_i^- are the pseudo-spin for the ith exciton defined as $\sigma_i^+=\sigma_x+i\sigma_y$ and $\sigma_i^-=\sigma_x-i\sigma_y$ with σ_x,σ_y being the Pauli matrices. g denotes the coupling strength between the exciton and the photon. Then, the generalized Dicke model is defined by:

$$H=\frac{\Omega}{2}\sum_{j=1}^{N}\sigma^z{}_{(j)}+\omega_0a^+a+\frac{g}{\sqrt{N}}\sum_{j=1}^{N}\left(a\sigma_{(j)}^++a^+\sigma_{(j)}^-\right)\qquad (2.5)$$

The collective exciton couples to a photon, single electromagnetic mode of frequency ω_0 in the cavity described by bosonic operators a, a^+ with effective coupling strength $g/\sqrt{N}$. The latter one is conveniently introduced when the density of atoms per unit volume is fixed since then the bare coupling one between any single atom and the atomic mode is effectively reduced with a growing number of atoms. g represents the coupling term between the photon and the exciton and Ω a single spin excitation.

Now we apply the result obtained in section one and derive the energy of polariton. Considering Eq. (1.24), the energy of polariton is written as

$$E(\omega)=1+\frac{2\Omega\omega^2g^2-\omega_0g^2\left(\Omega_\Delta^2+\Omega^2\right)}{\Omega_\Delta\left(\omega^2+\Omega_\Delta^2\right)\left(\omega^2+\omega_0^2\right)}\tanh\left(\frac{\beta\Omega_\Delta}{2}\right)+ \frac{g^4\Omega^2}{\Omega_\Delta^2\left(\omega^2+\Omega_\Delta^2\right)\left(\omega^2+\omega_0^2\right)}\tanh^2\left(\frac{\beta\Omega_\Delta}{2}\right) \quad (2.6)$$

Where the coefficients are defined as follow $\Omega_\Delta=\sqrt{\Omega^2+4g^2|\alpha_0|^2}$ and α_0 represents the Bose field constant. This excitation-energy spectrum of the polariton yields insight into the origin of the phase transition due to the competition between photon modes of a microcavity with an exciton.

3.2 Numerical Results

3.2.1 Temperature Effect on Polariton Energy

Numerical results of the energy are presented in this section. The objective is to analyze how energy behaves with coupling parameter and temperature.

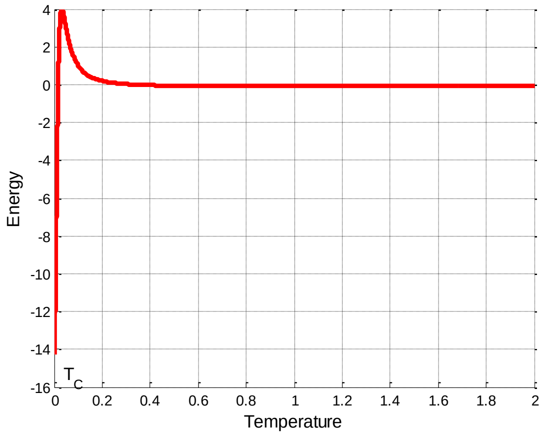

Fig. 1 – Polariton Energy versus Temperature

Figure 1 presents the evolution of Polariton energy as a function of temperature. The value of the Matsubara frequency is defined by $2\pi nT$ but here we have taken the Matsubara frequency as $8\pi T$. We choose $g=1.6$. It is seen that the transition appears, thus we observe that the system undergoes a phase transition from the normal phase (i.e. $T<T_c$) to the super-radiant phase (i.e. $T>T_C$). We can also see that the energy of the polariton in the normal phase is greater than that in the super-radiant phase. The polariton energy decreases with temperature after the critical point (T_C) around the absolute temperature, since the super-radiant phase is a phase which has a stable state. In this state, the polariton state can be detected. This state is very important, the polariton is more localized. We can also observe that in the super-radiant phase the polariton energy vanishes, it means that this quasi-particle is more stable in this phase than a single electron. Now one question is addressed, what will be the effect of Matsubara frequencies on the transitions?

3.2.2 The effect of Matsubara frequencies

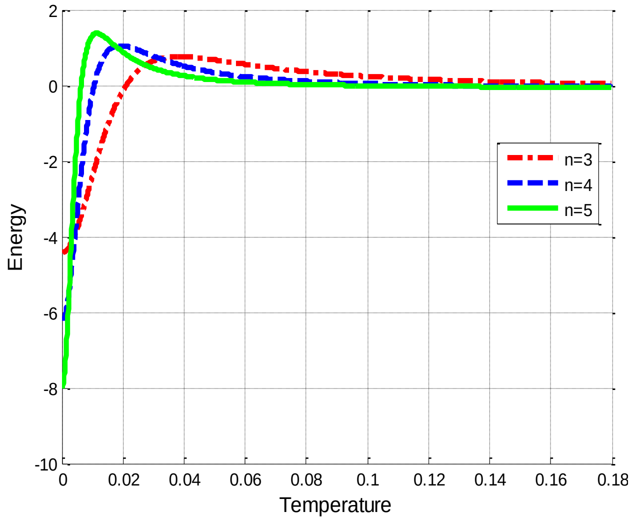

Fig. 2 – Polariton Energy versus temperature for different values of Matsubara frequencies

Figure 2 shows the numerical result of the energy of the polariton as a function of temperature for different values of Matsubara frequencies for weak coupling $(g<1)$. The values of the other parameters remain the same. We chose the coupling term $g=0.35$; n characterizes the different values of Matsubara frequency, since these frequencies are defined as $2\pi nT$. Like previously, we observe that the system also undergoes a phase transition from the normal phase (i.e. $T<T_C$) to the super-radiant phase (i.e. $T>T_C$). Here, we make the same observations as previously. We may also observe that the transition point (T_C) is closed to the absolute temperature. The more the matsubara frequency increases the more the critical shift toward the absolute temperature (for $n=5$, 4, 3 the critical points are $T_{C_5}<T_{C_4}<T_{C_3}$). We may observed that for $T>T_{C_3}$ the energy decreases with matsubara frequency, but

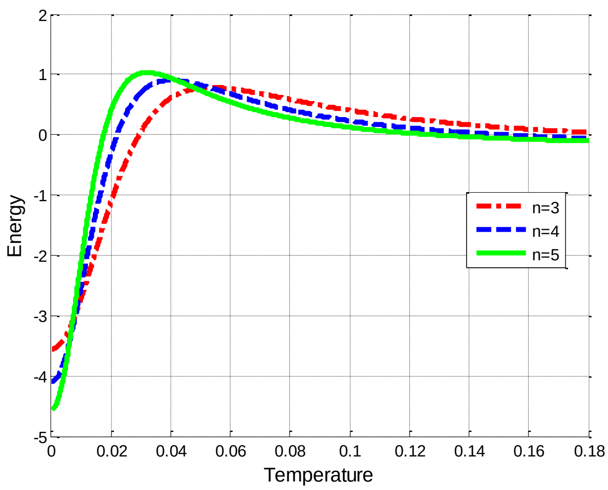

Fig. 3 – Energy of polariton versus temperature for different values of Matsubara frequency

increases for $T < T_{C_5}$. This behavior is different between T_{C_5} and T_{C_3}. For $n = 5$ polariton energy passes through the critical point T_{C_4}, for $n = 4$ it passes through T_{C_3}.

Figure 3 presents the evolution of the polariton energy versus temperature for different values of Matsubara frequency in the case of strong coupling $(g > 1)$. We keep the same values for other parameters. We choose the coupling term $g = 1.6$. In order to study the critical behavior of the polariton, we display the energy versus temperature. Note that the minimum position approaches the critical point, providing a convincing method to locate the critical point. This behavior strongly implies the sudden transition from the broken symmetry phase to the symmetry phase. We see that the energy of the polariton oscillates, thus there is a phase transition from the normal phase (i.e. $T < T_C$) to the super-radiant phase (i.e. $T > T_C$). This figure also presents some critical values of temperature T_C. We observe that the value of the critical temperature decreases with the Matsubara frequencies and the energy of the polariton increases with the increase of Matsubara frequencies. Contrary for the case of weak coupling we have a common point of the polariton energy for the three selected Matsubara frequencies values. Near this point the sign of the polariton energy changes with the Matsubara frequencies.

From the above, we observed that the Matsubara frequencies bring close the transition point from the absolute temperature. The critical temperature is more quickly reached with the increase of the Matsubara frequencies. Then, we are going to test if the breaking of symmetry still remains if there is a magnetic effect on the phase transition.

4. MAGNETOPOLARITON LASING IN SEMICONDUCTOR MICROCAVITIES

4.1 Magnetopolariton Energy

The introduction of a magnetic field in the semiconductor microcavity means that the polariton is magnetized. This is called a Magnetopolariton. Therefore, we can define a Magnetopolariton as a quasi-particle resulting from the strong coupling of the photon mode of a microcavity with an exciton resonance of the embedded semiconductor structure in the presence of a magnetic field. When a microcavity has a magnetic field, the system is composed of a quasi-particle (polariton) and the magnetic field. This field is characterized by the Zeeman energy. We can write the Hamiltonian of system as:

$$H_{Mag} = H_{zee} + H_{ph} + H_{ex} + H_{ex-ph} \qquad (3.1)$$

Here the first term describes the Zeeman energy which is defined as $H_{Zee} = \omega_B \sum_{i=1}^{N} \sigma_{(i)}^{z}$ where $\omega_B = \frac{1}{2}\lambda\mu_B B$ in this last relation, λ is the gyromagnetic factor; μ_B is the Bohr magneton and B is the magnetic field.

The second, third and fourth terms are defined in the previous part. Therefore, the Hamiltonian takes the following form:

$$H_{Mag} = \frac{\chi}{2}\sum_{j=1}^{N}\sigma^{z}{}_{(j)} + \omega_0 a^+ a + \frac{g}{\sqrt{N}}\sum_{j=1}^{N}\left(a\sigma_{(j)}^{+} + a^+\sigma_{(j)}^{-}\right) \qquad (3.2)$$

where $\chi = \Omega + \omega'_B$ with $\omega'_B = \lambda\mu_B B$.

Now we apply the result obtained in section one and derive the energy of Magnetopolariton. Then, the energy of Magnetopolariton is written as:

$$E(\omega) = 1 + \frac{2\chi\omega^2 g^2 - \omega_0 g^2\left(\chi_\Delta^2 + \chi^2\right)}{\chi_\Delta\left(\omega^2 + \chi_\Delta^2\right)\left(\omega^2 + \omega_0^2\right)}\tanh\left(\frac{\beta\chi_\Delta}{2}\right) + \frac{g^4\chi^2}{\chi_\Delta^2\left(\omega^2 + \chi_\Delta^2\right)\left(\omega^2 + \omega_0^2\right)}\tanh^2\left(\frac{\beta\chi_\Delta}{2}\right) \qquad (3.3)$$

Where the coefficients are defined as

$\chi_\Delta = \sqrt{\chi^2 + 4g^2\left|a_0\right|^2}$.

4.2 Numerical Results

4.2.1 Magnetopolariton energy in a weak coupling term

Figure 4 exhibits the evolution of the energy of the magnetopolariton as function of temperature for different values of the magnetic field (weak magnetic field).

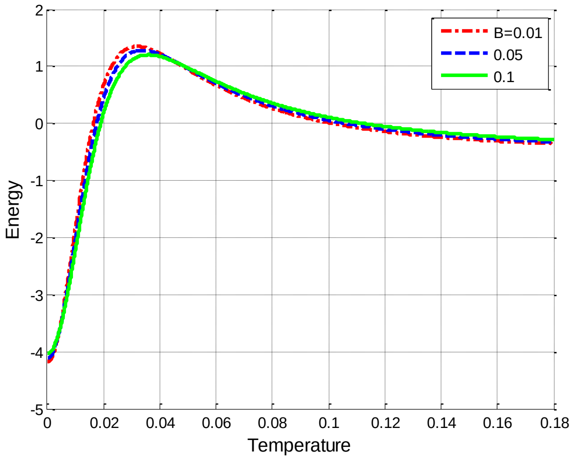

Fig. 4 – Energy of magnetopolariton versus temperature for different values of weak magnetic field

The parameters of this energy remain the same. The value of the weak coupling term is $g = 0.35$. It is seen that there is a transition point. We Observe a common point of the polariton energy for the three selected Matsubara frequencies values. Near this point the sign of the magnetopolariton energy changes with the Matsubara frequencies. A weak magnetic field have a weak impact on the Magnetopolariton.

4.2.1 Magnetopolariton Energy in a Weak Coupling Term

We observe that the system undergoes a phase transition. We may also see that the critical temperature appears at the same point for different values of the weak magnetic field. We also observe that the energy of the magnetopolariton slightly decreases with the magnetic field. We also observe that in the super-radiant phase, the energy of magnetopolariton vanishes. It means that this quasi-particle is more stable in this phase than a single electron. Then, compared to the Matsubara frequencies in the case of weak coupling, the transition points do not change.

4.2.2 Magnetopolariton energy in a strong coupling

Figure 5 exhibits the evolution of the energy of the magneto polariton versus temperature for different values of the magnetic field for strong coupling. The value of coupling term is $g = 1.6$. It is seen that the energy of the Magneto polariton oscillates, and thus, there is a transition point.

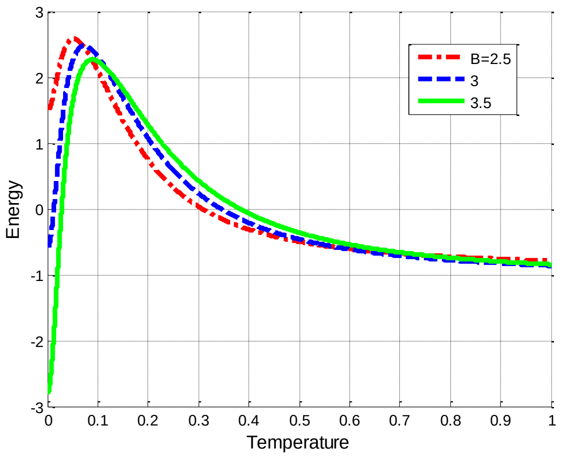

Fig. 5 – Energy of Magnetopolariton versus temperature for different values of strong magnetic field

We observe that the system also undergoes a phase transition. We may also see that the critical temperature appears at different points for different values of the magnetic field, but these critical temperatures are greater than the previous ones. We also observe that the energy of the magnetopolariton decreases with the magnetic field and the transition point moves away with the magnetic field contrary to the effect of Matsubara frequencies. For low temperatures, the behavior is parabolic, but it is linear after $T = 0.7$. Magnetopolariton have strong impact of strong magnetic field.

5. CONCLUSION

In this work, we have analyzed the polariton and magnetopolariton phase transitions via the path integral approach in the full Dicke model. We derived the polariton and magnetopolariton energies. Numerical results demonstrated that an observation of the phase transition from the normal phase (i.e. $T < T_C$) to the super-radiant phase (i.e. $T > T_C$) is indeed feasible for some parameters. It is seen that the critical temperature appears. We know that, the critical temperature is the stability and equilibrium point-driven phase transition. We have also observed that, when the coupling term is zero, nothing happens to our system, meaning that the exciton and photon interaction strongly affects the equilibrium states of the polariton. In other words, the Matsubara frequencies bring close the transition point from the temperature absolute. The critical temperature is more quickly reached with the increase of the Matsubara frequencies. The transition is thus affected by the coupling term, Matsubara frequencies and temperature. One of the important results is that the sudden transition is close to zero, which means that these three parameters have a strong effect on the polariton formation and stability.

However, when we applied a magnetic field in the semiconductor microcavity, there was still a phase transition. In a weak magnetic field, the transition point appears at the same point, while in a strong magnetic field, that critical temperature appears at different points of transition but the energy of the Magnetopolariton decreases with the magnetic field. Thus, the magnetic field does not affect the transition phase but makes a slight shift the critical temperature. It is obvious that the weak magnetic field does not have strong effect on the changing of the polariton phase even if its effect on the polariton amplitude is important whereas the Matsubara frequencies bring close the transition point from the temperature absolute. Strong magnetic field drastically decreases the polariton energy after the critical.

REFERENCES

1. C. Weisbuch, M. Nishioka, A. Ishikawa, Y. Arakawa, *Phys. Rev. Lett.* **69**, 3314 (1992).
2. A. Imamoglu, R.J. Ram, S. Pau, Y. Yamamoto, *Phys. Rev. A* **53**, 4250 (1996).
3. J.J. Baumberg, P.G. Savvidis, R.M. Stevenson, A.I. Tartakovskii, M.S. Skolnick, D.M. Whittaker, J.S. Roberts, *Phys. Rev. B* **62**, R16247 (2000)
4. L.C. Andreani, *Electrons and photons* (Ed. by E. Burstein, C. Weisbush) (Plenum press: New York: 1995).
5. C. Weisbuch, Mr. Nishioka, A.I. Shikawa, Y. Arakawa, *Phys. Rev. Lett.* **69**, 3314 (1992).
6. K.B. Tolpygo, *Zh. Eks. Teor. Fiz.* **20** No 6, 497 (1950) [in Russian].
7. M. Richard, J. Kasprzak, R. Andre, R. Romestein, Le Si Dang, G. Malpuech, A.V. Kavokin, *Phys. Rev. B* **72**, 201301 (2005).
8. P.G. Savvidis, J.J. Baumberg, R.M. Stevenson, M. Skolnick, D.M. Whittaker, J.S. Roberts, *Phys. Rev. Lett.* **84**, 1547 (2000).
9. A. Imamoglu, J.R. Ram, S. Pau, and Y. Yamamoto, *Phys. Rev. A* **53**, 4250 (1996).
10. S. Christopoulos, G. Baldassarri Höger von Högersthal, A.J.D. Grundy, P.G. Lagoudakis, A.V. Kavokin, J.J. Baumberg, G. Christmann, R. Butté, E. Feltin, J.-F. Carlin, N. Grandjean, *Phys. Rev. Lett.* **98**, 126405 (2007).
11. I. Carusotto, C. Ciuti, *Phys. Rev. Lett.* **93**, 166401 (2004).
12. R. Houdré, C. Weisbuch, R.P. Stanley, U. Oesterle, P. Pellandini, M. Ilegems, *Phys. Rev. Lett.* **73**, 2043 (1994).
13. S. Pau, et al., *Phys. Rev. A* **54**, R1789 (1996).
14. L.S. Dang, D. Heger, R. André, F. Bœuf, R. Romestain, *Phys. Rev. Lett.* **81**, 3920 (1998).

15. R.M. Stevenson, et al., *Phys. Rev. Lett.* **85**, 3680 (2000).
16. K. Hepp, E.H. Lieb, *Physica A* **8**, 2517 (1973).
17. Y.K. Wang, F.T. Hioe, *Phys. Rev. A* **7**, 831 (1973).
18. C. Emary, T. Brandes, *Phys. Rev. E* **67**, 066203 (2003).
19. C. Emary, T. Brandes, *Phys. Rev. Lett.* **90**, 044101 (2003).
20. N. Lambert, C. Emary, T. Brandes, *Phys. Rev. Lett.* **92**, 073602 (2004).
21. R.H. Dicke, *Phys. Rev.* **93**, 99 (1954).
22. J.A. Mlynek, A.A. Abdumalikov, C. Eichler, A. Wallraff, *Nature Comm.* **5**, 5186 (2014).
23. Sami Mohammad Al-Jaber, *J. Electromagn. Analys. Appl.* **3**, 184 (2011).
24. L.C. Fai, A. Fomethe, A. Fotue, S.C. Kenfack, J.T. Diffo, V.B. Mborong, S. Domngang, *J. Computation. Theoretic. Nanosci.* **7**, 1501 (2010).
25. L.C. Fai, A. Fomethe, V.B. Mborong, S.C. Kenfack, J.T. Diffo, S. Domngang, Ashok Mody, *Superlattice. Microstruct.* **47**, 631 (2010).
26. Emanuele G. Dalla Torre, et al., *Phys. Rev. A* **87**, 023831 (2013).
27. M. Aparicio Alcalde, A.L.L de Lemos, N.F. Svaiter, *J. Phys. A: Math. Theor.* **40** 11961 (2007).
28. M. Aparicio Alcalde, B.M. Pimentel, *Physica A* **390**, 3385 (2011).
29. M. Aparicio Alcalde, B.M. Pimentel, *Physica A* **392**, 3765 (2013).
30. M. Aparicio Alcalde, A.L.L. de Lemos, N.F. Svaiter, *J. Phys. A: Math. Theor.* **40**, 11961 (2007).
31. A.V. Kavokin, G. Malpuech, *Cavity Polariton* (Elesevier: Amsterdam: 2003).