Modeling Oscillations of Magnetizable Microdrops

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Developed a system of computer simulation of oscillations of magnetizable microdrops in a wide range of changing their parameters: surface tension, viscosity, magnetic permeability, density, and radius. Computational experiments of oscillations of magnetizable drops in an alternating magnetic field and the influence of various forces of nature (inertial, viscous, surface and magnetic) on the nature of the oscillations were carried out. Adequacy of the model, used as the basis for the developed system of computer simulation was shown on the basis of computational and experimental data.

Keywords: Magnetized drops, Magnetic fluids, Oscillations of drops, Modeling, Computational experiment.

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1. INTRODUCTION

Development of systems for computer modeling of oscillations of magnetizable drops is a topical problem related to the study of the sensitivity of their free surface to magnetic fields, for contactless control of magneto-fluidic nano disperse systems, which are currently widely used in control and measuring systems, technical devices and processes. Mathematical modeling of the processes occurring at the interfaces between fluids with different magnetic and electrical properties is a complex nonlinear problem of magnetic hydrodynamics and is not only practical but also scientific interest related to the development of the theory of dispersed phase equilibrium nanosystems. There are several approaches to solving this problem, one of them is based on the fact that the shape of the interphase boundary is unknown in advance and must be obtained in the process of solving the problem [1, 2]. This approach is associated with large computational complexity and is not acceptable when solving practical problems of managing fluid dispersed nanosystems in real-time. In another approach, the drop shape is considered to be known beforehand [3-6]. In the works of various authors [7-9] as a result of the larger number of experiments show that the shape of the magnetizable drops in a weak magnetic field corresponds to an ellipsoid of rotation. Using this fact allows to simplify the simulation and obtain results that can be used in engineering calculations.

2. DESCRIPTION OF THE OBJECT AND METHODS OF INVESTIGATION

To build a system of computer simulation of oscillations of magnetizable drops in a wide range of changes in the parameters of the liquid and the external field used theoretical description of the change in the shape of a magnetizable drops based on the energy method obtained in [10] based on the assumptions that the nature of the magnetization is linear and corresponds to the shape of a drop is elongated along the field ellipsoid of rotation. System of equations describing oscillations of magnetizable drops recorded in the form convenient for system development computer modeling of the form:

$$
\begin{cases}
\begin{aligned}
\frac{dV_\lambda(t)}{dt} &= \frac{2}{3} \cdot \frac{\lambda(t)^2 + 2}{\lambda(t) \cdot \left(2\lambda(t)^2 + 1\right)} \left(V_\lambda(t)\right)^2 - \frac{60 \cdot \eta}{\left(\rho_1 + \frac{\rho_2}{2}\right) \cdot r^2} \cdot \frac{\lambda(t)^{2/3}}{\left(2\lambda(t)^2 + 1\right)} \cdot V_\lambda(t) + \\
&- \frac{45 \cdot \sigma}{4r^3\left(\rho_1 + \frac{\rho_2}{2}\right)} \cdot \frac{\lambda(t)^3}{\left(2\lambda(t)^2 + 1\right) \cdot (\lambda(t)^2 - 1)} \left(\frac{\lambda(t)^2 - 4}{\sqrt{\lambda(t)^2 - 1}} \arcsin\left(\frac{\sqrt{\lambda(t)^2 - 1}}{\lambda(t)}\right) + \frac{2}{\lambda(t)^2} + 1\right) - \\
&- \frac{45\mu_0\left(\mu_i - \mu_e\right)^2 H(t)^2 \left(\lambda(t)^2 - 1\right)^2}{2r^2\left(\rho_1 + \frac{\rho_2}{2}\right)\left(\lambda^{-8/3} + 2\lambda^{-2/3}\right)} \times \\
&\frac{\left[\left(\ln\left(2\lambda(t)^2 + 2\lambda(t)\sqrt{\lambda(t)^2 - 1} - 1\right) - 2\sqrt{1 - \lambda(t)^{-2}}\right) \cdot \left(2\lambda(t)^2 + 1\right)\left(\lambda(t)^2 - 1\right)^{-3/2} - 2\lambda(t)^{-1}\right]}{\left(\lambda(t)(\mu_i / \mu_e - 1)\left(\ln\left(2\lambda(t)^2 + 2\lambda(t)\sqrt{\lambda(t)^2 - 1} - 1\right) - 2\sqrt{1 - \lambda(t)^{-2}}\right) + 2\left(\lambda(t)^2 - 1\right)^{3/2}\right)^2};
\end{aligned} \\
\qquad\qquad\qquad \frac{d\lambda(t)}{dt} = V_\lambda(t); \\
\qquad\qquad\qquad H(t) = H(t + T)
\end{cases}
\tag{1}
$$

Magnetic fluid microdrop has the following parameters: viscosity η; surface tension σ; density ρ_1; radius r; the magnetic permeability μ_i, and the magnetic fluid, in which is placed a drop, has a density of ρ_2 and the permeability μ_e.

The state of the magnetizable drops in the magnetic field strength $H(t)$, characterized by a parameter $\lambda(t)$ that represents the ratio of the semiaxes of the ellipsoid of rotation $a(t)$ and $b(t)$ with equal $\lambda(t) = a(t)/b(t)$ speed $\dot{\lambda}(t) = V_\lambda(t)$ and acceleration of the $\ddot{\lambda}(t) = \partial V_\lambda(t)/\partial t$.

The strength of the external magnetic field $H(t) = H(t+T)$ acting on the drop can vary as sinusoidal $H(t) = H_0 + H_a \cdot \sin(2\pi \cdot f \cdot t + \phi)$, and provide a sawtooth change $H(t) = (H_0 + H_a) - k \cdot t$ for $0 \le t \le T$.
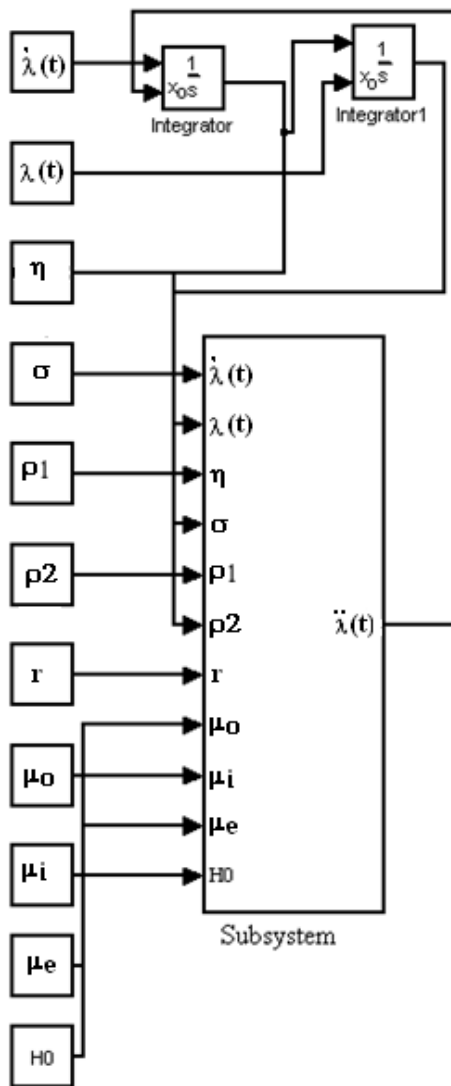
3. COMPUTER SIMULATION SYSTEM

Computer simulation system (Fig. 1) is implemented in Simulink matrix laboratory Matlab. The Simulink allows you to use modern technology visually-oriented programming for simulation block modeling of various systems and devices. Basic computer simulation system contains a subsystem "Subsystem". The subsystem "Subsystem" (Fig. 2) contains four modules to determine the contribution of forces of different nature on the nature of the oscillations.

The subsystem "1" represents a module, to calculate the first term of the right part of the first equation of the system of equations (1), the result of the calculation shown in the figure as $E(\lambda)$.

The second and the third module allow calculating respectively the second and third summands of the right part of the first equation of the system of equations (1). The result of the calculation of the second module is indicated in figure 2, as $E(\eta)/E(\rho)$.

It describes the action of viscous and inertial forces on the drop and is output to the evaluation unit Scope.
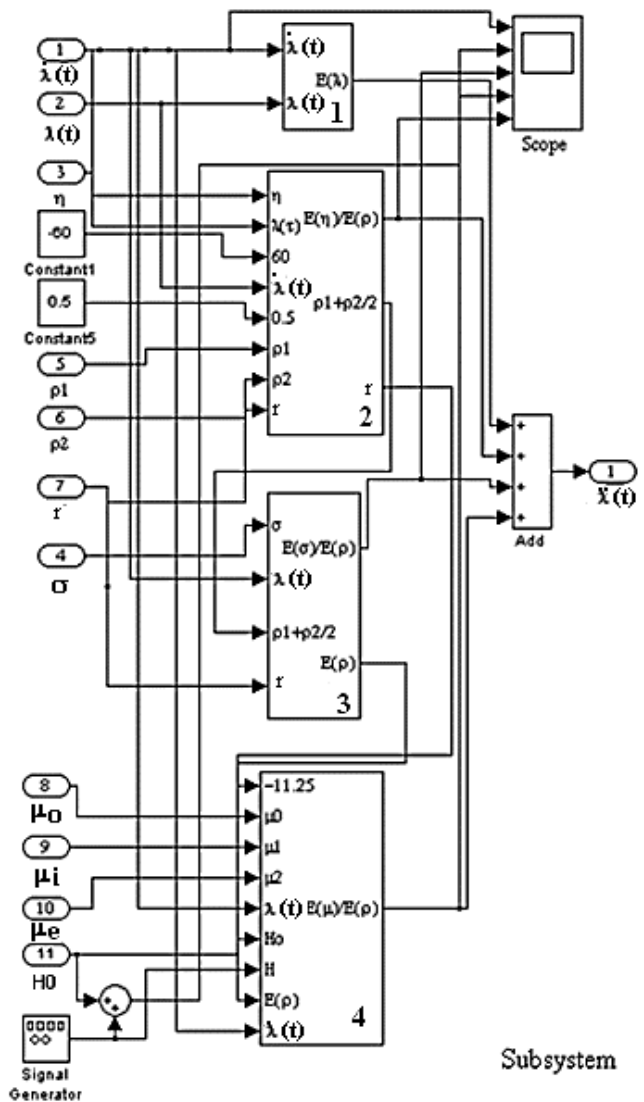

Fig. 1 – Diagram of computer simulation

Fig. 2 – Diagram "Subsystem"

$$\frac{E(\eta)}{E(\rho)} = \frac{60 \cdot \eta}{\left(\rho_1 + \dfrac{\rho_2}{2}\right) \cdot r^2} \cdot \frac{\lambda(t)^{2/3}}{\left(2\lambda(t)^2 + 1\right)} \qquad (2)$$

The result of the calculation of the third module is "3" (Fig. 2) $E(\sigma)/E(\rho)$, characterizes the effect of surface and inertial forces on the drop and also displayed on the device Scope.

$$\frac{E(\sigma)}{E(\rho)} = \frac{45 \cdot \sigma}{4r^3\left(\rho_1 + \dfrac{\rho_2}{2}\right)} \cdot \frac{\lambda(t)^3}{\left(2\lambda(t)^2 + 1\right) \cdot (\lambda(t)^2 - 1)} \times$$
$$\times \left(\frac{\lambda(t)^2 - 4}{\sqrt{\lambda(t)^2 - 1}} \arcsin\left(\frac{\sqrt{\lambda(t)^2 - 1}}{\lambda(t)} \right) + \frac{2}{\lambda(t)^2} + 1 \right) \qquad (3)$$

$$\frac{E(\mu)}{E(\rho)} = \frac{45\mu_0\left(\mu_i - \mu_e\right)^2 H(t)^2 \left(\lambda(t)^2 - 1\right)^2}{2r^2\left(\rho_1 + \dfrac{\rho_2}{2}\right)\left(\lambda^{-8/3} + 2\lambda^{-2/3}\right)} \times$$

$$\times \frac{\left[\left(\ln\left(2\lambda(t)^2 + 2\lambda(t)\sqrt{\lambda(t)^2 - 1} - 1\right) - 2\sqrt{1 - \lambda(t)^{-2}}\right) \cdot \left(2\lambda(t)^2 + 1\right)\left(\lambda(t)^2 - 1\right)^{-3/2} - 2\lambda(t)^{-1}\right]}{\left(\lambda(t)(\mu_i / \mu_e - 1)\left(\ln\left(2\lambda(t)^2 + 2\lambda(t)\sqrt{\lambda(t)^2 - 1} - 1\right) - 2\sqrt{1 - \lambda(t)^{-2}}\right) + 2\left(\lambda(t)^2 - 1\right)^{3/2}\right)^2} \qquad (4)$$

The result of the calculation of the fourth module $E(\mu)/E(\rho)$ (Fig. 2) is registered by the instrument Scope and characterizes the action of the magnetic and inertial forces on the drop. Signal generated by the device Generator.

4. COMPUTER SIMULATION

The developed system allows studying the nature of the oscillations of magnetizable microdroplets in a wide range of changes in interfacial tension σ from 10^{-7} N/m to 10^{-3} N/m and the strength of magnetic field acting on the drop, like a sine and a sawtooth. If the magnetic field acting on the microdrop, larger than the critical field jumpwise elongation H_1, the computer simulation system allows obtaining the hysteretic nature of forced oscillations. When $H(t) < H_1$ the result of numerical simulation correspond to nonhysteretic nature of forced oscillations.

Numerical simulation adequately describes obtained [11] when $H_0 > H_1$, $f = 0.02$ Hz and $f = 0.03$ Hz experimental results. Time dependence $\lambda(t)$ is periodic, microdroplets have time to change shape during the half-period of oscillations of the external field, the period of time when $\lambda(t) \approx 1$ is reduced with increasing frequency.

Figures 3 -5 present the simulation results of forced oscillations microdroplets with parameters: $\rho_1 = 1900$ kg/m³; $\rho_2 = 1060$ kg/m³; $\mu_e = 1$; $\mu_i = 50$; $\sigma = 3{,}7 \cdot 10^{-7}$ N/m, $\eta = 0{,}068$ Pa·s. Curves 1 correspond to the values $\lambda(t)$; curves 2 correspond to the shape of an external magnetic field; curves 3, 4 и 5 present respectively the dependency relations $E(\sigma)/E(\rho)$, $E(\mu)/E(\rho)$ and $E(\eta)/E(\rho)$ from time.

Fig. 3 presents the simulation results obtained in the sawtooth field at $H_0 = 0$; $H_a = 160$ A/m, $r = 5$ mkm.

Fig. 4 presents the simulation results obtained in sinusoidal field at $H_0 = 0$; $H_a = 240$ A/m, $r = 5{,}6$ mkm.

Fig. 5 shows the simulation results obtained in the alternating magnetic field with magnetic bias when $H_0 = 80$ A/m; $H_a = 240$ A/m; $f = 0{,}02$ Hz, $r = 5{,}6$ mkm.
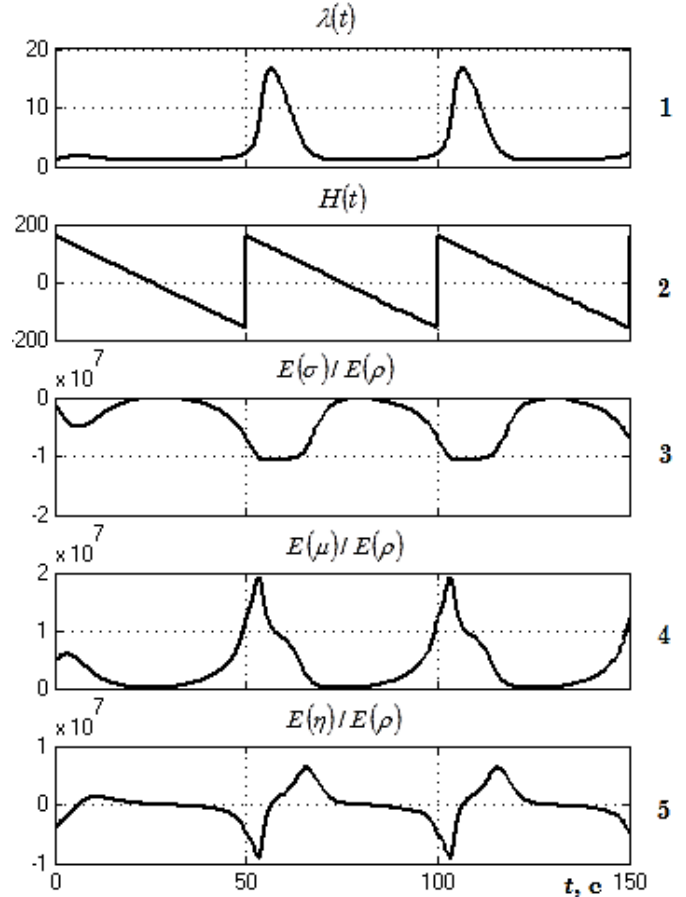

Fig. 3 – Time functions in a sawtooth magnetic field

5. CONCLUSION

The calculation results which are shown in figures 3-5 correspond to the conception of the competition between capillary and magnetic stresses. Minimum instantaneous values $\lambda(t)$ correspond to a maximum contribution of surface forces.
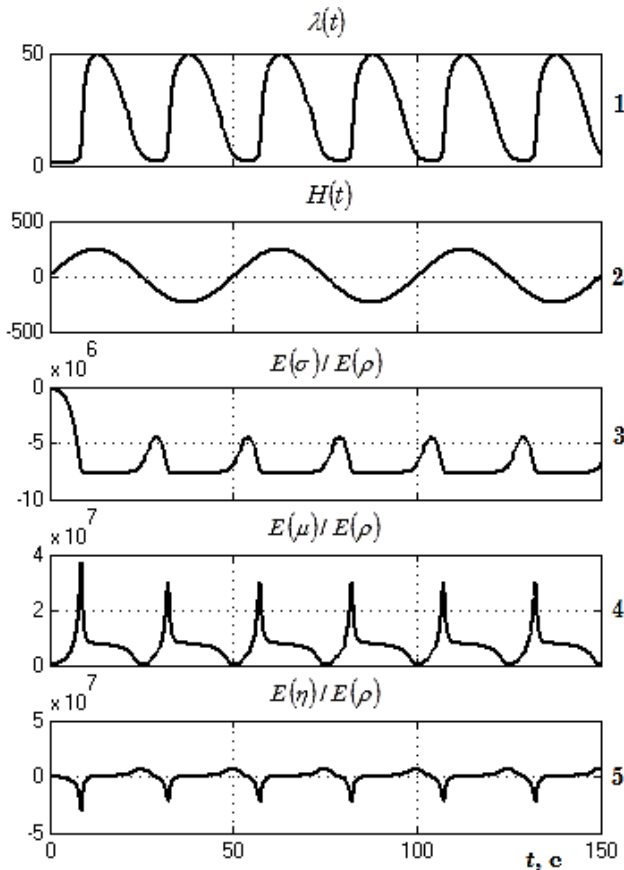
Fig. 4 – Time functions in a sinusoidal magnetic field

Fig. 5 – Time functions in a alternating magnetic field with magnetic bias

The use of a sinusoidal field with a magnetic bias allows us to manage the form of droplets so as, during halftime of the external sinusoidal field on microdrop to the sum of fixed and variable components, and in another of half their difference.

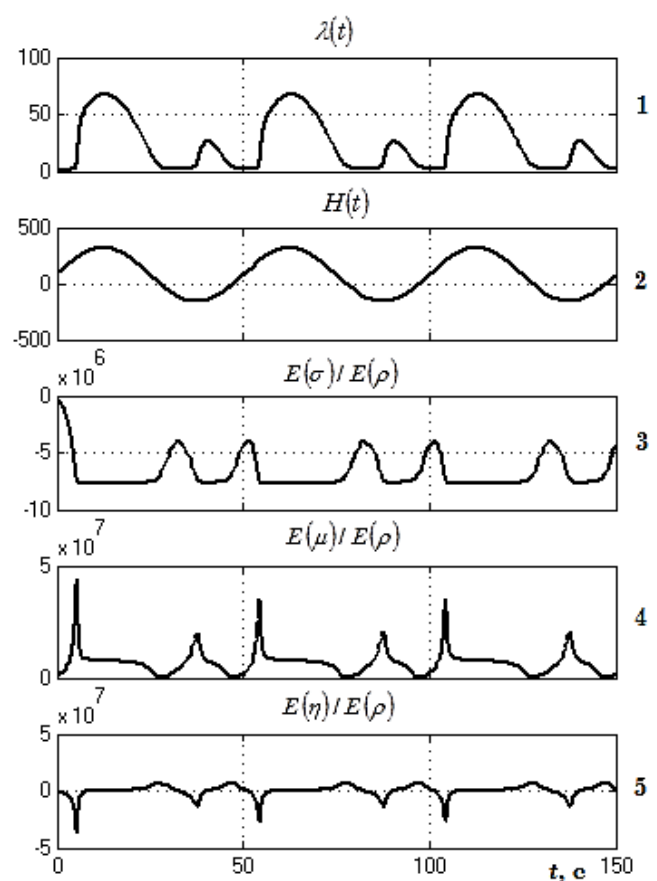The increase of instantaneous values $\lambda(t)$ corre-

sponds to an increase of the contribution of magnetic forces to the sum of the forces acting on microdrops. The maximum value of the ratio of viscous and inertial forces corresponds to a transition of microdrops to its undeformed state in which the influence of inertial forces is minimal.

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