APPLYING ECONOMICS AND MATHEMATIC METHODS FOR SYSTEM OF COMMODITY TURNOVER OPTIMIZATION AT MANUFACTURING AND TRADE ENTERPRISES

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Introduction

Market offers to the particular manufacturer different opportunities for products realization and restrict from some actions at the same time. Manufacture who is interested in effective sailing out, must organize effective system of distribution, which would be aimed at decreasing general logistics expenses and expenses for inventories. Indeed, in modern conditions of market economy using elements of logistic concepts to make goods turnover function efficiently and establishing system in order to determine the optimal level of the cost of the inventory in warehouses and during transportation of products is one of the most important prerequisites for the success of each manufacturer. That is why, to improve formation of concrete advantages we need to plan, form, organize, regulate, manage and control functioning of the commodity turnover system productively. That will ensure meeting needs and demands of consumers and generating a profitability of enterprise.

While working out the most perspective variants of turnover system development at the manufacturing and trade enterprises scientific importance to economics and mathematic methods applying is growing. That opens wide opportunities for taking scientifically based decisions

1. The latest scientific works analysis and aim setting

Theoretical and methodological principles of formation and management of goods turnover are highlighted in the works of such foreign and native scientists as: V. Apopy, I. Mishchuk, V. Rebytskyy[1], L. Balabanova, A. Germanchyuk[2], D. Bauersoks, J. Kloss[3], A. Voychak[4], A. Hadzhynskyy[5], N. Holoshubova[6], E. Holubin[7], M. Gordon, S. Karnaukhov[8], A. Kalchenko[9], M. Oklander[10]and others. The usage of economic and mathematical modeling and mathematical programming for optimal planning of economic processes studded by such scientists as A. Beh, T. Horodnia, A.

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[1] *Organizaciya torgivli: pidruchnik* / V.V. Apopij, I.P. Mishhuk, V.M. Rebickij ta in. / Za red. V.V. Apopiya. 2-ge vid., pererob. ta dop. K.: Centr navchalnoi literaturi, 2005.

[2] Balabanova L.V. *Kommercheskaya deyatelnost': marketing i logistika.* / L.V. Balabanova, A.N. Germanchuk. Doneck: DonGUET im. M. Tugan-Baranovskogo, 2003.

[3] Donald J. Bowersox, David J. Closs *Logistical Management: The Integrated Supply Chain Process* 2nd edition. M.: Olimp-Business, 2005.

[4] Vojchak A.V. *Suchasni tendenciï rozvitku kanaliv rozpodilu*. Marketing v Ukraïni. 2000. nr 2.

[5] Gadzhinskij A.M. *Logistika.* M.: 2012.

[6] Goloshubova N.O. *Organizaciya torgivli. K.: Kniga, 2004.*

[7] Golubin E.D. *Distribuciya. Formirovanie i optimizaciya kanalov sbyta*. M.: Vershina, 2006.

[8] Gordon M.P. *Logistika tovarodvizheniya* / M.P. Gordon, S.B. Karnauhov. M.: Centr ekonomiki i marketinga, 2001.

[9] Kalchenko A.G. *Logistika.* K.: KNEU, 2004.

[10] Oklander M.A. *Logistika.* K.: Centr uchbovoi literaturi. – 2008.

Shcherbakov[11], M. Buhir[12], M. Hlushyk, I. Kopych, A. Pentsak, V. Sorokivskyy[13], A. Zamkov, A. Tolstopyatenko, Y. Cheremnykh[14], V. Zarubin[15], M. Kuchma[16], Ulyanchenko O. [17]and others.

However, processes of improvement of goods turnover at manufacturing and sailing enterprises are still not developed enough. The main aim of this article is optimization of the system of manufacturing and sailing turnover at the enterprise by using elements of logistic conception under competitive conditions. Subject of study is presented by processes of manufacturing and sailing turnover at the enterprise as a component of logistic system. Object of study is both a complex of scientific and methodological methods of formation and improvement of turnover system and decrease of general logistic costs of the enterprise.

To reach the aim the following tasks were set:

- To analyze, systematize and justify choice of economic and mathematic models and methods for supplying and distribution system;
- Considering level of economic development and specific of manufacturing and trade enterprises functioning to optimize turnover system by solving transporting problem of linear programming by method of potentials.

2. The essence and peculiarities of economical and mathematic modulation

Applying methods of economic and mathematic modeling demands solving numerous economical tasks, for example: distribution of limited resources, location of enterprises, improvement of manufacturing structure, optimization of company`s program, technology, stocks, cutting materials, consistency, transporting expenses and others. All points mentioned are very important for successful evolution of modern firms and add to the whole picture of the company organization.

As stated at source [15]economic and mathematic models commonly solved in the following order:

1. Gathering information needed and creating conceptual model. The correctly formulated model is a preliminary, approximate representation of the object or process, which captures the most important parameters and how they interact. On this stage the main aims, variants and alternatives of how to realize them and, general requirements and limitations are identified.

2. Taking into consideration the peculiarities of problems statement, logistics and structural links, correlations of quantity and functional relations should be stated at this level. That would let to check if previously set hypothesis were right.

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[11] *Zbirnik zadach z matematichnogo programuvannya.* / O.V. Beh, T.A. Gorodnya, A.F. Shherbak. Lviv: Magnoliya, 2007.

[12] Bugir M. K. *Matematika dlya ekonomistiv. K. : Akademiya. 1998.*

[13] *Matematichne programuvannya* / M.M. Glushik, I.M. Kopich, O.S. Pencak, V.M. Sorokivski. Lviv: "Novij-Svit- 2000" 2006.

[14] Zamkov O. O. *Matematicheskie metody v ekonomike* / O. O. Zamkov, A. V. Tolstopyatenko, Yu. N. Cheremnyh. M. : DIS, 2004.

[15] Zarubin V.S. *Matematicheskoe modelirovanie*. M.: MGTU im. N.E. Baumana, 2001.

[16] Kuchma M.I. *Matematichne programuvannya: prikladi i zadachi*. L'viv: «Novij Svit-2000», 2007.

[17] Ulyanchenko O.V. *Doslidzhennya operacij v ekonomici*. Harkiv: Grif, 2002.

The correctness of model building is checked at all stages of models design, starting with designing models and finishing with its approbation. If the model is used to make a decision on possible energy consumption and production in the region, some inaccurate is allowed. The correctness of the solutions will satisfy necessary productive economic conditions of in a particular region. In case if model is applied to concrete enterprise, consumers and manufactories it must be precise[18].

3. The rightness of choice is identified by kind of tasks and subject area for which the algorithm feats best. When using mathematic models you will find a solution with the help of checked optimization methods[19].

4. On the last stage of models adjustment required changes and correctives are made. The major form of research is to taste models on practice changing conditions of models functioning and observing results. To ensure correctness of examination important to check correctness of results received during computer imitation. Researcher checks out results after making sure that model was constructed correctly on the previous level.

Accurateness of resaved results can be evaluated by comparing with real information. That requires experimental confirmation and enlargement of the legal framework of research.

5. In the end, we need to analyze results of solutions, evaluate correctness and preciseness of information, calculations and any other information received. Original model becomes better.

Development of economic and mathematic modulation leads to creation great variety of mathematic models for solving economical, managerial and technological processes[13].

3. Characteristics and peculiarities of choosing models and methods economical and mathematic modulation

While classifying economic and mathematical models (Table 1), it worth to remember that different approaches to creating models and methods of solution exist. We compared analysis of economic and mathematical models, the use of which is always defined by essence of setting objectives (Table 1).

Table 1. Characteristics of economic and mathematic models

Model	*Advantages*	*Disadvantages*
1	2	3
According to the purpose		
Theoretical and analytical	Used in researches general attributes and regularity of economical processes, characterizes functional connections between expenses and amount of production manufactured	Model does not solve real economic problems; it is impossible to choose criteria of optimality
Practical	Used when solving concrete economical tasks (models of economic analyses, predictions and	It is impossible to choose criteria of optimality

[18] Zelenskij A.N. *Osnovy matematicheskogo modelirovaniya*. K.: UMKVO, 1995.
[19] Katrenko A.V. *Doslidzhennya operacij*. Lviv: "Magnoliya 2006", 2007.

	management)	

1	2	3
According to depiction methods of internal and external parties		
Functional	Used in economic regulations(productive function)	It is impossible to choose criteria of optimality
Structural	Used while studding correlations between fields of manufacturing (models of fields relations)	It is impossible to choose criteria of optimality
According to description of reality		
Descriptive	Used for description and explanation of facts(real situation) or forecasts	It is impossible to choose criteria of optimality
Prescriptive	Used for finding something needed (subject, for example)	It is impossible to choose criteria of optimality
According to way of integration with the external environment		
Opened	Interaction of process with external environment used frequently	It is impossible to choose criteria of optimality
Closed	Influence micro environment and its factors; used in linear programming	Impact of the environment is not considered
According to impact of the external environment		
Macro models	Formalizes economy as a single unit; analyzes the structure and dynamics; forecasts its trends; determines an effectiveness of state economic regulation; possible to choose the most effective option economic development of country	Internal relations and structure f the object are usually ignored. On this level only incoming and outgoing connections and their mutual dependence are studied. Operates cost indicators (for example gross investments) It is impossible to choose criteria of optimality.
Micro model	Characterized by comprehensive technical and economical indexes. Used for planning and operating in particular enterprises or associations (reasonable distribution of resources, replacement of equipment, control for resources, service, transportation costs, etc.)	Without influence of macro-economic factors; cannot chose criteria of optimality
According to depiction of relations between causes and consequences		
Determination	Tasks in case of total; elements of probability are missed, the number of outputs defined by inputs; is a part probability and stochastic models	Does not involve accidental variables and options, which take values in accordance with the distribution function; impossible to choose criteria of optimality.
Stochastic	Tasks in conditions of inaccurate information, uncertainty and risks	It is impossible to choose criteria of optimality. The probability to obtain any possible accurate results is equal to 0
According to the influence of temporary factors		
Static	Considered the same processes of problem solutions; used in linear models of resources distribution	Without of time criterion; impossible to choose criteria of optimality

1	2	3
Dynamic	We consider complex solution search process, taking into account time factors; changes in the economic process in some period of time are observed. This processes are used in linear models of enterprise resource allocation	It is impossible to choose criteria of optimality.
According to relations between variables		
Linear	Using the best approach to planning and management, flexibility, alternative industrial and economic situations, routing; ability to choose criteria of optimality that meets the objectives of the problem: maximum profit, profitability, minimum cost, etc. Method takes into account possibility of internal and external conditions of production activity; simple to solve and obtaining optimization solutions; economic opportunity interpretation; universal method of solution (simplex method) exists, the traffic problem can be solved by potentials or Hungarian method, etc. Solves "the problem of choice" minimizing during the goods or cargo transportation; imposes an additional condition of whole numbers. The solution search process includes a lot of steps, takes into account time factor; solves tasks both in condition of full and part-certainty; multicriterial; quality of the coefficients used and information received: proportional, additive, inalienable	Tasks can be solved only with linear elementary functions
Not linear	Solves tasks with 2 and more functions, where one of them is linear function and other not. Processes of search recues step by step solutions, where time factor is taken into account. Tasks can be solved both in terms of complete and incomplete certainty; Multicriterial	Difficult to solve; no universal method of solution; quality of the coefficients used and information received not always: proportional, additive, inalienable

Source: own elaboration.

We propose to choose linear model of mathematic modeling on the assumption of analyze of economic a mathematic models creation, their advantages and disadvantages for goods turnover system upgrading (table 1). Task of distribution or transporting task should be solved previously. Transporting task is task how to economize on identical cargo transportation from one place to another. Solving this task is the easiest way to cut down on expenses and increase profits for manufacture[20].

Transporting task has following *features*:

1)has solution;

2) number of linear independent equations in the system of limits (1-3 equals to m+n-1[20]:

[20] Kulyan V.R. *Matematicheskoe programmirovanie*. K.: MAUP, 2000.

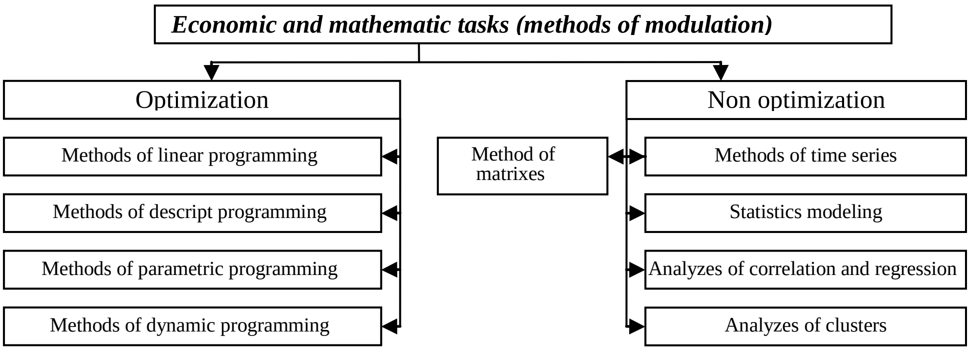
$$f = \sum_{i=1}^{n} \sum_{j-1}^{m} c_{ij} x_{ij} \ , \qquad (1)$$

$$\sum_{j=1}^{n} x_{ij} = a_i \ , \quad (i= 1,m), \qquad \sum_{i=1}^{m} x_{ij} = b_j \ , \quad (j=1,n) \qquad (2)$$

$$x_{ij} \geq 0 \qquad (i= 1,m,\ j=1,n) \qquad (3)$$

3) if a_i(i = 1,m), b_j (j=1, n) –aims, that means that among solutions one is whole numbers solution.

Economic and mathematical model solution is obtained by testing mathematical methods[21]. To choose numerical method correctly we should compare optimization methods because it forms the criterion of optimality as specified by the task (depends on the objective function). Two big groups of economic and mathematical tasks exist: optimization and not optimization (Picture 1)[22].



Picture 1. Classification of methods of solving problems

Source: Kuchma M.I. *Matematichne programuvannya: prikladi i zadachi*. L'viv: «Novij Svit-2000», 2007. Marmoza A.T. *Praktikum z matematichnogo programuvannya*. K.: Kondor, 2004.

For solving transporting task we need to use optimization models, linear in particular, as this method allows to understand if the method of transportation from manufactures to stocks is effective or not and minimize transportation costs. It would be possible providing that an amount of goods produced feats requests of consumers and wholly sold out (analyze of optimization methods is given in table. 2).

Table 2. Comparison of optimization methods

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[21] Prokopov S.V. *Ekonomiko-matematicheskoe modelirovanie v proizvodstvennom menedzhmente*. K.: KNUTD, 2004.

[22] Marmoza A.T. *Praktikum z matematichnogo programuvannya*. K.: Kondor, 2004.

Methods	***Advantages***		***Disadvantages***
1	2	3	4
Linear programming	Opportunity to fulfill plan	*Approximation of conditionally optimal plan*: illusion of optimal plan is built and then led into permissible boundaries of criterion of optimality (method of resolving multipliers, Hungarian and simplex method).	
		Step-by-step plan improvement: optimal solution is find by repeating steps (iterations), during which basic solution are build until a plan is not optimal (distribution method, potential and expansion method for solving large dimensions).	
Discrete programming	Variables and Parameters of tasks are discrete numbers	*Pruning:* its essence lies in the fact that firstly problem is solved without considering integers. When we receives plan of not whole numbers we should add to the system of limitations new linear restriction, which as though satisfies integer plan. These actions we repeat until getting integer plan.	Solving by this method is uncomfortable and not advised to use while solving transporting tasks
		Combinatorial: bases in sorting possible variants of tasks solution by replacing complete sorting plan to partial. It is characterized by such type of sorting, which excludes intentionally non optimal variants of plan without its previous observation.	Solving by this method is uncomfortable and not advised to use while solving transporting tasks
		Approximation: its essence lies in the usage of random search. When we randomly choose point, "make a step" on unknown rout and care not to go out of the task limits. Should we consider any positive or negative changes we retune to the beginning and make new random step.	Solving by this method is uncomfortable and not advised to use while solving transporting tasks

1	2	3	4
Parametric programming	The method bases on the fact that coefficients of the objective function are either numerical descriptions of restrictions that are not constants or functions dependent on a numerous parameters (that could be presented simultaneously). Tasks of this method: 1) the linear forms depend on one parameter – solves by method of continuous improvement of the plan with special rule of vector selection and specialized features of the decision completeness; 2) Components of the restriction vector linearly dependent on solving by method of approximation of conditionally optimal plan with special rule of vector selection and special features of the decision completeness; 3)Task, which is a generalization of the first two groups; parameter is maintained in all elements of the expanded matrix conditions, very much connected with parameter and can be determined by any analytical functions.		Solving by this methods is more graphical and hard to use, that is why not used for transporting problem solving
Dynamic programming	The main feature of the method is distribution of planned operations in a number of steps. The process becomes multi-step and each time the objective function is optimized only in one single step. Optimization task is created by relations that are consistently linked. The decision is based on calculation scheme, when instead of one task with many variables we construct many tasks with lower number of variables in each. This greatly reduces an amount of calculations. This advantage can be achieved only under 2 circumstances : 1) When criterion of optimality is additive (general optimal solution is the sum of optimal solutions of each step); 2) when future results do not dependents on the previous state of the system in which decision is taken.		Solving by this method is uncomfortable and not advised to use while solving transporting tasks

Source: own elaboration.

While analyzing optimization methods of economic and mathematic tasks by determination their advantages and disadvantages we assumed that the most comfortable to use, less cumbersome and more accurate (universal) method for solving transport problem is method of linear programming. Using techniques developed by this method we are able to receive optimal plan of goods turnover.

Making more concrete method of linear programming we found that method of plan continuous improvement is the most complex, efficient and thus optimal. Feathers of this method with the help of model created and transport tasks solving we can minimize transportation costs and maximize profits of enterprise.

4. Substantiation of chosen method for forming sales and supply system

To form a system of supply and sales, the purpose of which is to find such an option distribution (plan), which ensures the most economic effect (maximizes profits and minimizes costs), the choice of action defines by information about limitations that requests to the plan of actions. To use mathematical programming methods for finding the optimal plan, we should make a record of economic problems with the help of mathematical formulas (equations, inequalities). The objective and restrictions is represented as functions of variables. Independent variables can be of 2 types: 1) controlled [xj, (j = 1, n)], the value of which can be changed in a certain range; 2) uncontrolled, the value of which does not depend on the will of the people, but determined by the environment.

To build mathematic model we need to have clear understanding of the aim of system functioning which is observed. This aim will then turn into result received by choosing and realization certain program of actions (in our case either minimization of expenses or maximization of incomes).

Mathematic model in such tasks has the following values x_{ij} ($i = \overline{1,m};\ j=\overline{1,n}$), with the help of which system of limits works[17]:

$$\sum_{j=1}^{n} a_{ij} x_{ij} = a_i, \quad (i = \overline{1,m}), \qquad \sum_{i=1}^{m} b_{ij} x_{ij} \le b_j, \quad (j=\overline{1,n}) \tag{4}$$

Function (1) reaches its extreme meaning (the biggest or the smallest); a_i, b_j, a_{ij}, b_{ij}, c_{ij}, x_{ij} ($i = \overline{1,m};\ j=\overline{1,n}$) – known values.

While planning the number of cargoes transportations task of their optimal (for special criteria) organization evaluation always appears. It presumes such planning when price and time for transportation would be minimal. Transporting task always requires certain details about the rout, which connects certain stations.

Consider statement of the transporting task by criterion of transportation cost: in stations of departure A_1, A_2,…,A_m stays (in the same order) a_1,a_2,…,a_m units of the similar cargo, for delivering to *n* number of consumers B_1,B_2,…,B_n in needed amount b_l,b_2,…,b_n units for each. Transporting expenses for delivering units of cargo from *i* point of manufacturing to j point of consumption; c_{ij}- taxes, should be as low as possible. x_{ij} – amount of cargo,that is planned to carry from *i* point of departure to j point of consumption (arrival)[17].

Clauses of transporting task are presented in the table (table 3).

Table 3. Distribution of transporting task

Supplier	Consumers			Stocks of goods
	B_1	…	B_n	
A_1	C_{11} X_{11}	…	C_{1n} X_{1n}	a_1
…	…	…	…	…
A_n	C_{m1} X_{m1}	…	C_{mn} X_{mn}	a_m
Needs in goods	b_1	…	b_n	

Source: Kuchma M.I. *Matematichne programuvannya: prikladi i zadachi*. L'viv: «Novij Svit-2000», 2007.

Term 1. *Plan of transportation* is matrix $\{x_{ij}\}_{m\ n}$, elements of which defines amount of cargo to convey from *i* point of departure to j point of arrival to satisfy customers demands completely, but to keep aggregate transportation costs minimal. We present general aggregate expenses, which are connected with plan of transportation realization, by objective function (1):

Term 2. Matrix with elements of $c_{ij\ mxn}(i = \overline{1, m};\ j = \overline{1, n})$ is called *matrix of taxes* or *matrix of transporting costs* (expenses on transportation of one unit of cargo from i provider to j consumer).

Task narrows to finding general expenses caused by some plan of transportation $\{x_{ij}\}$; that expenses should satisfy conditions and defined by objective function (1):

Variables x_{ij} have to satisfy conditions of the amount of existing stocks by requests (needs) for an amount of cargo and natural condition of inseparability. Formed conditions we records as an equation[17]:

$$\sum_{j=1}^{n} x_{ij} \le a_i, \quad (i = 1,m), \tag{5}$$

$$\sum_{i=1}^{m} x_{ij} = b_j, \quad (j=1,n) \tag{6}$$

Tasks of goods turnover plan optimization by the lowest price is formed on the following way: to find such plan of goods turnover $\{x_{ij}\}$ $(i = \overline{1, m};\ j = \overline{1, n})$, that makes objective function to reach its minimum in terms of limitation (5, 6, 3). Task has *m n* unknown and *m+n* limits.

Variables x_{ij} must satisfy limits of stocks and needs; can be mathematically presented as a system of equations (3):

$$\begin{cases} \sum_{j=1}^{n} x_{ij} \le a_i \,(i = 1, m), \\ \sum_{i=1}^{m} x_{ij} \ge b_j \,(j = 1, n), \end{cases} \tag{7}$$

Mathematic formulation of the transporting task is to find such solution of the system which will minimize function (1).

At this article we lights up transporting task only by *criteria of costs for transporting planning,* because optimization by criterion of the shortest period of sales is not narrowed to tasks of linear planning. The setting of such problems is also complicated by additional expenses for routes transporting capacity, possible restrictions on the goods quantity for some vehicles and other complications.

Two types of transporting tasks that include needed and sufficient conditions of acceptable plan exist[17]:

1. Balance (closed) in which general amount of cargo (stocks) at delivery offices equals to general amount of requests (demand) – **term 3.** Mathematically it can be recorded as:

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j \ , \qquad (8)$$

2. Unbalanced (opened) in which general amount of suppliers is higher or lower of general consumers demand – **term 4.** Mathematically fourth term can be recorded as[17]:

$$\sum_{i=1}^{m} a_i > \sum_{j=1}^{n} b_j \ , \text{ або } \qquad \sum_{i=1}^{m} a_i < \sum_{j=1}^{n} b_j \ , \qquad (9)$$

Unbalanced model is always can be turned into balanced. If $\sum_{i=1}^{m} a_i > \sum_{j=1}^{n} b_j$, than we need to add fictitious (n+1) clause of destination B_{n+1} in mathematical model of transporting task. After that we adds additional column to the matrix of conditions, which corresponds to fictitious clause of destination and has needs equal to difference between general amount of supplies and real demands of customers[16]:

$$b_{n+1} = \sum_{i=1}^{m} a_i - \sum_{j=1}^{n} b_j \ , \qquad (10)$$

Taxes for goods transportation to the fictitious place of destination should be marked and regard as equal to 0. As a result, opened model of transporting becomes closed. The objective function of both tasks is the same because additional transportation has zero costs.

If $\sum_{i=1}^{m} a_i < \sum_{j=1}^{n} b_j$, than mathematical model of transporting task should be added to fictitious (*m+1*) point of departure A_{m+1} and stock of goods for consider equal[20]:

$$a_{m+1} = \sum_{j=1}^{n} b_j - \sum_{i=1}^{m} a_i \ , \qquad (11)$$

Taxes for cargo transporting from fictitious place consider as equal to 0. To the matrix of transporting task adds one line, objective function does not change and the system of tasks limits becomes compatible[13].

For balanced tasks conditions of equation (2) are implemented.

Further we consider method of closed type for solving transporting tasks by the criterion of transportation cost. It is used in accordance with the foregoing, and for unbalanced models.

Theorem 1. In order to have acceptable plans for transporting task, it is necessary and sufficient to perform equality[20]:

Theorem 2. The rank of transportation task matrix is smaller than the number of linearly independent equations in one unit. It means that rank is equal to m + n-1 or any system of limitation has m + n-1 of basic variables[16].

As stated by the theorem 2, every basic plan should has (m - 1)(n -1) independent variables that equals to 0 and $m+n$—1basic (occupied) variables. System of limitations has m*n unknowns $\{x_{ij}\}$ (i $=\overline{1,m}$; j$=\overline{1,n}$).

Plan of transportation we calculate with the help of table 3. Assumes, that if variable x_{ij} become $a_{ij} \neq 0$, so we put the value in specific sections (i;j) , if x_{ij} = 0, so section (i**;** j) is empty[11].

In case if the number of filled sections less than $m+n-1$, that means that we must fill empty sections with zeroes to make its sum be equal to $m+n$-. Requirements to the basic plan should be also fulfilled.

Basic plans should satisfy requirements connected with cycles.

Term 5. A set of sections of transporting matrix in which only two adjacent sections are located in one line or one column and the last section is set in the same line or column with the first one and called *closed cycle*, which mathematically can be recorded as[17]:

$$(i_1 j_1) \to (i_1 j_k) \to (i_k j_k) \to ...(i_s j_s) \to (i_s j_1). \qquad (12)$$

Graphically cycle can be presented as closed broken line, sections of which are only in lines or columns and every section connects two sections in the cycle.

The idea of solving the transportation task is similar to the general idea of the simplex method (SM). However, due to the specific constraints of the task, to solve it we should complete the procedures step by step[17]:

I. Creation of the initial plan:

1. Formation of mathematic model (implementation of markings and limits);

2. Setting of economic problem (defining objective function and type of transporting task);

3. Submission of transporting task data in the form of ordinary distribution table (3) and search for initial acceptable (basic) transportation plan. That plan should meet the demand of each customer (Vj) and ensure delivering of entire cargo from every provider (Ai).

It is important to mention that if planned transportation from one place i to another j cargo is in x_{ij}>0 amount, that means we puts x_{ij} in section at the intersection of i line and j column. If supply there is not planed, x_{ij}=0, so section (i,j) stays empty. Section $m+n-1$ should be filed.

In case if during realization of the algorithm creation of initial (basic) appears the situation that after filling (i, j) section i stock is exhausted and demands are fully satisfied j (if it is not in the last step), so we should add 0 in one of sections that have the lowest rate in a line or column. If there is at least one zero component in basic plan, the transporting task is *degenerate*.

II. Control over (evaluation) received plan for optimality (improvement of the existing plan). If conditions of plan optimality are achieved, transporting task is solved and calculated expenses tends to minimum. If plan is not optimal cargo should be divided once more to cut down on transporting costs (transfer from one basic plan to other and returning to the second phase)[16].

When task is hard to solve because of great number of suppliers and consumers, it is better to use method of sequential plan development. Using simplex method in that case is not rational. Comparison of planes is given in table 4.

After proper analyzes and compression of advantages and disadvantages of method of sequential plan development the author claims, that initial plan is much chipper than plan created with the help of method of northwest corner. But its price is bigger than the price of plan, which was created with the help of method of double profit.

Table 4. Methods characteristics of sequential improvement of the plan

Method		***Advantages***	***Disadvantages***
1	2	3	4
Direct	Of northwest corner	Initial basic plan is the easiest to create. It does not require difficult calculations. Basic plan is always started with filling upper left section (A_1B_1) of the distribution table, where the smallest between two numbers a_1 and $b_1$is recorded. Than the next section in the line (till stocks of suppliers are not over) \column(till needs of customers is not fully satisfied) is filled. We finishes filing the table at the lower right section A_mB_n	While finding an amount of cargo needed to deliver regulations of costs (transporting expenses, taxes) are not taken into consideration. Basic plan is not optimal
	Minimum element of line	During determination of amount of goods to transport we consider min costs regulations in every line starting with the first one. The main peculiarity of the method is that for every single line we find minimal taxes in the distributional table encircle it and make delivery. With the reduction of supplier stocks we move on to another section to find taxes, encircle it make delivery. The action should be repeated unless all stocks are drawn out. If there are few min elements we choose whichever	Initial basic plan needs more time to be created than plan of northwest corner but less than needed for plan of min costs matrix or plan of double profit creation. Basic plan found by those methods requires further improvements and as a result worse than plan created with the help of line\column method
	Minimum element of column	While determination of amount of goods to transport we consider min costs regulations in every column starting with the first one. The main peculiarity of the method is that for every single column we find minimal taxes in the distributional table encircle it and make delivery. Encircle min taxes and make delivery. If demand is not high enough, we find smaller taxes at this column and make delivery. That process continues until demand is not fully satisfied. If there are	

		few min elements we choose whichever.	

1	2	3	4
	Minimum cost	Determining the number of transportations from the whole table we take into account regulations costs for transporting. We fill section with the lowest costs for transportation of one unit in every step of task solving. These actions repeat until all the products will be distributed between suppliers and customers	Initial basic plan requires more time to be created but closer to the optimal than plan of northwest corner and of line\column method
	Double advantage	Determining the number of transportations on the routes the cost of transportation regulations calculates twice (on the column and on the line) choosing the most economical route among possible. Basic initial plan is faster to create by this method then plan by indirect method. It is the most optimal plan. Peculiarities: firstly we find the minimum taxes and note it with checkmark in each selected line in the distribution table of transporting task. Than the minimal element in every column an put checkmark near as well. Find the section with two checkmarks, encircle it and deliver goods to that section. If there are no sections with 2 checkmarks left we make goods delivery to the section with one checkmark. When all groups are not redistributed we continue analyze using method of minimal element of matrix(method of minimal expenses)	That method requires much more time to be created than any other
Not direct	Fogel	Basic plan is the closest to the optimal and as a result the cheapest. Calculation of penalty cost because of lack of the cheapest routes transportations. Peculiarities: 1. we calculate penalties in every line and column, which allows finding an amount of penalties because of lack of the cheapest routes transportations. 2. Chose line\ column with max penalty and section with the min taxes (C_{ij}) for transporting expenses for the column\ line and place the max possible cargo. That action prevents from high penalties. 3. Correct balances of supply and demand. Supplier who has no capacity left or consumer with no requests should not be Considered any longer. After that we return to the first step and define costs for penalties (without filled and crossed out routs).	Requires a lot of additional calculations

Source: own elaboration.

Systematization and analyze of optimization models shows that for improvement of goods turnover system and solving transporting task we need to choose method of continuous plan improvement (dual benefits). That method allows optimizing goods turnover among great number of wholesale enterprises and warehouses (consumers) and manufacturers (suppliers).

Initial basic plan of transporting task should be checked for optimality. Checking is hold by *method of potentials.*

We compere some estimates α_i (i = 1,$\overline{m}$) i β_j(j=1, n) for every supplier and every consumer to check initial basic plan. Those mathematical expressions are called suppliers and consumers potentials accordingly, if:

1) plan $\vec{X}$ = {x_{ij}} optimal, its means that plan corresponds to system of n + m numbers α_i and β_j, which satisfies conditions:

$$C_{ij}-(\alpha_i + \beta_j) \geq 0 \qquad (13),$$

C_{ij} – price for transporting some amount of products *i* –by supplier *j*- –by customer ;

α_i - potential of suppliers (i = 1,m);

β_j – potential of customers;

a) For each filled section $\alpha_i + \beta_j = C_{ij}$ for $x_{ij} > 0$ (i = 1,m), (14);

b) For each not filled section $\alpha_i + \beta_j \leq C_{ij}$ for $x_{ij} = 0$ (i = 1,m), (15);

c) Characteristics of not filled sections $E_{ij} = C_{ij}-(\alpha_i + \beta_j) \geq 0$ (16)

2) if initial basic plan is formed, it should be optimized (to calculate characteristics of empty sections) (E_{ij}): if $E_i < 0$, we need to continue calculating; if $E_{ij} > 0$, plan is optimal, we should analyze it and make conclusions about results. In case if received E_{ij} are positive, but at least one E_{ij}=0 alternative plan is also exists. Such optimal plan of chocolate turnover fits minimal expenses and maximum profits[16].

Algorisms of transporting task solution by method of potentials:

1. To create initial basic plan of transporting task.
2. To prepare distribution table and highlight taxes, that corresponds to the location of existing component support plan.
3. To record and solve the system of linear equations (14) for calculation of potentials. One value can be chosen randomly.
4. To calculate elements of matrix of pseudo taxes (characteristics of empty sections E_{ij}) , using formula (16). If every found E_{ij} is positive, than plan is optimal.
5. When E_{ij} are negative, search for optimal plan is continued. Choose section with the smallest number, build cycle, calculate new basic plan. When the top of the cycle is calculated, we turn to new basic plan.
6. Algorism is carried out from the point 2 to the calculation of optimal basic plan.

Summary
The essence and peculiarities of economic and mathematic modeling to improve the system of goods turnover at manufacturing and trade enterprises is observed at the article. Models and optimization methods are compared and choice of method for creation of supply and distribution system is proved. The author proposes the model for improvement of the system of goods turnover at the enterprise, which will ensure minimization of logistics expenses and maximization of profits after solving transporting task.

Using methods of linear programming, method of consistent improvement in particular, author shows an approach to solving transporting program. That approach helps to reduce expenses for stocks and logistics expenses in general and to form correcting plans of goods turnover (optimal and alternative).

As a result, received information can be used by modern consulting companies and manufacturing and trade enterprises in order to analyze, to improve or to create new and more effective system of goods turnover.

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