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**MATHEMATICAL MODELING OF NONLINEAR  
PLATINUM DYNAMICS IN UKRAINE**

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*This article is devoted to investigation of platinum market dynamics in Ukraine by nonlinear dynamics methods. The Hurst exponent, correlation dimension and correlation entropy for platinum courses in Ukrainian market during 2002–2015 are calculated. According to the results of the calculation we indicate the presence of the chaotic processes in the dynamics of the system. It allows better to understand a structure and behavior features of platinum market.*

**Keywords:** *attractor, fractal, Hurst exponent, dimension of phase space, correlation dimension, correlation entropy, nonlinear dynamics, chaos theory, financial time series, precious metal.*

**Introduction.** Providing of dynamic and effective financial market development is an important task of the state on the stage of market reformation of economy. To develop an efficient financial market, in basis of its development it is necessary to put modern scientifically reasonable methods and adjusting principles, provide functioning of his various elements, including the market of precious metals, including platinum, it plays an important role for stable development of economy.

**Analysis of recent research and publications.** Ukrainian and foreign scientists, who were engaged in research of problems of applied application of methods of nonlinear dynamics and the theory of chaos: Andryenko, V.M. [1], Zaika, V. I. [2], Solovev, Yu. L [3], Strogatz, S. H. [4], Chiarella, C. [5], Panchenko, V. [6] and other.

**Previously unsettled problem constituent.** Lately scientists have made a special focus on research of financial time series from the point of view of chaos theory. A new area that contains the section of mathematical methods of economy that are actively developing is sufficient. There is gaining popularity a point of view,

that the precious metals market, in addition to the random chaotic fluctuations, is exposed to the fluctuations occurring under the action of long-term regularities, or so-called deterministic chaos. In conditions of unstable economic and political situation in a country, when sudden changes of precious metals rates can be as indicators of deepening of the crisis, an all-round analysis of platinum market by the methods of nonlinear dynamics remains relevant, because it allows better to understand a structure and behavior features of this market.

**Main purpose of the article.** The aim of the article is to investigate non-equilibrium processes at the platinum market in Ukraine by the methods of nonlinear dynamics. In process the method of economic-mathematical modeling, methods of nonlinear dynamics, chaos theory and fractal mathematics are used.

**Results and discussions.** The prices on precious metals depend on an economic and political situation in a less measure, than the other means of accumulation, and are not the obligation of any state, that is why they are the most effective protector of the inlaid facilities from depreciation. The presence of a sufficiently strong direct correlation between individual precious metals markets means that the precious metals markets largely support each other. In addition, remark that increased demand for precious metals usually can be observed in periods of significant fluctuations in other markets, primarily financial.

During the research the analysis of platinum rate dynamics in Ukraine for period from 02.04.2002 to 20.11.2015 was carried out. Fig.1. depicts the dynamic series constructed on the basis of the change in the price of platinum according to the National Bank of Ukraine [7].

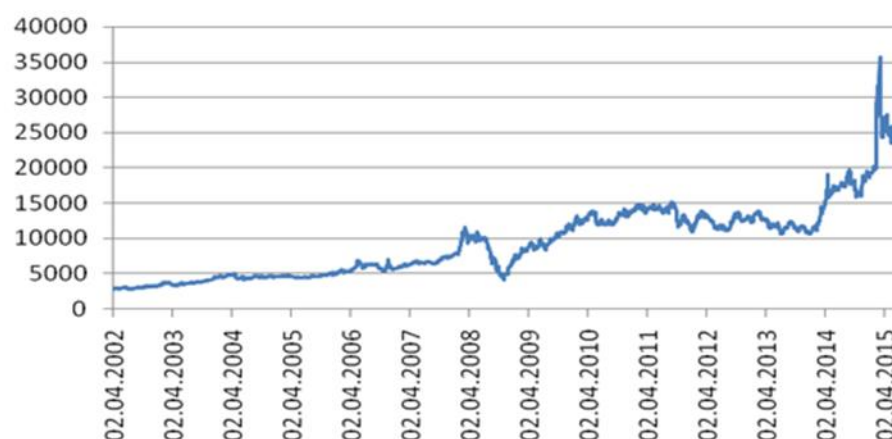


Fig. 1. Dynamics of platinum rate according to the NBU for the period from 02.04.2002 to 20.11.2015 (Source: The National Bank of Ukraine)

During 2002–2008 the rates of precious metals were growing. With the onset of the crisis in 2009–2011, this growth gained pace, driven by a sharp fall of the hryvnia to the U.S. dollar in the fall of 2008, and inflation expectations, which led to the

growth of demand. The rate of platinum looks the most stable among rates of precious metals. The value of platinum is related to its industrial utilization and its value is largely determined by the real economy than, for example, the price of platinum.

For example, platinum is used in fuel treatment systems in diesel engines of modern cars. For this reason, the fall in vehicle production (for example, due to the recession) reduces the demand for platinum in the auto catalyst sector (40% of total platinum demand) that may reduce the price. This situation could be observed in 2008 and in the first eight months of 2012. The platinum rate has also undergone the sharp fluctuations in 2015. There was a sharp growth of its value in February 2015 with a record value 35682,1 on February 26, which is on 164% higher compared to the rate of the corresponding date of the previous year.

So, the analysis of the dynamics of platinum price in the market of precious metals gives grounds to hypothesize about the existence of nonlinear components in the dynamics, and, consequently, the chaotic nature of the market. Visual analysis of time series does not allow distinguishing stochastic and nonlinear dynamics, it is therefore advisable to use special methods, in particular, mathematical methods in the theory of nonlinear dynamics.

One of the most common methods, which allow identifying the fractal structure and non-linear cycles in a time series, that is, to distinguish random from non-random series, is the method of normalized range of Hurst or R/S analysis. If the value of Hurst exponent corresponds to a persistent time series, it is possible to continue research using the theory of nonlinear dynamics. The Hurst exponent is associated with the ratio normalized range ( $R / S$ ), where  $R$  is the "range" of a time series, and  $S$  is the standard deviation. For finding the Hurst exponent  $H$ , the dependence of  $R / S = f(N)$  is built in double logarithmic scale, then the experimental points approximate by direct, which angular coefficient is  $N$ . The whole area of fractal dimensions is limited by the lines  $H = 0$  and  $H = 1$  and the line  $H = 0.5$  divides it into persistent and antipersistent area [1, pp. 100-101].

The Hurst exponent is used as a measure of persistence - it is the existence of a trend, which is absent in normal Brownian motion. The value of  $H > 1/2$  means that the dynamics of the process in the past, likely, result into continuation of movement in the same direction. If  $H < 1/2$ , we consider that in the future the process will change the direction.  $H = 1/2$  means uncertainty, i.e. Brownian motion.

During the research it was calculated the Hurst exponent for stationary time series, constructed on the basis of the series of the platinum dynamics in the home market during the period from 02.04.2002 to 20.11.2015 with using software R (Fig. 2). Checking the series for stationarity implemented by the test of Dickey-fuller. The Hurst exponent is 0,8917, which suggests that the series is persistent and trendstuck, has fractal properties. This result allows us to apply for further research of

development trends of the platinum the methods of the theory of nonlinear dynamics.

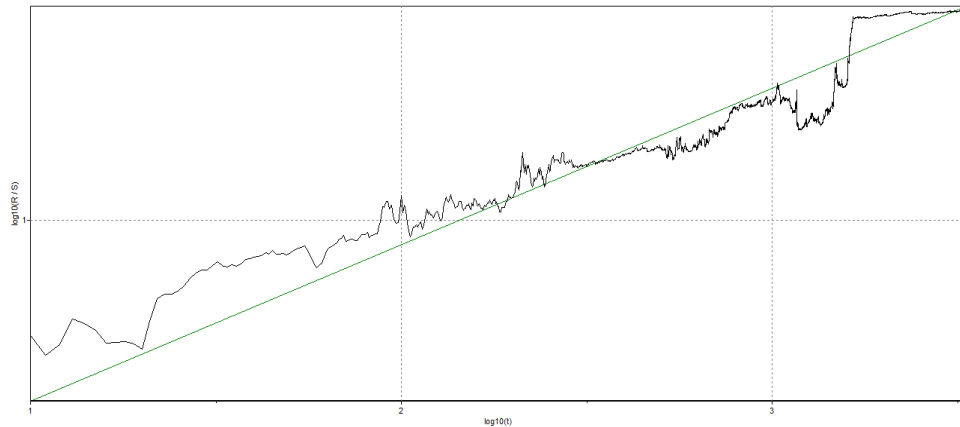


Fig. 2. **The Hurst exponent for the platinum rate** (Source: designed by the authors)

The main idea of applying the methods of chaotic dynamics to the analysis of time series is that the basic structure of a chaotic system that contains all the information about the system and its attractor, can be reconstructed through the measurement of only one observable parameter of this dynamical system, fixed as the time series [2, p. 60–62].

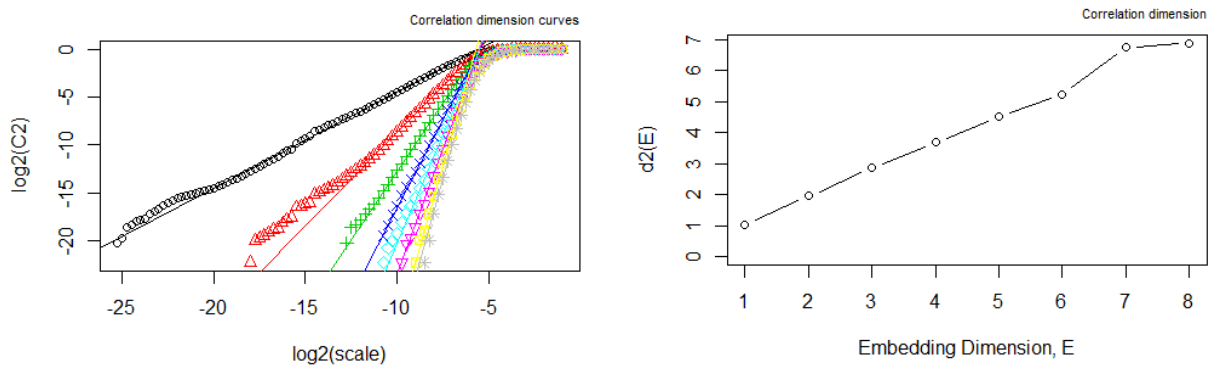
The dimension of attachment (dimension of phase space)  $m$  is called a smallest dimension of the space containing the whole attractor. It corresponds to the number of independent variables that uniquely define the prescribed motion of the dynamic system. Thus, by setting the dimension of the attachment, we receive information about the complexity of the system. This implies the ability to distinguish a dynamic system with complex behavior (but characterized by a finite  $m$ ), and random (stochastic) noise that is described by (theoretically) infinitely large number of independent variables.

The most important characteristics of the attractor are: Hausdorff dimension  $D=D_0$  and the Kolmogorov entropy  $K=K_1$ . The calculation of these exponents is extremely time-consuming and virtually impossible for attractors in higher dimensions. In practice, therefore, use their lower bounds: correlation dimension  $D_2 \leq D_0$  and correlation entropy  $K_2 \leq K_1$ . which can be evaluated directly for a discrete sequence of points of the trajectory [1].

In experimental data we usually do not know the dimension of the phase space of the system and have information about only one coordinate of the points on attractor. Therefore, all calculations are carried out for several dimensions of the phase space  $m = 1, 2, 3, \dots$ . Thus, the correlation dimension of the attractor  $D_2(m)$  at first increases, but then usually goes on a constant level. By such method, we get the required correlation dimension  $D_2$  of the attractor and the estimate for the dimension of the phase space. For random data  $D_2(m)$  monotonically increases with increasing

the dimension the attachment  $m$ . Therefore, the correlation dimension  $D2(m)$  are used to validate the presence of a chaotic component [9, p. 25–26].

According to the results of calculations (Fig. 3) correlation dimension saturates with the dimension of the phase space  $m = 5$ , and the value of the correlation dimension  $D2 = 4,315$ . Based on the obtained value of the correlation dimension we conclude that the platinum market in Ukraine as a dynamic system has a chaotic component, that behavior is deterministic.



**Fig. 3. The correlation integral and the dependence of correlation dimension and the dimension of the phase space** (Source: designed by the authors)

Correlation entropy  $K2$  can be calculated quite simply. To do this, calculate the correlation integral too, but it is consider not only its dependence on the distance  $\varepsilon$  but also on the dimension of phase space  $n$ . Thus, it is consider that:

$$C(\varepsilon, n) \approx \varepsilon^{D_2} \exp(-nK_2)$$

Where  $C(\varepsilon)$  – the correlation integral, that can be evaluated directly for a discrete sequence of points of the trajectory ;  $D2$  - correlation dimension.

This suggests, that:

$$K_2(\varepsilon, n) = \ln \frac{C(\varepsilon, n)}{C(\varepsilon, n+1)}$$

The entropy  $K2$  is approximated in an acceptable range of values  $\varepsilon$  and  $n$  [8, p]. The numerical value of the entropy is a quantitative characteristic of the degree of randomness of the system. Because the correlation entropy is a lower estimate of the Kolmogorov entropy, we are interested in approximation its value to zero. If the minimum of the Kolmogorov entropy  $K$  is greater than zero, then the entropy is also positive, indicating the presence of chaotic processes in the dynamics of the system. When  $K2 \rightarrow 0$  we can see the transition to the regular process [8, p 202–203].

The value of the correlation entropy for time series constructed on the basis of dynamics of prices on the platinum market in Ukraine,  $K2 = 0,335$ , and consequently in the dynamics of the system exist the chaotic processes.

**Conclusions and further researches directions.**

Thus, analyzing the dynamics of the platinum market in Ukraine, we can draw the following conclusions: stationary time series constructed on the basis of series of the dynamics of platinum on the home market during the period from 02.04.2002 to 20.11.2015, has fractal properties and is persistent and trendstuck, that is evidenced by the value of the Hurst exponent. According to the results of the calculation of the correlation dimension we conclude that the system has a chaotic component, which behavior is deterministic. The value of the correlation entropy also indicates the presence of the chaotic processes in the dynamics of the systems.

The obtained results allow us to conduct further research and forecasting of the prices in the platinum market in Ukraine using the methods of nonlinear dynamics and theory of chaos. It is also advisable to conduct similar studies for markets of other precious metals, namely silver, gold and palladium. This will allow to figure out whether the obtained results are typical for the market of precious metals in general.

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**МАТЕМАТИЧНЕ МОДЕЛЮВАННЯ  
НЕЛІНІЙНОЇ ДИНАМІКИ ПЛАТИНИ В УКРАЇНІ**

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*У статті досліджено динаміку ринку платини в Україні методами нелінійної динаміки. Для курсів платини в Україні за період 2002-2015 років обчислено: показник Херста, кореляційну розмірність, кореляційну ентропію. Згідно з результатами обчислень виявлено наявність хаотичних процесів у динаміці системи. Це дозволяє краще зрозуміти структуру та особливості поведінки ринку платини.*

***Ключові слова:** фрактал, аттрактор, показник Херста, розмірність фазового простору, кореляційна розмірність, кореляційна ентропія, нелінійна динаміка, теорія хаосу, фінансовий часовий ряд, дорогоцінний метал.*

**МАТЕМАТИЧЕСКОЕ МОДЕЛИРОВАНИЕ  
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***Ключевые слова:** фрактал, аттрактор, показатель Херста, размерность фазового пространства, корреляционная размерность, корреляционная энтропия, нелинейная динамика, теория хаоса, финансовый временной ряд, драгоценный метал.*