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COMPLEX ANALYSIS  
AND RELATED TOPICS

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ABSTRACTS

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We say that function  $f \in \mathcal{H}_0$  is of strongly regular growth (s. r. gr.) with respect to the function  $v \in L$  if for all  $\theta \in [0, 2\pi]$ , perhaps, with the exception of  $\theta$  belonging to a countable set, the limit

$$\lim_{r \rightarrow +\infty}^* (\ln f(re^{i\theta}) - N(r))/v(r) = H(\theta, f)$$

exists. Here  $\lim_{r \rightarrow +\infty}^*$  indicates that  $r$  tends to infinity outside some  $C_0$ -set. We denote the class of functions of s. r. gr. by  $\mathcal{H}_0^*(v)$ .

**Theorem 1.** Suppose that  $f \in \mathcal{H}_0$  and for some numbers  $p \in [1, +\infty)$ ,  $b_0 \in \mathbb{R}$  and a function  $G \in L^p[0, 2\pi]$  the conditions

$$\left\| \frac{\ln |f(re^{i\theta})|}{v_1(r)} - b_0 \right\|_p \rightarrow 0, \left\| \frac{\arg f(re^{i\theta})}{v(r)} - G(\theta) \right\|_p \rightarrow 0, r \rightarrow +\infty, \quad (1)$$

hold. Then  $f \in \mathcal{H}_0^*(v)$ ,  $H(\theta, f) = iG(\theta)$  for almost all  $\theta \in [0, 2\pi]$ .

Conversely, if  $f \in \mathcal{H}_0^*(v)$  and zeros of  $f$  are located on a finite system of rays, then for arbitrary  $p \in [1, +\infty)$  condition (1) holds with  $G(\theta) = -iH(\theta, f)$ ,  $b_0 = \lim_{r \rightarrow +\infty} n(r)/v(r)$ .

**Theorem 2.** There exists a function  $f \in \mathcal{H}_0^*(v)$ , for which at least one of the relations (1) does not hold.

Thus, the condition of that zeros of  $f$  are distributed on a finite system of rays in Theorem 1 is essential.

## Canonical functions of gamma-admissible measures in half-plane

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For a  $(\gamma, \varepsilon)$ -admissible measure in the upper half-plane [1] the concept of a canonical function is introduced. This concept is a generalization of Nevanlinna's canonical product for analytic in half-plane

functions of a finite order. It is shown that for a function whose growth is defined by a proximate order in the sense of Valiron, the canonical function and Nevanlinna's canonical product coincide.

1. Malyutin K. G., Kozlova I. I. Subharmonic functions of finite  $(\gamma, \varepsilon)$ -type in a half-plane // *Mat. Stud.* – 2012. – V. 38, No 2.– P. 154-161.

### Wiman's type inequality and Levy's phenomenon for random analytic functions in the unit disk

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Let  $\mathcal{L}$  be the class of positive continuous functions on the interval  $(0, 1)$  increasing to  $+\infty$  and such that  $\int_{r_0}^1 h(r)dr = +\infty$ ,  $r_0 \in (0, 1)$ . For a measurable set  $E \subset (0, 1)$  and  $h \in \mathcal{L}$  the  $h$ -measure of  $E$  is defined by  $h\text{-meas}(E) \stackrel{\text{def}}{=} \int_E h(r)dr$ .

Let  $f$  be an analytic function in the unit disc  $\mathbb{D} = \{z: |z| < 1\}$  of the form  $f(z) = \sum_{n=0}^{+\infty} a_n z^n$ ,  $Z = (Z_n(t))$ ,  $t \in [0, 1]$ , be a complex sequence of random variables such that  $Z$  is multiplicative system (MS) uniformly bounded by the number 1 ([1]) on the Steinhaus probability space, and  $K(f, Z)$  be the class of random entire functions of the form  $f_t(z) = f(z, t) = \sum_{n=0}^{+\infty} a_n Z_n(t) z^n$ . For  $r \in (0, 1)$  we denote  $M_f(r) = \max\{|f(z)|: |z| = r\}$ ,  $\mu_f(r) = \max\{|a_n| r^n: n \geq 0\}$ ,

$$\Delta_h(r, f) = \frac{\ln M_f(r) - \ln \mu_f(r)}{2 \ln h(r) + \ln \ln(h(r)\mu_f(r))}.$$

From a result proved in [2] it follows that in the case when  $h(r) = (1 - r)^{-1}$ , for every analytic function  $f$  in  $\mathbb{D}$  there exists a set  $E \subset (0, 1)$  of finite logarithmic measure, i.e.  $h\text{-meas}(E) < +\infty$  for the function  $h(r) = (1 - r)^{-1}$  such that  $\overline{\lim}_{r \rightarrow 1-0, r \notin E} \Delta_h(r, f) \leq \frac{1}{2}$ .