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About the Simplex Form of the Polyhedron of Arrangements

O.O. Iemets, O.O. Yemets`, I.M. Polyakov
Poltava University of Economics and Trade, yemetsli@ukr.net

Abstract **– The simplex form of the general polyhedron of arrangements, which is used in linear programming problems in combinatorial cutting methods is obtained and it increases the efficiency of cutting methods.**

Keywords **– Euclidean combinatorial optimization, arrangements, the simplex form of a polyhedron, the polyhedron of arrangements.**

I. Introduction

When using linear programming problems in Euclidean combinatorial optimization as auxiliary in cutting methods [1-5], the simplex form of the polyhedron is required.

II. Main Part

In the report the obtaining of the simplex form for the polyhedron of k-arrangements from the elements of the multiset $G=\{g_1, ..., g_\eta\}$, which is given by the system

$$\begin{cases} \sum_{i\in\omega} x_i \ge \sum_{i=1}^{|\omega|} g_i \quad \forall\omega\subset J_k; \\ \sum_{i\in\Omega} x_i \le \sum_{i=1}^{|\Omega|} g_{\eta-i+1} \quad \forall\Omega\subset J_k; \end{cases}$$

under the condition

$$g_1 \le g_2 \le ... \le g_\eta.$$

is considered.

Here and below $|\omega|$ denotes the number of elements in the set ω.

It is proved that it has the form of the system

$$\left(U-\sum_{i=1}^{|\omega|} g_i\right)\sum_{i\in\omega} X_i - \left(U+\sum_{i=1}^{|\omega|} g_i\right)Y_\omega -$$

$$-\sum_{i=1}^{|\omega|} g_i\left(\sum_{i\in J_k\setminus\omega} X_i + \sum_{\substack{\Omega\subset J_k \\ \Omega\ne\omega}} Y_\Omega + \sum_{\forall\Omega\subset J_k} Z_\Omega + V\right) -$$

$$-\alpha_{|\omega|} W_\omega^\alpha = 0,\ \forall\omega\subset J_k,$$

$$\left(U-\sum_{i=1}^{|\Omega|} g_{\eta-i+1}\right)\left(\sum_{i\in\Omega} X_i + Z_\Omega\right) - \sum_{i=1}^{|\Omega|} g_{\eta-i+1} \times$$

$$\times\left(\sum_{i\in J_k\setminus\Omega} X_i + \sum_{\forall\omega\subset J_k} Y_\omega + \sum_{\substack{\forall\omega\subset J_k \\ \omega\ne\Omega}} Z_\omega + V\right) - \beta_{|\Omega|} W_\Omega^\beta = 0,$$

$$\forall\Omega\subset J_k,$$

$$\sum_{i=1}^{k} X_i + \sum_{\omega\subset J_k} Y_\omega + \sum_{\Omega\subset J_k} Z_\Omega + \sum_{\omega\subset J_k} W_\omega^\alpha +$$

$$+\sum_{\Omega\subset J_k} W_\Omega^\beta + V = 1,$$

$$Y_\omega \ge 0;\ Z_\omega \ge 0;\ W_\omega^\alpha \ge 0;\ W_\Omega^\beta \ge 0\ \ \forall\omega\subset J_k.$$

Parameters and variables of the system are set by the following conditions:

$$X_j = x_j\cdot U^{-1}\ \ \forall j\in J_k;\ Y_i = y_i\cdot U^{-1}\ \ \forall i\in J_r;$$

$$V = u\cdot U^{-1};$$

$$\sum_{i\in\omega} x_i - y_\omega = \sum_{i=1}^{|\omega|} g_i\ \ \forall\omega\subset J_k;$$

$$\sum_{i\in\Omega} x_i + z_\Omega = \sum_{i=1}^{|\Omega|} g_{\eta-i+1}\ \ \forall\Omega\subset J_k;$$

$$\sum_{i=1}^{k} x_i + \sum_{\omega\subset J_k} y_\omega + \sum_{\Omega\subset J_k} z_\Omega + u = U;$$

$$U=\sum_{i=1}^{k} g_{\eta-i+1}+2\sum_{j=1}^{k}\left[C_k^j\left(\sum_{i=1}^{j} g_{\eta-i+1}-\sum_{i=1}^{j} g_i\right)\right];$$

$$\alpha_{|\omega|}=(|\omega|-1)U-(2^{k+1}+k-1)\sum_{i=1}^{|\omega|} g_i ;$$

$$\beta_{|\Omega|}=(|\Omega|+1)U-(2^{k+1}+k-1)\sum_{i=1}^{|\Omega|} g_{\eta-i+1} .$$

III. EXAMPLE

Let us consider the example of the simplex form of the polyhedron of arrangements. Let $k=3$ $G=\{e_1,e_2,e_2,e_3,e_3\}$, that is $\eta=5$: $n=3$, $g_1=e_1$, $g_2=g_3=e_2$, $g_4=g_5=e_3$, $e_1<e_2<e_3$. That is the polyhedron of arrangements has the form

$$x_1\ge g_1;\ x_2\ge g_1;\ x_3\ge g_1;\ x_1+x_2\ge g_1+g_2;$$

$$x_1+x_3\ge g_1+g_2;\ x_2+x_3\ge g_1+g_2;$$

$$x_1+x_2+x_3\ge g_1+g_2+g_3;\ x_1\le g_5;\ x_2\le g_5;$$

$$x_3\le g_5;\ x_1+x_2\le g_5+g_4;\ x_1+x_3\le g_5+g_4;$$

$$x_2+x_3\le g_5+g_4;\ x_1+x_2+x_3\le g_5+g_4+g_3.$$

Parameters in the simplex form are:

$$U=24e_3-7e_2-14e_1,\ \alpha_1=-18g_1;$$

$$\alpha_2=15g_5+9g_4+g_3-26g_2-32g_1;$$

$$\alpha_3=30g_5+18g_4-16g_3-34g_2-46g_1;$$

$$\beta_1=12g_5+18g_4+2g_3-16g_2-28g_1;$$

$$\beta_2=27g_5+9g_4+3g_3-24g_2-42g_1;$$

$$\beta_3=42g_5+18g_4-14g_3-32g_2-56g_1.$$

The simplex form of this polyhedron is:

$$X_1(U-g_1)-Y_1(U+g_1)-g_1(X_2+X_3+Y_2+$$

$$+Y_3+Y_{12}+Y_{13}+Y_{23}+Y_{123}+Y_{12}+Y_{13}+Y_{23}+Y+$$

$$+Z_1+Z_2+Z_3+Z_{12}+Z_{13}+Z_{23}+Z_{123}+V)-$$

$$-\alpha_1W_1^{\alpha}=0;$$

$$X_2(U-g_1)-Y_2(U+g_1)-g_1(X_1+X_3+Y_1+$$

$$+Y_3+Y_{12}+Y_{13}+Y_{23}+Y_{123}+Z_1+Z_2+Z_3+$$

$$+Z_{12}+Z_{13}+Z_{23}+Z_{123}+V)-\alpha_1W_2^{\alpha}=0;$$

$$X_3(U-g_1)-Y_3(U+g_1)-g_1(X_1+X_2+Y_1+$$

$$+Y_2+Y_{12}+Y_{13}+Y_{23}+Y_{123}+Z_1+Z_2+Z_3+$$

$$+Z_{12}+Z_{13}+Z_{23}+Z_{123}+V)-\alpha_1W_3^{\alpha}=0;$$

$$(X_1+X_2)(U-(g_1+g_2))-Y_{12}(U+g_1+g_2)-$$

$$-(g_1+g_2)(X_3+Y_1+Y_2+Y_3+Y_{13}+Y_{23}+$$

$$+Y_{123}+Z_1+Z_2+Z_3+Z_{12}+Z_{13}+Z_{23}+Z_{123}+$$

$$+V)-\alpha_2W_{12}^{\alpha}=0;$$

$$(X_1+X_3)(U-(g_1+g_2))-Y_{13}(U+g_1+g_2)-$$

$$-(g_1+g_2)(X_2+Y_1+Y_2+Y_3+Y_{12}+Y_{23}+$$

$$+Y_{123}+Z_1+Z_2+Z_3+Z_{12}+Z_{13}+Z_{23}+Z_{123}+$$

$$+V)-\alpha_2W_{13}^{\alpha}=0;$$

$$(X_2+X_3)(U-(g_1+g_2))-Y_{23}(U+g_1+g_2)-$$

$$-(g_1+g_2)(X_1+Y_1+Y_2+Y_3+Y_{12}+Y_{13}+Y_{123}+$$

$$+Z_1+Z_2+Z_3+Z_{12}+Z_{13}+Z_{23}+Z_{123}+V)-$$

$$-\alpha_2W_{23}^{\alpha}=0;$$

$$(X_1+X_2+X_3)(U-(g_1+g_2+g_3))-Y_{123}(U+$$
$$+g_1+g_2+g_3)-(g_1+g_2+g_3)(Y_1+Y_2+Y_3+$$
$$+Y_{12}+Y_{13}+Y_{23}+Z_1+Z_2+Z_3+Z_{12}+Z_{13}+$$
$$+Z_{13}+Z_{23}+Z_{123}+V)-\alpha_3 W_{123}^{\alpha}=0;$$
$$(X_1+Z_1)(U-g_5)-g_5(X_2+X_3+Y_1+Y_2+Y_3+$$
$$+Y_{12}+Y_{13}+Y_{23}+Y_{123}+Z_2+Z_3+$$
$$+Z_{12}+Z_{13}+Z_{23}+Z_{123}+V)-\beta_1 W_1^{\beta}=0;$$
$$(X_2+Z_2)(U-g_5)-g_5(X_1+X_3+Y_1+Y_2+Y_3+$$
$$+Y_{12}+Y_{13}+Y_{23}+Y_{123}+Z_1+Z_3+$$
$$+Z_{12}+Z_{13}+Z_{23}+Z_{123}+V)-\beta_1 W_2^{\beta}=0;$$
$$(X_3+Z_3)(U-g_5)-g_5(X_1+X_2+Y_1+Y_2+Y_3+$$
$$+Y_{12}+Y_{13}+Y_{23}+Y_{123}+Z_1+Z_2+Z_{12}+$$
$$+Z_{13}+Z_{23}+Z_{123}+V)-\beta_1 W_3^{\beta}=0;$$
$$(X_1+X_2+Z_{12})(U-(g_5+g_4))-(g_5+g_4)\times$$
$$\times(X_3+Y_1+Y_2+Y_3+Y_{12}+Y_{13}+Y_{23}+Y_{123}+Z_1+$$
$$+Z_2+Z_3+Z_{13}+Z_{23}+Z_{123}+V)-\beta_2 W_{12}^{\beta}=0;$$
$$(X_1+X_3+Z_{13})(U-(g_5+g_4))-(g_5+g_4)\times$$
$$\times(X_2+Y_1+Y_2+Y_3+Y_{12}+Y_{13}+Y_{23}+Y_{123}+Z_1+$$
$$+Z_2+Z_3+Z_{12}+Z_{23}+Z_{123}+V)-\beta_2 W_{13}^{\beta}=0;$$
$$(X_2+X_3+Z_{23})(U-(g_5+g_4))-(g_5+g_4)\times$$
$$\times(X_1+Y_1+Y_2+Y_3+Y_{12}+Y_{13}+Y_{23}+Y_{123}+Z_1+$$
$$+Z_2+Z_3+Z_{12}+Z_{13}+Z_{123}+V)-\beta_2 W_{23}^{\beta}=0;$$
$$(X_1+X_2+X_3+Z_{123})(U-(g_5+g_4+g_3))-$$
$$-(g_5+g_4+g_3)(Y_1+Y_2+Y_3+Y_{12}+Y_{13}+Y_{23}+$$
$$+Y_{123}+Z_1+Z_2+Z_3+Z_{12}+Z_{13}+Z_{23}+V)-$$
$$-\beta_3 W_{123}^{\beta}=0;$$
$$X_1+X_2+X_3+Y_1+Y_2+Y_3+Y_{12}+Y_{13}+Y_{23}+$$
$$+Y_{123}+Z_1+Z_2+Z_3+Z_{12}+Z_{13}+Z_{23}+Z_{123}+$$
$$+V+W_1^{\alpha}+W_2^{\alpha}+W_3^{\alpha}+W_{12}^{\alpha}+W_{13}^{\alpha}+W_{23}^{\alpha}+W_{123}^{\alpha}+$$
$$+W_1^{\beta}+W_2^{\beta}+W_3^{\beta}+W_{12}^{\beta}+W_{13}^{\beta}+W_{23}^{\beta}+W_{123}^{\beta}=1.$$

Conclusions

In this paper the simplex form of the general polyhedron of arrangements is obtained. This form of the polyhedron of arrangements is necessary for applying of Karmarkar`s polynomial algorithm in solving auxiliary problems of linear programming in combinatorial cutting methods. The increase of the effectiveness of cutting methods is to be expected, in consequence of using this form.

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