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STUDY OF BIFURCATION IN SEMI-CLASSICAL MODELS OF SOLID-STATE LASERS

S.V. Kolomiets
Sumy National Agrarian University,
160 Kirova Str., Sumy 40021 Ukraine, e- mail: s_kolomiets@mail.ru

Dynamics of semi-classical model of solid-state laser is studied by method of bifurcation of cycle initiation, criteria of stability of limiting cycles, which occur in consequence of Hopf's bifurcation, are obtained, intervals of stability for pumping parameter, when stationary solution is considered as a supplementary parameter, are made. Joseph's method is used to integrate the system of four differential equations, which describe the dynamics of laser model, asymptotically. By means of reduction of four-dimensional problem to the two-dimensional one, periodical solution of the system of the differential equations is found.

This paper is devoted to the theoretical research of ways of initiation of non-linear dynamic modes in lasers with modulation of parameters, to the study of influence of change of solid-state laser parameters, which characterize its structure or serve as the control parameters of Q-spoiler to laser dynamics. The objective of the work is to determine the conditions of initiation of Hopf's bifurcation in dynamic system, to study stability of periodical oscillations, which arise as a result of the loss of stationary solution stability, when one of the model's parameters passes through its own bifurcation value and to formulate a periodical solution of the system of differential equations.

To integrate the system of differential equations asymptotically, two are used. The first one is bifurcation algorithm of cycle initiation, which requires the transition to the Poincare canonical form and the second one is D. Joseph's technique, which does not require such transition, and gives an advantage in case, when the system includes more than two equations.

Method of bifurcation of cycle initiation is used to analyze the system of differential equations, which describes the dynamics of semi-classical type laser model [1]:

$$\begin{cases} \dot{x} = Gx\left(y(q-yk)^{-1} - 1 - \varphi(x,a,b)\right) \\ \dot{y} = A - y - xy(q-yk)^{-1}, q = 1 + k + \dfrac{\Delta^2}{1+k} \end{cases}, \quad (1)$$

where x - is intensity of photon radiation field, y - is the difference of atom level occupancies (inversion), k - is ratio of the field constants' relaxation to polarization of atomic system, $\Delta = (\omega_0 - \omega_c)\upsilon_a^{-1}$, υ_a - is rate of relaxation of atomic polarization, ω_c - is center of spectral line, ω_0 - is proper frequency of resonator, G - is high parameter in the theory of class B lasers. The system (1) is obtained from the more complicated one by means of adiabatic exclusion of three fast phase coordinates, consequently some conditions for its correctness should be fulfilled, among them: $\upsilon_a >> \upsilon_i$, $\upsilon_a >> \upsilon_r$, where υ_i - is relaxation rate of inversion, υ_r - is rate of field decrease in the resonator. Function $\psi(x) = 1 + \varphi(x)$ describes the dependence of losses of the resonator on radiating power, where the constants a and b - are the control parameters.

Let's set out in writing basic ideas of the method of theory of bifurcation of cycle initiation [2]. In the beginning, the stationary solution of the system (1) is to be found, whereupon the origin of coordinates is to be moved to the found point. Jacoby's matrix of the right parts of the system is to be calculated within the found stationary solution, and then its proper values, which correspond to the case of stable focus, in other words, complex proper values with negative real part, are to be found. According to E. Hopf's theorem, the periodical oscillations can occur in the system (1), if the real part of the proper values, when increasing, passes through zero and further takes positive value, that corresponds to the loss of stability of the stationary solution. Value of one of the parameters, at which the proper numbers of Jacoby's matrix are merely imaginary ones, is called bifurcation value. As applied to the system (1) it means that the trace of Jacoby's matrix is equal to zero, whereof it is just possible to find the corresponding parameter value. Then Jacoby's matrix is calculated at bifurcation value of the parameter, whereupon we find its proper vector, which corresponds to the proper value of $i\omega_0$, and matrix of transformation is formed of the real and imaginary part of the vector that allows us to proceed to the new variables in the system (1).

Method of bifurcation of cycle initiation allows us not only to find periodical solution of the dynamic system, but also to solve the problem concerning its stability, by way of determination of Floquet's ratio sign. Presence of the high parameter G in the models of solid-state lasers allows us to simplify considerably the expression for Floquet's ratio sign determination that comes to the following stability criterion: $\operatorname{Re} \Phi < 0$, whereof we can obtain the interval of stability for one of the parameters of laser model. The periodical solution by itself is formed according to formulae obtained in [2].

Joseph's technique [3] is used for asymptotic integration of the system of four differential equations, which describes the dynamics of laser model of semi-classical type. Four-dimensional problem is reduced to the two-dimensional one by means of plane projection of phase space, formed by the proper vectors of linear part of operator, which correspond to the two imaginary proper values of Jacoby's matrix, calculated in stationary point. As the rest of the proper values have negative real part, their contribution to the plane projection of the solution is not taken into account.

Bifurcation parameter is represented in the form of the sum of its bifurcation value and of the unknown disturbing parameter, which expands into a series in powers of low parameter. In the same way, unknown frequency of oscillations and vector of phase coordinates expands into a series that allows making a transition from the initial system of non-linear equations to the succession of system of linear equations, since non-linear part of each of them becomes known vector-function, the previous system being integrated, at the same time the first of them is homogeneous and linear one and has the solution in the form of linear combination of periodical vectors. Unknown elements of expansion of frequency and disturbing parameter are found from the algebraic system of linear equations, obtained by means of application of Fredholm alternative to the right part of each system: the solutions of homogeneous conjugate system should be orthogonal to the right part of the corresponding system.

For the solution of the problem concerning stability of periodical solution, the factorization theorem is applied, according to which one Floquet's ratio is represented in the form of product of two efficients; the second of the efficients is equal to derivative of disturbing parameter with respect to the low parameter. Values to determine Floquet's ratio sign are also found.

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