

Credit Risk Optimization

Risk management is a core activity in asset allocation conducted by banks, insurance and investment companies, or any financial institution that evaluates risks. Credit risk is the risk of a trading partner not fulfilling their obligations in full on the due date or at any time thereafter.

Losses can result both from counterparty default, and from a decline in market value stemming from the credit quality migration of an issuer or counterparty.

Traditionally used tools for assessing and optimizing market risk assume that the portfolio return-loss is normally distributed.

With this assumption, the two statistical measures, mean and standard deviation, can be used to balance return and risk. The optimal portfolio is selected on the “efficient frontier”, the set of portfolios that have the best mean-variance profile. In other words, this is the set of Pareto optimal points with two conflicting criteria: mean and variance.

To cope with skewed return-loss distributions we consider Conditional Value-at-Risk (CVaR) as the risk measure. This measure is also called Mean Excess Loss, Mean Shortfall, or Tail VaR.

By definition, β -CVaR is the expected loss exceeding - Value-at-Risk (VaR), i.e., it is the mean value of the worst $(1 - \beta) * 100\%$ losses. For instance, at $\beta = 0.95$, CVaR is the average of the 5% worst losses. CVaR is a currency-denominated measure of significant undesirable changes in the value of the portfolio.

CVaR may be compared with the widely accepted VaR risk performance measure for which various estimation techniques have been proposed. E.g. VaR answers question what the maximum loss with the confidence level $\beta * 100\%$ over a given time horizon is. Thus, its calculation reveals that the loss will exceed VaR with

likelihood $(1 - \beta) * 100\%$, but no information is provided on the amount of the excess loss, which may be significantly greater.

Mathematically, VaR has serious limitations. In the case of a finite number of scenarios, it is a nonsmooth, nonconvex, and multiextremum function (with respect to positions), making it difficult to control and optimize. Also, VaR has some other undesirable properties, such as the lack of sub-additivity.

By contrast, CVaR is considered a more consistent measure of risk than VaR. CVaR supplements the information provided by VaR and calculates the quantity of the excess loss. Since CVaR is greater than or equal to VaR, portfolios with a low CVaR also have a low VaR.

Under quite general conditions, CVaR is a convex function with respect to positions, allowing the construction of efficient optimization algorithms. Since this can be accomplished by linear programming, a large number of instruments and scenarios can be handled.