JOURNAL OF NANO- AND ELECTRONIC PHYSICS
Vol. **9** No 3, 03033(4pp) (2017)

ЖУРНАЛ НАНО- ТА ЕЛЕКТРОННОЇ ФІЗИКИ
Том **9** № 3, 03033(4cc) (2017)

Kinetics of Strongly Nonequilibrium Magnon Gas Leading to Bose-Einstein Condensation

D.V. Slobodianiuk*, O.V. Prokopenko

Taras Shevchenko National University of Kyiv, 64/13, Volodymyrska Str., 01601 Kyiv, Ukraine

(Received 21 March 2017; revised manuscript received 26 April 2017; published online 30 June 2017)

Kinetic equations describing nonequilibrium magnon gas under the action of parametric pumping are considered with an account of kinetic instability processes. These processes strongly influence condition of magnon Bose-Einstein condensate formation. It is shown that the number of condensed magnons increases and the threshold of condensate formation decreases when kinetic instability processes are present in a system. Obtained results are important for further development of Bose-Einstein condensation theory for nonequilibrium magnons in magnetics.

Keywords: Kinetic equation, Bose-Einstein condensation, Magnon, Kinetic instability.

DOI: 10.21272/jnep.9(3).03033 PACS numbers: 75.30.Ds, 75.45. + j

1. INTRODUCTION

Magnons are quasiparticles associated with elementary excitations of magnetic subsystem of the crystal [1]. Magnons are bosons, thus they are described by Bose-Einstein quantum statistics. It is well known that Bose particles can exhibit so-called condensation, when at some system parameters number of particles in lowest energy state of the system rapidly increases [2]. For Bose gases of atoms Bose-Einstein condensation (BEC) is observed at rather low temperatures. However, for magnons this approach is not valid – number of magnons decreases with temperature [1]. Thus another approach to observe BEC in magnon gases was developed – strong increase in magnon density due to external parametric pumping [1, 3].

To describe magnon system with magnon BEC one need to take into account the whole spin-wave spectrum of weakly interacting magnons. Moreover, kinetics of condensation processes can not be described by classic thermodynamics laws – the system is far from the equilibrium and large group of parametrically pumped magnons is present at some point of the spectrum [1]. Thermalization of these magnons via magnon-magnon scattering processes results in BEC of bottom magnons [4 – 6].

Another group of processes one need to take into account is so-called kinetic instability processes [7]. These processes lead to a direct scattering of parametrically pumped magnons into bottom of the spectrum,

$$2\omega_p = \omega_{bottom} + \omega_{top}, \tag{1}$$

where ω_p is the pumping magnon frequency, ω_{bottom} is the lowest spin-wave spectrum frequency and ω_{top} is some spin-wave frequency of "up" magnons formed due to the scattering processes. These processes can effectively mediate BEC and also should be accounted.

The goal of this work is to investigate kinetics of strongly nonequilibrium magnon gas leading to BEC and analyze influence of kinetic instability processes on the formation of magnon condensate.

2. THEORETICAL MODEL

To describe evolution of magnon gas under the parametric pumping we will further develop approach described in [8]. In this work magnon gas is described as a sum of interacting different magnon groups that correspond to the different frequency and wave-vector regions of magnon spectrum (see Eqs. (2)).

First group of magnons is parametric magnons with population N_p created by an external electromagnetic pumping. It should be noted, that we will consider system dynamics after pumping is switched off leaving the system in a strongly nonequilibrium state with $N_p >> 1$ at $t = 0$. Exact details of pumping magnon group time evolution, while pumping is switched on, leading to the formation of given number of parametric magnons will not be considered in our model.

Next group of magnons is BEC magnons, having population N_c. Main feature of this magnon group is the threshold excitation of BEC in a system [2], that will be discussed later.

To take into account kinetic instability processes one need to consider two separate magnon groups N_b and N_t. The first group corresponds to the magnons close to the "bottom" of the spin-wave spectrum having the frequency close to ω_{min} and the second group is connected with "bottom" via kinetic instability processes having frequency about $2\omega_p - \omega_{min}$, where ω_p is the pumping frequency. Also magnon group N_b plays important role in Brillouin light scattering (BLS) signal formation from the bottom of magnon spectrum.

$$\begin{aligned}
\frac{dN_p}{dt} &= -\Gamma_p N_p - A_{kin} N_p^2 \left(N_b + N_t\right), \\
\frac{dN_b}{dt} &= -\Gamma N_b + A_{kin} N_p^2 N_b + A_{gb} N_g^3 - A_{bg} N_b^3 - A_{bc}\left(N_b^3 - N_{cr}^3\right)\Theta\left(N_b - N_{cr}\right), \\
\frac{dN_t}{dt} &= -\Gamma N_t + A_{kin} N_p^2 N_t + A_{gt} N_g^3 - A_{tg} N_t^3,
\end{aligned} \tag{2}$$

---

* denslobod@ukr.net

$$\frac{dN_g}{dt} = -\Gamma N_g + \Gamma_p N_p - A_{gt}N_g^3 + A_{tg}N_t^3 - A_{gb}N_g^3 + A_{bg}N_b^3,$$
$$\frac{dN_c}{dt} = -\Gamma N_c + A_{bc}\left(N_b^3 - N_{cr}^3\right)\Theta\left(N_b - N_{cr}\right).$$

Finally, all other magnons in a system can be described by magnon group N_g. Total number of magnons in a system is given as $N = N_b + N_c + N_t + N_p + N_g$. This value is conserved in conservative case, thus it can be used to verify numerical solution.

Magnon flux from i to j subsystem is proportional to N_j^3 with coefficient A_{ij} arising from 4-magnon scattering process in a system [1]. Described magnon flux between magnon groups is "natural" phenomena in interacting Bose magnon gas leading to energy redistribution through the spin-wave spectrum and thermalization. Mentioned earlier kinetic instability processes are specific processes arising from parametrically pumped magnons interacting with two exact points of spin-wave spectrum, having frequencies ω_{min} and $2\omega_p - \omega_{min}$. To account these processes in our kinetic equations for group amplitudes N_i we include additional terms in equations for N_b, N_t and N_p that are proportional to the phenomenological constant A_{kin}.

Finally, phenomenological damping in a system is introduced with terms $\Gamma_i N_i$.

Equations describing evolution of given magnon system (2) also take into account the fact that BEC formation is a threshold process, observed at $N_c > N_{cr}$ [2]. Thus, Heaviside function Θ $(N_c - N_{cr})$ is included in the corresponding equations. For simplicity we assumed that all relaxation rates of N_b ,N_c, N_t and N_g are equal. This simplification allow us to use only one parameter Γ that can be estimated experimentally while measuring the threshold of spin-wave parametric excitation [1]. Generally speaking those Gilbert damping coefficients can be different for different spin-wave spectrum regions and are proportional to the spin-wave frequency [1]. Damping of parametrically pumped magnon group is Γ_p and is usually larger then Γ [1, 8].

3. RESULTS AND DISCUSSION

To analyze the magnon dynamics the system of equations (2) was solved numerically. We used the following numerical parameters for constants in the equations: dissipation of pumping magnons $\Gamma_p = 26{,}3\ \text{s}^{-1}$, dissipation of other magnon groups $\Gamma = 6.1\text{s}^{-1}$. Magnon flux coefficients A_{ij}: $A_{bg} = A_{bc} = A_{tg} = 14250\ \text{s}^{-1}$, $A_{gb} = A_{gt} = 43\ \text{s}^{-1}$. These parameters are similar to ones used in work [8]. Finally, kinetic constant A_{kin} was varied in our calculation. We need to stress here, that in typical experiment kinetic instability processes and their efficiency strongly depend on external magnetic field applied to a sample [7]. By choosing an appropriate magnetic field value one can totally suppress kinetic instability processes or maximize its influence on a system [7].

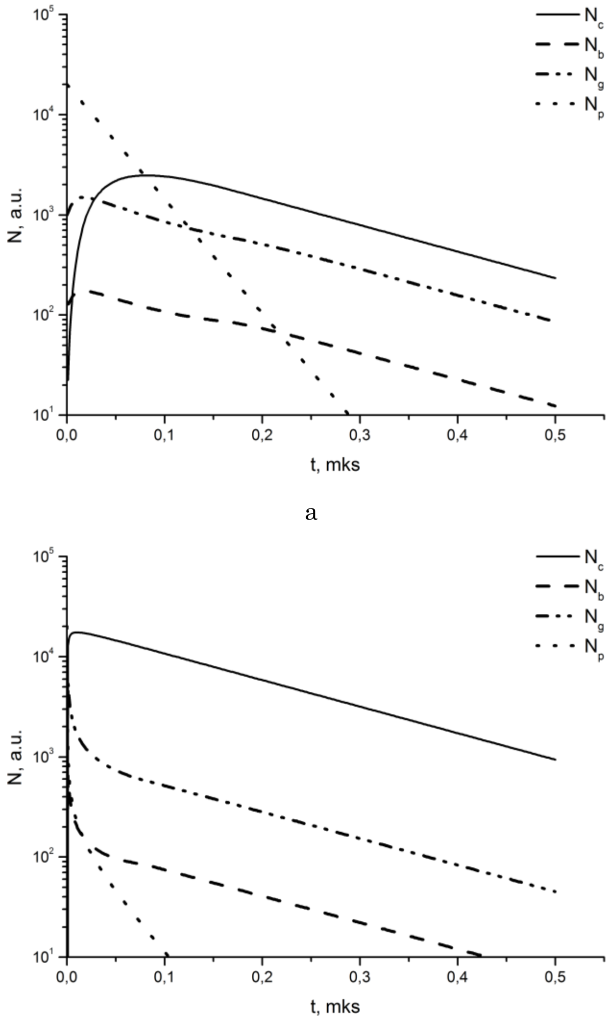
We used the following initial conditions for magnon groups' amplitudes:

$$\begin{aligned} &N_b(0) = 135;\ N_g(0) = 1000; \\ &N_t(0) = 80; N_c(0) = 0; \\ &N_{cr} = 135. \end{aligned} \tag{3}$$

Initial number of pumped magnons N_p was varied to simulate different pumping powers in a system.

Fig. 1 shows time evolution of magnon groups for two separate cases: when kinetic instability processes are absent in a system (a), and when they are included (b). As one can see in the first case magnon flux from the pumped magnon group N_p is much less prominent, then in the second one. Kinetic instability processes, as it can be seen from equations (2), stimulate additional dissipation for pumping magnons leading to a more rapid decay of this group with time. This leads to another redistribution of magnons through the whole magnon spectrum: in the case of kinetic instability processes (see Fig. 1b) number of condensed magnons also is tens times higher than in the case (a).

To further investigate this effect, Fig. 2 shows dependence of the N_c/N_p ratio on pumping magnons number



a

b

Fig. 1 – Time evolution of magnon groups for $N_p = 20000$, $A_{kin} = 0$ (a), $A_{kin} = 10A_{gb}$ (b)

for different values of kinetic instability constant. This ratio can be threatened as an order parameter of a system [9], and shows a degree of magnon condensation. Here we need to stress, that most magnon BEC experiments are carried using BLS technique [10, 11]. Due to a finite wave-vector and frequency resolution of this method information of magnon condensate is received not from a single point of a spectrum, that corresponds to the minimum spin-wave frequency *(ω_{min}, k_{min})*, but from some finite frequency and wave-vector region [10, 11]. This leads to the fact, that resulting BLS signal contains information not only about condensed magnons N_c but also from magnons in the vicinity of $\omega_{min} - N_b$. Separating these two contributions in the resulting BLS signal is a challenging task, yet to be solved.

To investigate system behavior under the action of varying external parametric pumping, and thus varying pumping group starting condition N_p we need to modify initial conditions (3) of our kinetic equations. They are written for the case of given number of pumping magnons $N_p = 20000$. In the case of no pumping $N_p = 0$, all other magnon groups are in thermodynamic equilibrium, thus $N_i(0) = 0$. Obviously in a case of intermediate pumping levels starting numbers of magnons groups will differ from (3).

To take this fact into account we can modify our starting conditions in a following way:

$$\begin{aligned} &N_b(0) = q_b N_p, \\ &N_g(0) = q_g N_p, \\ &N_t(0) = q_t N_p, \\ &N_c(0) = 0, \\ &N_{cr} = 135. \end{aligned} \tag{4}$$

Here we proposed a simple linear relation between the starting magnon numbers and the pumping magnon number. Coefficients q are chosen to satisfy conditions (3) when $N_p = 20000$ and, obviously, all $N_i(0) = 0$ when $N_p = 0$.

As one can see from Fig. 2 kinetic instability process leads to the decrease of condensate formation threshold. It is clear from the solution of equations (2), these processes effectively mediate transfer from pumping magnon group to bottom magnons, leading to the condensate formation.

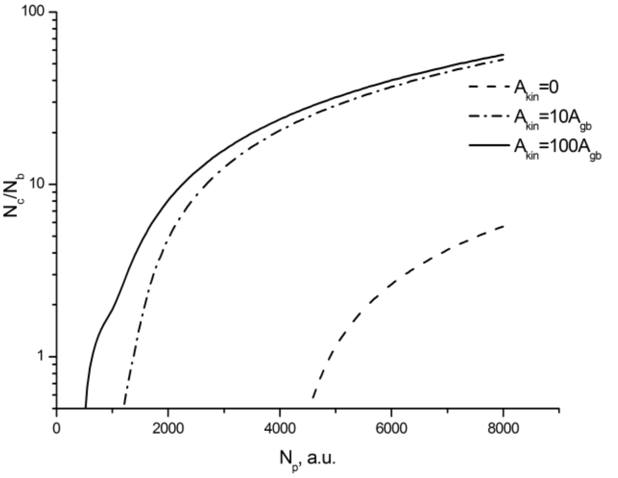


Fig. 2 – Dependence of N_c/N_b ratio on pumping magnons number

Second prominent feature of the system with allowed kinetic instability processes is the significant increase of N_c/N_p ratio. As one can see for given pumping power this ratio is about 10 times larger in the case of kinetic instability processes taken into account.

4. CONCLUSION

In present work kinetics of strongly nonequilibrium magnon gas under the action of parametric pumping was analyzed. System dynamics was described using phenomenological kinetic equations describing change of magnon populations in different magnon groups. Additional terms, taking into account kinetic instability processes in magnon gas, were introduced. It was shown, that mentioned processes play important role in magnon BEC formation. The obtained results are important for understanding of quantum effects in Bose gases and nonequilibruim magnon dynamics.

ACKNOWLEDGEMENTS

Authors are grateful to Dr. D. Bozhko for helpful discussions. Publication is based on the research provided by the grant support of the State Fund For Fundamental Research of Ukraine, Taras Shevchenko National University of Kyiv (Grant 16BF052-01) and National Academy of Sciences of Ukraine.

Кінетика сильно нерівноважного магнонного газу за умов Бозе-Ейнштейнівської конденсації

Д.В. Слободянюк, О.В. Прокопенко

Київський національний університет імені Тараса Шевченка,
вул. Володимирська, 64/13, 01601 Київ, Україна

В роботі проаналізовані кінетичні рівняння, що описують нерівноважний магнонний газ під дією параметричної накачки з урахуванням процесів кінетичної нестійкості. Встановлено, що ці процеси суттєво впливають на умови формування Бозе-Ейнштейнівського конденсату магнонів. Показано, що кількість сконденсованих магнонів зростає, а поріг утворення конденсату зменшується за наявності в системі процесів кінетичної нестійкості. Одержані результати важливі для подальшого розвитку теорії Бозе-Ейнштейнівської конденсації нерівноважних магнонів в магнетиках.

Ключові слова: Кінетичне рівняння, Бозе-Ейнштейнівська конденсація, Магнон, Кінетична нестійкість.

Кинетика сильно неравновестного магнонного газа в условиях Бозе-Эйнштейновской конденсации

Д.В. Слободянюк, О.В. Прокопенко

Киевский национальный университет имени Тараса Шевченка,
ул. Владимирская, 64/13, 01601 Киев, Украина

В роботе проанализированы кинетические уравнения, которые описывают сильно неравновесный магнонный газ под действием параметрической накачки с учетом процессов кинетической неустойчивости. Установлено, что эти процессы существенно влияют на условия формирования Бозе-Эйнштейновского конденсата магнонов. Показано, что число сконденсированных магнонов увеличивается, а порог формирования конденсата уменьшается при наличии процессов кинетической неустойчивости в системе. Полученные результаты важны для последующего развития теории Бозе-Эйнштейновской конденсации неравновесных магнонов в магнетиках.

Ключевые слова: Кинетическое уравнение, Бозе-Эйнштейновская конденсация, Магнон, Кинетическая неустойчивость.

REFERENCES

1. S.O. Demokritov, V.E. Demidov, O. Dzyapko, G.A. Melkov, A. A. Serga, B. Hillebrands, A.N. Slavin *Nature* **443**, 430 (2006).
2. L.D. Landau, E.M. Lifshitz, *Statistical Physics*, 3rd ed. (Elsevier: 2013).
3. T. Giamarchi, C. Rüegg, O. Tchernyshyov, *Nat. Phys.* **4**, 198 (2008).
4. D.A. Bozhko, P. Clausen, A.V. Chumak, Yu.V. Kobljanskyj, B. Hillebrands, A.A. Serga, *Low Temp. Phys.* **41**, 1024 (2015).
5. O. Dzyapko, V.E. Demidov, S.O. Demokritov, *Phys.-Usp.* **53**, 853 (2010).
6. A. Serga, et al, *Nature Commun.* **5**, 3452 (2014).
7. G.A. Melkov, V.L. Safonov, S.V. Sholom *J. Magn. Magn. Mat.* **132**, 180 (1994).
8. D. Bozhko, A. Serga, P. Clausen, V. Vasyuchka, F. Heussner, G. Melkov, A. Pomyalov, V. L'vov, B. Hillebrands, *Nat. Phys.* **12**, 1057 (2016).
9. S.M. Rezende, *Phys. Rev. B* **79**, 174411 (2009).
10. O. Büttner, M. Bauer, S.O. Demokritov, B. Hillebrands, Yu.S. Kivshar, V. Grimalsky, Y. Rapoport, A.N. Slavin, *Phys. Rev. B* **61**, 11576-11587 (2000).
11. C.W. Sandweg, B. Jungfleisch, V. Vasyucka, A. Serga, P. Clausen, H. Schultheiss, B. Hillebrands, A. Kreisel, P. Kopietz, *Rev. Sci. Instrum.* **81**, 073902 (2010)