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Energy Spectrum of a Particle in an Infinitely Deep Potential Well with a Non-flat Bottom

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To model the quantum behavior of current carriers in heterogeneous nanostructures, different potential wells and barriers are widely used. Of particular interest are those (including artificial ones) for which the corresponding Schrödinger equation can be solved exactly. One of them is an infinitely deep potential well with a flat bottom defined as $U(x) = \infty$ for $|x| \geq a$ and $U(x) = 0$ for $|x| < a$. Here, we report on exact solution of the stationary Schrödinger equation in the case, when the potential bottom is non-flat, i.e., $U(x) = -fx$ $(f > 0)$ if $|x| < a$. Physically, this means that the particle in the well is subjected to an external force f in the x-direction.

For this potential, the stationary Schrödinger equation reads

$$\frac{d^2}{dx^2}\psi(x) + \frac{2m}{\hbar^2}(E + fx)\psi(x) = 0, \tag{1}$$

where $\psi(x)$ is the wave function of the particle of mass m, $\hbar$ is the Planck constant, and E is the particle energy. Introducing new variables

$$y = -\left(x + \frac{E}{f}\right)\left(\frac{2mf}{\hbar^2}\right)^{1/3}, \quad \Psi(y) = \psi\left(-\frac{E}{f} - y\left(\frac{\hbar^2}{2mf}\right)^{1/3}\right), \tag{2}$$

Eq. (1) is reduced to the well-known equation

$$\frac{d^2}{dy^2}\Psi(y) - y\Psi(y) = 0. \tag{3}$$

Its general solution is expressed through the Airy functions $\mathrm{Ai}(y)$ and $\mathrm{Bi}(y)$ as $\Psi(y) = \alpha\mathrm{Ai}(y) + \beta\mathrm{Bi}(y)$. The parameters α and β and the energy E are uniquely determined from the boundary conditions $\alpha\mathrm{Ai}(y_\pm) + \beta\mathrm{Bi}(y_\pm) = 0$, where $y_\pm = (\pm a - E/f)(2mf/\hbar^2)^{1/3}$, and the normalization condition $(\hbar^2/2mf)^{1/3}\int_{y_-}^{y_+}|\Psi(y)|^2\,dy = 1$. For example, the discrete energy spectrum of the particle is determined as real solutions of the equation

$$\mathrm{Ai}(y_-)\mathrm{Bi}(y_+) - \mathrm{Ai}(y_+)\mathrm{Bi}(y_-) = 0 \tag{4}$$

with respect to E. In particular, if $f \to 0$ then Eq. (4) yields the known result $E_n = \pi^2\hbar^2 n^2/8ma^2$ $(n = 0,1,2,\ldots)$, and if $f \to \infty$ then

$$\boldsymbol{E_n = -af + \left(\frac{9\pi^2\hbar^2 f}{8m}\right)^{1/3}\left(n + \frac{3}{4}\right)^{2/3}}. \tag{5}$$