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# Minimal Set of Equations for Drift of Ferromagnetic Nanoparticles Induced by Magnetic Fields in Fluids

Kvasnina O.V., *Student*; Yermolenko A.S., *Student*;
Lyutyy T.V., *Associate Professor*; Denisov S.I., *Professor*
Sumy State University, Sumy

Recently, it has been established that ferromagnetic nanoparticles subjected to a periodic force and a non-uniformly rotating magnetic field can drift in a viscous fluid due to the Magnus effect. Because the drift phenomenon is of interest for applications such as particle separation, in this work we present a minimal set of equations for describing this phenomenon when a periodic force is induced by a gradient magnetic field.

We consider a spherical particle of radius $a$ which is under the action of the gradient magnetic field $\mathbf{H}_g = gx\sin(\Omega t - \phi)\,\mathbf{e}_x$ and non-uniformly rotating magnetic field $\mathbf{H} = H_m(\cos\psi\,\mathbf{e}_x + \sin\psi\,\mathbf{e}_y)$. Here, $g$ is the magnetic field gradient, $x$ is the space coordinate, $\Omega$ and $\phi$ are the frequency and initial phase, respectively, $H_m$ is the rotating field magnitude, the azimuthal angle $\psi$ is a given periodic function of time with period $2\pi/\Omega$, and $\mathbf{e}_x$ and $\mathbf{e}_y$ are the unit vectors along the axes $x$ and $y$. Assuming that the Reynolds numbers are small, the particle is single-domain, its magnetization $\mathbf{M} = M\mathbf{m}$ ($|\mathbf{M}| = M$) is "frozen" into the body and $\mathbf{m} = \cos\varphi\,\mathbf{e}_x + \sin\varphi\,\mathbf{e}_y$, we derived the following set of equations

$$\begin{aligned} \mathbf{u} &= \big(\mathbf{e}_x + (\gamma/\alpha)\dot{\varphi}\mathbf{e}_y\big)\cos\varphi\sin(2\pi\tau - \phi), \\ \dot{\varphi} &= \alpha[\sin(\psi - \varphi) - q\sin\varphi\sin(2\pi\tau - \phi)] \end{aligned} \tag{1}$$

for the dimensionless particle velocity $\mathbf{u} = \mathbf{v}/v_0$ ($\mathbf{v}$ is the dimension velocity, $v_0 = 2Mga^2/9\eta$, $\eta$ is the dynamic viscosity of the fluid) and the azimuthal angle $\varphi$ of $\mathbf{M}$. Here, the overdot denotes the derivative with respect to the dimensionless time $\tau = \Omega t/2\pi$, $\alpha = \pi M H_m/3\eta\Omega$ and $\gamma = \rho a^2 M H_m/36\eta^2$ ($\rho$ is the fluid density) are the dimensionless parameters, and the time-dependent parameter $q$ is defined as

$$q = (g/H_m)\big[x(0) + (2\pi v_0/\Omega)\int_0^\tau u_x(\tau')d\tau'\big], \tag{2}$$

where $\boldsymbol{x}(\mathbf{0})$ is the initial $\boldsymbol{x}$ coordinate of the particle. The dimensionless particle drift velocity in the steady state, $\mathbf{u}_{dr} = \mathbf{lim}_{n\to\infty}\int_n^{n+1}\mathbf{u}(\boldsymbol{\tau}')d\boldsymbol{\tau}'$, can be calculated by solving Eqs. (1) together with (2).