Cryst. Res. Technol. 41, No. 4, 388 – 399 (2006) / DOI 10.1002/crat.200510592

Conductivity and temperature coefficient of resistance of multilayered polycrystalline films

A. Chornous[1], L. Dekhtyaruk[1], M. Marszalek[2], and I. Protsenko*[1]

[1] Sumy State University, R. Corsakova, 2, 40007, Sumy, Ukraine
[2] The Henryk Niewodnichanski Institute of Nuclear Physics PAN, Radzikovskiego, 152, 31-342 Krakov, Poland

Received 14 April 2005, revised 17 August 2005, accepted 19 October 2005
Published online 15 March 2006

Key words multilayered films, conductivity, temperature coefficient of resistance, Mayadas – Shatzkes model, grain boundary, transport phenomena.
PACS 68.55.Jk, 72.15.Eb, 72.15.Lh, 72.15.Qm

We calculate the electric conductivity and the temperature coefficient of resistance (TCR) of a multilayered film consisting of the alternating polycrystalline metal layers of different thickness and purity within the relaxation time formalism. In the case of Cr, Cu and Co-based multilayered films we perform verification of our analytical formulas and demonstrate a qualitative agreement between the theoretically calculated values of the TCR and experiment.

1 Introduction

In the last years the electron transport properties of multilayered structures consisting of the alternating metal layers are being investigated intensively both experimentally and theoretically. Firstly, thin-film systems are used widely in the modern microelectronic devices. Secondly, these systems are very promising objects: combining metals with the different physical properties one may obtain conductors with the substantially new electric characteristics which are differed radically from the properties of usual bulk metals. In particular, the multilayered structures consisting of the alternating magnetic and non-magnetic layers exhibit a giant magnetoresistance effect, when applying of an external weak magnetic field leads to significant changes in the resistance of the device [1,2].

In this paper we investigate the electric conductivity, σ, and the temperature coefficient of resistance (TCR), β, of a multilayered film consisting of the alternating polycrystalline metal layers within the relaxation time formalism and modified Mayadas – Shatzkes model (MS model) [3]. We derived the exact analytical expressions for the conductivity σ and the TCR β within this model. We analyze their asymptotical behavior in the cases when metal layers are either thick or thin as to compare with the electron mean-free path. The temperature coefficient of resistance was calculated taking into account the effects of temperature variations both of the layer thickness and an average crystalline size along the layer plane. We performed also the detailed numerical calculation both of the σ and β for the various parameters of electron scattering in the bulk of the metal layer and at the interlayer interface. We obtained a good qualitative agreement between the theoretically calculated values of the TCR and experiment in the case of Cr, Cu and Co-based multilayered films.

2 Problem statement: Conductivity of the multilayered film

Let us consider a multilayered periodic film (ML, see figure 1) consisting of the alternating polycrystalline metal layers of the different thickness and different purity. Let x-axis be perpendicular to the interface. We take y- and z-axes are directed along the layers, which supposed to be infinite. (Consequently, electrons move along

* Corresponding author: e-mail: protsenko@aph.sumdu.edu.ua, dekhtyaruk@mail.ru

the yz-plane as in a bulk metal). Let an external electric field, $\mathbf{E} = (0, E_y, 0)$, and the temperature gradient, $\nabla T = \left(0, \partial T/\partial y, 0\right)$, are applied in parallel to the interface. Obviously, the considered system is a periodical structure with the bi-layer of the thickness $d = d_1 + d_2$ as a multilayer repeat period. Consequently, we may reduce the problem to the calculating of the longitudinal conductivity and the TCR of a bi-layer film with the periodical boundary conditions.

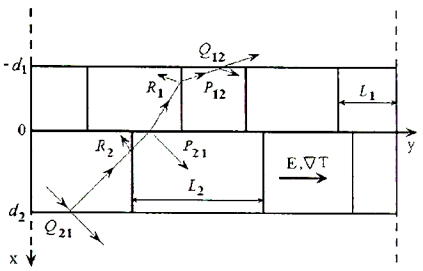

Fig. 1 Schematic view of the multilayered film consisting of the polycrystalline layers of different thickness $(d_i \neq d_j)$ and purity $(\lambda_i \neq \lambda_j)$: a possible electron trajectory within the repeat period is shown by a broken line.

The current density **J** is given by

$$\mathbf{J} = \frac{2e}{dh^3} \sum_{i=1}^{2} \int_0^{d_i} dx \int d^3p \, \mathbf{v}_i f_i(|x|, \mathbf{p}) . \tag{1}$$

To find **J**, one have resolved the Boltzmann kinetic equation for the electron distribution function $f_i(\mathbf{r}, \mathbf{p})$ in each layer of the ML repeat period. The function $f_i(\mathbf{r}, \mathbf{p})$ may be written in the form

$$f_i(\mathbf{r}, \mathbf{p}) = f_0(\varepsilon_i) - \frac{\partial f_0}{\partial \varepsilon_i} \Psi_i(\mathbf{r}, \mathbf{p}) . \tag{2}$$

Within the relaxation time approximation model for the collision term, we obtain

$$\frac{\partial \Psi_i}{\partial t} + \frac{\Psi_i}{\tau_i} = e \mathbf{v}_i \left\{ \mathbf{E} + \frac{1}{e} \frac{\varepsilon_i - \mu}{T} \nabla T \right\} \equiv g_i(\mathbf{p}). \tag{3}$$

Here e the electron charge, $\mathbf{v}_i$ and $\varepsilon_i l$ are the velocity and the energy, respectively, of an electron in the i-th layer; **r** and **p** are its coordinate and the quasi-momentum; μ is the chemical potential, T is the temperature, h is the Planck constant. $f_0(\varepsilon_i)$ is the Fermi distribution function, and $t = x/v_x$ is the time of flight of an electron along the trajectory. Within MS model [3], the "effective" electron relaxation time τ_i (which describes the electron scattering in the bulk of the sample and scattering at the grain boundaries) can be written as

$$\frac{1}{\tau_i} = \frac{1}{\tau_{0i}} \left\{ 1 + \alpha_i \frac{p_{Fi}}{|p_{yi}|} \right\}, \tag{4}$$

where τ_{0i} is the characteristic time of the elastic electron scattering in the bulk of the sample, i.e. it is the "memory breaking" time; p_{Fi} is the Fermi quasi-momentum; p_{yi} is the component of the electron quasi-momentum which is perpendicular to the grain boundaries. The grain-boundary parameter, $\alpha_i = \frac{\lambda_i}{L_i} \frac{R_i}{1 - R_i}$, determines the structure of the sample depending on the ratio between the electron mean-free path, λ_i, and the average crystallite size in the layer plane, L_i. On the other hand, this parameter determines the characteristic properties of the electron scattering at the grain boundaries (since R_i is a probability of specular scattering of electrons by the grain boundaries).

The general solution of Eq.(3) is given by

$$\Psi_i(\mathbf{r},\mathbf{p}) = F_i e^{\frac{\lambda_i^* - t}{\tau_i}} + \int_{\lambda_i^*}^{t} dt' g_i(\mathbf{p}) e^{\frac{t'-t}{\tau_i}}, \tag{5}$$

where constants F_i are determined by the boundary conditions; λ_i^* is the time of the last scattering event of an electron at the interlayer interface, $x_s = (-d_1, 0, d_2)$, $\lambda_i^* < t$

$$\lambda_i^* = t - \left| \frac{x - x_s}{v_{xi}} \right|. \tag{6}$$

For the sake of simplicity, we assume that the Fermi surface in each metal layer is spherical. In this case, we may neglect by the renormalization of the chemical potential of the charge carriers due to their interaction with the interlayer interface [5]. Neglecting also by the edge effects, we may write the boundary conditions for the functions $\Psi_i(\mathbf{r},\mathbf{p})$ in the following form [4, 5]

$$\Psi_i^{s_j}(s_i d_i, \mathbf{p}) = P_{ij} \Psi_i^{s_i}(s_i d_i, \mathbf{p}') + Q_{ji} \tilde{\Psi}_j^{s_j}(s_i d_i, \mathbf{p}''), \tag{7}$$

$$\Psi_i^{s_j}(0, \mathbf{p}) = P_{ij} \Psi_i^{s_j}(0, \mathbf{p}') + Q_{ji} \Psi_j^{s_i}(0, \mathbf{p}''). \tag{8}$$

Here, $i \neq j = 1,2$; $P_{ij} = const$ is the probability of specular reflection of electrons from the interface between i-th and j-th layers, $Q_{ji} = const$ is the probability of electron transmission from j-th layer into the i-th layer without scattering. These parameters satisfy the following condition $P_{ij} + Q_{ji} \leq 1$. Quasi-momenta $\mathbf{p}$, $\mathbf{p}'$ and $\mathbf{p}''$ are related to each other by the conditions of conservation of the energy and the tangential (parallel to the interface) component of the electron quasi-momentum; $s_i = sign\, v_{xi}$ determines the sign of the component of the charge carriers in i-th layer, v_{xi}, normal to the interface $(s_1 = -, s_2 = +)$. Symbol "tilde" in the second term in the right-hand-side of Eq.(7) means that function $\tilde{\Psi}_j$ describes the charge-carrier distribution in layers which are adjacent to the bi-layer (the ML repeat period) that is considered.

Matching functions $\Psi_i(\mathbf{r},\mathbf{p})$ at the interlayer interfaces (by substituting Eq.(5) into Eqs.(7,8)), we obtain a set of linear algebraic equations for the coefficients F_i whose solution allows us to determine functions $\Psi_i(\mathbf{r},\mathbf{p})$ and calculate the current density $\mathbf{J}$ (see Eq.(1)). Consequently, comparison of the results of our calculation with the well-known formula

$$\mathbf{J} = \sigma \mathbf{E}, \tag{9}$$

yields the following expression for the conductivity of the multilayered polycrystalline film

$$\sigma = \frac{1}{d} \sum_{i=1}^{2} d_i \sigma_{0i} \Phi_i. \tag{10}$$

Here, σ_{0i} is the conductivity of a bulk single-crystal sample. Functions Φ_i depend on the thicknesses of the metal layers

$$\Phi_i = f(\alpha_i) - \langle G_i \rangle, \tag{11}$$

$$G_i = 1 - \frac{1}{\Delta} \left\{ (1 + P_{ij}^2 E_i)(1 + P_{ji}^2 E_j) - Q_{ij} Q_{ji} E_i E_j \right\} \left\{ C_i (1 - P_{ji} E_j) + Q_{ji} \tau_{j,i} E_j C_j \right\} \equiv 1 - \frac{A B_i}{\Delta},$$

$$C_i = P_{ij}(1 - E_i) + Q_{ji} \tau_{j,i} (1 - E_j),$$

$$\Delta = 1 - P_{ij}^2 E_i^2 - P_{ji}^2 E_j^2 - 2 Q_{ij} Q_{ji} E_i E_j + (Q_{ij} Q_{ji} - P_{ij} P_{ji})^2 E_i^2 E_j^2, \tag{12}$$

$$E_i = \exp\left\{-\frac{k_i H_i}{x}\right\},\quad H_i = 1+\frac{\alpha_i}{\cos\varphi\sqrt{1-x^2}},\quad k_i = \frac{d_i}{\lambda_i},\quad \tau_{j,i} = \frac{\tau_{0j}}{\tau_{0i}}\frac{H_i}{H_j} \equiv \tau_{0j,i}H_{i,j}, \tag{13}$$

$$\langle ... \rangle = \frac{6}{\pi k_i}\int_0^{\pi/2} d\varphi\cos^2\varphi \int_0^1 dx \frac{(x-x^3)(1-E_i)}{H_i^2}\left\{\ \ldots\ \right\}. \tag{14}$$

Functions $f(\alpha_i)$ describe the conductivity of a bulk polycrystalline sample in the MS model [3]

$$f(\alpha_i) = 1-\frac{3}{2}\alpha_i + 3\alpha_i^2 - 3\alpha_i^3\ln\left(1+\frac{1}{\alpha_i}\right) \cong \begin{cases} 1-\dfrac{3}{2}\alpha_i + 3\alpha_i^2, & \alpha_i << 1, \\ \dfrac{3}{4\alpha_i} - \dfrac{3}{5\alpha_i^2}, & \alpha_i >> 1. \end{cases} \tag{15}$$

Consequently, in the general case, the conductivity of the ML polycrystalline film is given by Eqs.(10)-(14). The resulting formulas are overcomplicated to be calculated analytically. Consequently, the further analysis is based on numerical calculations. Nevertheless, in the limiting cases, when parameters k_i and α_i are large (or small) as to compare with unity, we obtained simple analytical formulas for the conductivity σ. It allows one to compare theory and experiment easily.

Firstly, let us discuss the case, when $k_i >> 1$, i.e. the electron mean-free path λ_i is much smaller than the layer thickness d_i. Then the exponential terms in Eq.(11) are vanishing and we can perform the integration over φ and x variables. In this case, the conductivity of the ML film, σ, is given by Eq.(10) where the thickness-depending function Φ_i may be written as

$$\Phi_i = f(\alpha_i) - \frac{3}{8k_i}\left\{\left(1-P_{ij}\right)\Gamma_{1,i} - Q_{ji}\tau_{0j,i}\Gamma_{2,i}\right\}, \tag{16}$$

$$\Gamma_{1,i} = 1-\frac{32}{3\pi}\alpha_i + 12\alpha_i^2 + \frac{16}{\pi}\left\{5-\left(4-5\alpha_i^2\right)I_i\right\}\alpha_i^3 - 40\alpha_i^4, \tag{17}$$

$$\Gamma_{2,i} = 1-\frac{16}{3\pi}\left\{\alpha_i+\alpha_j - \frac{3\pi}{4}\left(\alpha_i^2+\alpha_i\alpha_j+\alpha_j^2\right) - 3\left(\alpha_i^3+\alpha_i^2\alpha_j+\alpha_i\alpha_j^2+\alpha_j^3\right) + \right.$$
$$\left. + \frac{3\pi}{2}\left(\alpha_i^4+\alpha_i^3\alpha_j+\alpha_i^2\alpha_j^2+\alpha_i\alpha_j^3+\alpha_j^4\right) + \frac{3}{\alpha_i-\alpha_j}\left[\alpha_i^4\left(1-\alpha_i^2\right)I_i - \alpha_j^4\left(1-\alpha_j^2\right)I_j\right]\right\}, \tag{18}$$

$$I_i = \begin{cases} \dfrac{1}{\sqrt{1-\alpha_i^2}}\ln\dfrac{1+\sqrt{1-\alpha_i^2}}{\alpha_i}, & \alpha_i \le 1, \\ \dfrac{\arccos\left(\dfrac{1}{\alpha_i}\right)}{\sqrt{\alpha_i^2-1}}, & \alpha_i > 1. \end{cases}$$

We may simplify additionally this equation in the following cases. Let, either the electron mean-free path is less than the average grain size ($L_i >> \lambda_i$) or the grain boundaries are almost transparent for electrons ($R_i << 1$). Then, the grain-boundary parameter α_i is small enough, $\alpha_i << 1$, and Φ_i is given by

$$\Phi_i = 1-\frac{3}{2}\alpha_i - \frac{3}{8k_i}\left\{\left(1-P_{ij}\right)\left(1-\frac{32}{3\pi}\alpha_i\right) - Q_{ji}\tau_{0j,i}\left(1-\frac{16}{3\pi}\left(\alpha_i+\alpha_j\right)\right)\right\},\quad \alpha_i << 1. \tag{19}$$

In the opposite case, $\alpha_i >> 1$, when metal layers have a fine-grained structure ($L_i << \lambda_i$) or the grain boundaries are almost opaque for charge carriers ($1-R_i << 1$), we obtain for the thickness-depending function Φ_i

$$\Phi_i = \frac{3}{4\alpha_i}\left\{1-\frac{1}{4k_i\alpha_i}\left[\left(1-P_{ij}\right)\left(1-\frac{512}{105\pi\alpha_i}\right)-Q_{ji}\tau_{0j,i}\frac{\alpha_i}{\alpha_j}\left(1-\frac{256}{105\pi}\frac{\alpha_i+\alpha_j}{\alpha_i\alpha_j}\right)\right]\right\}, \quad \alpha >> 1. \tag{20}$$

Let discuss the other limiting case, $k_i << 1$, when λ_i is much larger than the layer thickness d_i. Then we obtain the following approximated result

$$\Phi_i \cong \frac{3}{4}\frac{\left(1+P_{ij}\right)\left(1-P_{ji}\right)+Q_{ij}Q_{ji}+2Q_{ji}d_{j,i}}{\left(1-P_{ij}\right)\left(1-P_{ji}\right)-Q_{ij}Q_{ji}}k_i \begin{cases} \ln\dfrac{1}{k_i}, & \alpha_i \le k_i, \quad (21a) \\ \ln\dfrac{1}{k_i}-\dfrac{4}{\pi}\alpha_i, & k_i < \alpha_i << 1, \quad (21b) \\ \ln\dfrac{1}{\alpha_i k_i}, & 1 < \alpha_i << \dfrac{1}{k_i}. \quad (21c) \end{cases}$$

where $d_{j,i} = d_j / d_i$ is the layer-thickness ratio.

In this case, like to the case of a thin single-crystal film [6], the value of the function Φ_i is determined by the small group of "effective" electrons. Grazing along the interlayer interface these electrons do not undergo scattering events during the time of the order of τ_{0i} (where τ_{0i} is the electron relaxation time in a bulk single-crystal sample). It is precisely these electrons that are determined the value of the effect (see, e.g. the Pippard "inefficiency" conception [7]). The relative number of these electrons is of the order of k_i. The logarithmic factor, $\ln(1/k_i)$, is the contribution of the surface scattering of the electrons which propagate at small angles (which are of the order of $\varphi_i << d_i / l_i$) to the interlayer interface.

Note, if $\alpha_i \le k_i$, electron scattering at the interlayer interfaces is the main relaxation mechanism and we may neglect the polycrystalline nature of the metal layers. In other words, formally, we may consider these layers as single-crystal layers. Indeed, in this case the contribution of the grain-boundary scattering to the resistivity of the ML film is vanishing (see Eq.(21a) and Ref.[8]). Increasing of the grain-boundary parameter α_i leads to the increase of the role of the grain-boundary scattering (the relative number of the electrons which undergoes this type of the scattering is of the order of $(4/\pi)\alpha_i$, see Eq.(21b)). At the further increase of the parameter α_i, when $\alpha_i >> 1/k_i$, the grain-boundary scattering dominates. In this case, the ML film may be considered as an effectively bulk (or thick) sample because the electron scattering at the layer interfaces (the "external" size-effect) is vanishing as to compare with the intrinsic scattering at the grain boundaries (the "internal" size-effect).

Comparing the asymptotical formulas for the conductivity of the multilayered film with the formulas for the conductivity of the two-layered film we may conclude the following. Formally, we may consider the ML film as a two-layered polycrystalline film (see [9,10]) where electron scattering at the external surfaces is described by the "effective" parameter of specular reflection q_{eff}. For example, when our ML film is consisted of thin metal layers ($k_i << 1$) and the interface scattering is almost diffusive $\left(P_{ij}+Q_{ji} << 1\right)$ we may consider this ML system as a two-layer plate with the specular electron reflection at the external surfaces, $q_{eff} = 1$. If the ML film is consisted of the thick metal layers ($k_i >> 1$) we also may consider this system as a two-layer plate with the following "effective" parameter of the specular reflection, $q_{eff} = P_{ij} + Q_{ji}\tau_{0j,i}$.

To perform numerical calculation we rewrite our exact results (10-14) in the following form

$$\frac{\sigma}{\sigma_{01}} = \frac{\Phi_1}{1+d_{2,1}}\left\{1+D_{2,1}\right\}. \tag{22}$$

If $d_{j,i} << 1$, we expand Eq.(22) in a series with the small parameter $d_{j,i}$ and keep the main terms

$$\sigma = \sigma_{0i}\Phi_i\left\{1-d_{j,i}+D_{j,i}\right\}, \tag{23}$$

where $D_{j,i} = \frac{d_j \sigma_{0j} \Phi_j}{d_i \sigma_{0i} \Phi_i}$. As may be seen from Eq.(23), in the limiting cases of the layer-thickness ratio $d_{2,1}$ $(d_1 = const)$, the conductivity of the ML film may be written in the form

$$\frac{\sigma}{\sigma_{01}} \cong \begin{cases} \Phi_1, & d_{2,1} \to 0, \\ \dfrac{\sigma_{02}}{\sigma_{01}} \Phi_2 \approx \dfrac{\lambda_2}{\lambda_1} f(\alpha_2), & d_{2,1} \to \infty. \end{cases} \tag{24}$$

Here we take into account, firstly, that $\sigma_{02} / \sigma_{01} \cong \lambda_2 / \lambda_1$ and, secondly, that the metal layer with index "2" becomes thick enough when $d_{2,1} \to \infty$. Consequently, its conductivity should be calculated from the solution given by Eq.(15).

To illustrate the behavior of the conductivity of the ML film we computed σ (in units of σ_{01}) as a function of the layer-thickness ratio at various values of the parameters k_2 (where $k_2 = k_1 d_{2,1} \lambda_{2,1}$) using the exact solution (see Eq.(22)). The results of these calculations are displayed in figure 2. As follows form the numerical calculation (see curves a-f, figure 2), the conductivity of the ML film at $d_{2,1} << 1$ is determined by the conductivity of the layer with the index "1" (see Eq.(24)), and the value of it is determined by the electron scattering at the interlayer interfaces. When $d_{2,1} >> 1$, as it follows from Eq.(24), the ML conductivity is determined both by the ratio of the electron mean-free paths in the adjacent layers and by the bulk conductivity of the layer with the index "2", i.e. $\sigma / \sigma_{01} \cong (\lambda_2 / \lambda_1) f(\alpha_2)$.

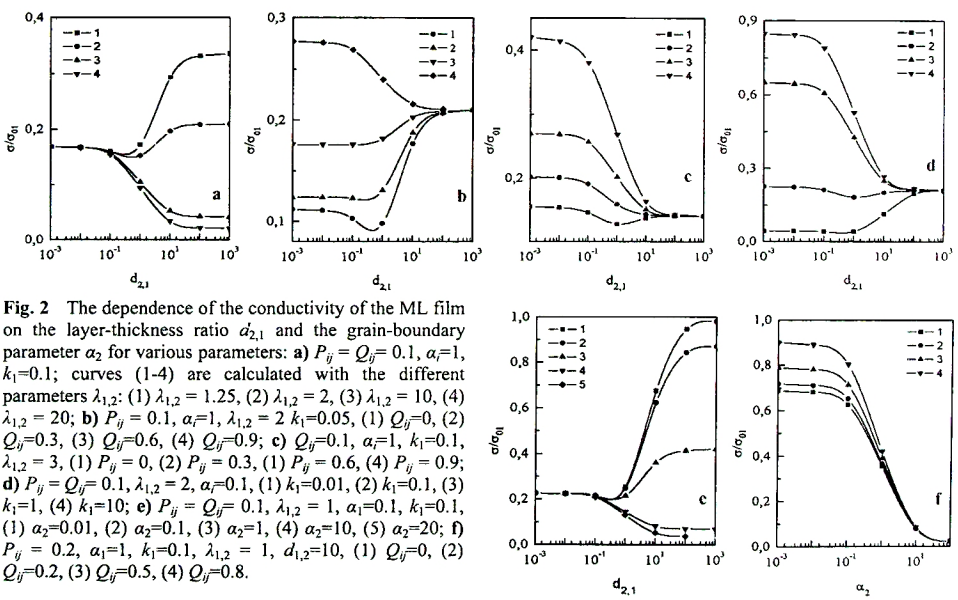

Fig. 2 The dependence of the conductivity of the ML film on the layer-thickness ratio $d_{2,1}$ and the grain-boundary parameter α_2 for various parameters: **a)** $P_{ij} = Q_{ij} = 0.1$, $\alpha_i = 1$, $k_1 = 0.1$; curves (1-4) are calculated with the different parameters $\lambda_{1,2}$: (1) $\lambda_{1,2} = 1.25$, (2) $\lambda_{1,2} = 2$, (3) $\lambda_{1,2} = 10$, (4) $\lambda_{1,2} = 20$; **b)** $P_{ij} = 0.1$, $\alpha_i = 1$, $\lambda_{1,2} = 2$ $k_1 = 0.05$, (1) $Q_{ij} = 0$, (2) $Q_{ij} = 0.3$, (3) $Q_{ij} = 0.6$, (4) $Q_{ij} = 0.9$; **c)** $Q_{ij} = 0.1$, $\alpha_i = 1$, $k_1 = 0.1$, $\lambda_{1,2} = 3$, (1) $P_{ij} = 0$, (2) $P_{ij} = 0.3$, (1) $P_{ij} = 0.6$, (4) $P_{ij} = 0.9$; **d)** $P_{ij} = Q_{ij} = 0.1$, $\lambda_{1,2} = 2$, $\alpha_i = 0.1$, (1) $k_1 = 0.01$, (2) $k_1 = 0.1$, (3) $k_1 = 1$, (4) $k_1 = 10$; **e)** $P_{ij} = Q_{ij} = 0.1$, $\lambda_{1,2} = 1$, $\alpha_1 = 0.1$, $k_1 = 0.1$, (1) $\alpha_2 = 0.01$, (2) $\alpha_2 = 0.1$, (3) $\alpha_2 = 1$, (4) $\alpha_2 = 10$, (5) $\alpha_2 = 20$; **f)** $P_{ij} = 0.2$, $\alpha_1 = 1$, $k_1 = 0.1$, $\lambda_{1,2} = 1$, $d_{1,2} = 10$, (1) $Q_{ij} = 0$, (2) $Q_{ij} = 0.2$, (3) $Q_{ij} = 0.5$, (4) $Q_{ij} = 0.8$.

It is evident from figure 2, the behavior of the function $\sigma(d_{2,1})$ depends essentially on the ratio between the ML conductivities at small and large values of $d_{2,1}$. The function $\sigma(d_{2,1})$ decreases monotonically when $\sigma(d_{2,1} << 1) >> \sigma(d_{2,1} >> 1)$ (see figure 2a, curves 3 and 4). However, if $\sigma(d_{2,1} << 1) << \sigma(d_{2,1} >> 1)$, the conductivity of the ML film increases monotonically with $d_{2,1}$ (see figure 2b, curve 4). Finally, if

$\sigma(d_{2,1} << 1) \sim \sigma(d_{2,1} >> 1)$, the function $\sigma(d_{2,1})/\sigma_{01}$ has a minimum due to the diffusive scattering of electrons at the layer interfaces (see figure 2a-2e).

To simplify the interpretation of our numerical results let us assume that metal layers have a large-grain structure $(L_i >> l_i)$, the grain boundaries are almost transparent for charge carriers $(R_i << 1)$ and the interface scattering is diffusive $(P_{ij} + Q_{ij} << 1)$. Then, we obtain $\Phi_i \sim k_i$, and $D_{2,1} \sim d_{2,1}^2$. Consequently, we may rewrite Eq.(22) in the following form (see Ref.[8])

$$\frac{\sigma}{\sigma_{01}} \cong \frac{k_1}{1+d_{2,1}}\left\{1+d_{2,1}^2\right\}. \tag{25}$$

Analyzing the right-hand side of Eq.(25), we obtain that the conductivity of the ML film has a minimum at $d_{2,1\,\min} \cong 0.414$ that is supported by our numerical calculation. The shift of the $d_{2,1\,\min}$ up to the larger values is due to the contribution of grazing electrons which propagate almost parallel to the interlayer interface.

This minimum is disappeared with increasing one of the following parameters, viz. (i) the probability of the transmission between the adjacent layers without scattering (figure 2b); (ii) the speculiarity of the interlayer interface scattering (figure 2c); (iii) the normalized thickness of the layer with index "1", i.e. k_1 (figure 2d); and (iv) the grain-boundary parameter α_2 (figure 2e). Then the conductivity of the ML film either decreases monotonically or increases monotonically with increasing of $d_{2,1}$. Figure 2f depicts the dependence of the ML conductivity on the grain-boundary parameter α_2 at different values of the electron transmission between the adjacent layers.

2 Temperature coefficient of the resistance

Under the conditions of external and internal size-effects the temperature dependence of the resistance R of the multilayered polycrystalline film determined mainly by the temperature variations of the electron mean-free path λ_i. On the other hand, the temperature behavior of the average size of grains, L_i, and the geometric size of a ML sample are also affect on the T-dependence of the resistance (we assume that parameters which determine the interface scattering are temperature independent). According to the standard definition (see, e.g. [11]) the temperature coefficient of the resistance (TCR) of a multilayered film is

$$\beta = \frac{d\ln R}{dT}, \tag{26}$$

$$R = \frac{1}{\sigma}\frac{a_1}{a_2 d}. \tag{27}$$

Here, a_1 and a_2 are the length and width of the metal layers, respectively; d is the multilayer repeat period, i.e. the thickness of the bi-layer; σ is the conductivity of the ML film given by Eqs.(10-14). Using Eqs.(10), (27) and (26) we obtain general analytic expressions of the TCR of the polycrystalline ML film under the conditions of external and internal size-effects. These expressions take into account temperature changes in the grain size and variation of the layer thicknesses due to the thermal expansion

$$\beta = \sum_{i\neq j}\frac{\beta_{0i}}{1+D_{j,i}}\left\{1-\left(1+\frac{2\chi_{di}}{\beta_{0i}}\right)\frac{\partial\ln\Phi_i}{\partial\ln k_i}-\beta_{0j,i}\left(1+\frac{2\chi_{dj}}{\beta_{0j}}\right)\frac{\partial\ln\Phi_i}{\partial\ln k_j}+\left(1+\frac{\chi_{di}}{\beta_{0i}}+\frac{\chi_{Li}}{\beta_{0i}}\right)\frac{\partial\ln\Phi_i}{\partial\ln\alpha_i}\right.$$
$$\left.+\beta_{0j,i}\left(1+\frac{\chi_{dj}}{\beta_{0j}}+\frac{\chi_{Lj}}{\beta_{0j}}\right)\frac{\partial\ln\Phi_i}{\partial\ln\alpha_j}-\left(\left(1+\frac{\chi_{di}}{\beta_{0i}}\right)-\beta_{0j,i}\left(1+\frac{\chi_{dj}}{\beta_{0j}}\right)\right)\frac{\partial\ln\Phi_i}{\partial\ln\tau_{0j,i}}\right\}. \tag{28}$$

Here, $\beta_{0j,i} = \beta_{0j}/\beta_{0i}$, $\chi_{di} = \frac{d\ln d_i}{dT}$, $\chi_{Li} = \frac{d\ln L_i}{dT}$ are phenomenological parameters which determine changes of the average layer thickness d_i and the average grain size L_i with temperature increasing. The TCR of a bulk

sample, β_{0i}, (see Refs. [10, 11]) is given by

$$\beta_{0i} = -\frac{d\ln\sigma_{0i}}{dT} - \chi_{di}, \tag{29}$$

In the frames of our model, substituting the thickness-depending function Φ_i (see Eq. (11)) into Eq.(28) we obtain the exact expression for the TCR of the polycrystalline sample at an arbitrary ratio between electron mean-free path λ_i and the layers thickness d_i for arbitrary types of the electron scattering at the interlayer interfaces and at the grain boundaries

$$\beta = \sum_{i\neq j}\frac{\beta_{0i}}{1+D_{j,i}}\left\{M_i - \beta_{0j,i}M_i^*\right\}, \tag{30}$$

$$M_i = 2\left(1+\frac{\chi_{di}}{\beta_{0i}}\right) - \frac{1}{\Phi_i}\left\{\left(1+\frac{2\chi_{di}}{\beta_{0i}}\right)J_{di} - \left(1+\frac{\chi_{di}}{\beta_{0i}}+\frac{\chi_{Li}}{\beta_{0i}}\right)J_{\alpha i} + \left(1+\frac{\chi_{di}}{\beta_{0i}}\right)J_{\tau i}\right\}, \tag{31}$$

$$M_i^* = \frac{1}{\Phi_i}\left\{\left(1+\frac{\chi_{dj}}{\beta_{0j}}+\frac{\chi_{Lj}}{\beta_{0j}}\right)J_{\alpha i}^* - \left(1+\frac{2\chi_{dj}}{\beta_{0j}}\right)J_{dj}^* - \left(1+\frac{\chi_{dj}}{\beta_{0j}}\right)J_{\tau i}\right\}, \tag{32}$$

$$J_{di} = f(\alpha_i) - \left\langle\frac{k_iE_iH_i}{x}\left\{G_i(1-E_i)^{-1} - \Theta_i\right\}\right\rangle,\ J_{di}^* = \left\langle\frac{k_jE_jH_j}{x}\Theta_i^*\right\rangle, \tag{33}$$

$$J_{\alpha i} = f^*(\alpha_i) + \left\langle\frac{k_iE_i}{x}(H_i-1)\left\{G_i(1-E_i)^{-1} - \Theta_i - \frac{x}{k_iE_iH_i}(\Lambda_i+2G_i)\right\}\right\rangle, \tag{34}$$

$$J_{\alpha i}^* = \left\langle\frac{k_jE_j}{x}(H_j-1)\left\{\Theta_i^* + \frac{x}{k_jE_jH_j}\Lambda_i\right\}\right\rangle,\ J_{\tau i} = \left\langle\Lambda_i\right\rangle, \tag{35}$$

$$f^*(\alpha_i) = \frac{3}{2}\alpha_i - \frac{3\alpha_i^2(2+3\alpha_i)}{1+\alpha_i} + 9\alpha_i^3\ln\left(1+\frac{1}{\alpha_i}\right) \cong \begin{cases}\dfrac{3}{2}\alpha_i - 6\alpha_i^2, & \alpha_i << 1,\\ \dfrac{3}{4\alpha_i} - \dfrac{6}{5\alpha_i^2}, & \alpha_i >> 1,\end{cases} \tag{36}$$

$$\Theta_i = \left\{P_{ij}(A-B_i) + \left(Q_{ij}Q_{ji} - P_{ij}P_{ji}\right)(A+B_i)E_j\right\}\Delta^{-1} - B_i\Xi_i, \tag{37}$$

$$\Theta_i^* = \left\{A\left(\left(Q_{ij}Q_{ji} - P_{ij}P_{ji}\right)(1-E_i) - Q_{ji}\tau_{j,i}\right) + B_i\left(P_{ij} - \left(Q_{ij}Q_{ji} - P_{ij}P_{ji}\right)E_i\right)\right\}\Delta^{-1} + B_i\Xi_j, \tag{38}$$

$$\Xi_i = 2A\left\{P_{ij}P_{ji}E_i + Q_{ij}Q_{ji}E_j - \left(Q_{ij}Q_{ji} - P_{ij}P_{ji}\right)^2E_iE_j^2\right\}\Delta^{-2},\ \Lambda_i = Q_{ji}\tau_{j,i}(1-E_j)A\Delta^{-1}. \tag{39}$$

Functions G_i, A and B_i were defined by Eq.(12); the angular brackets are given by Eq.(14).

To simplify comparison between theory and experiment, we obtain asymptotical results for the TCR (see Eq.(30)) in the limiting cases of parameters k_i and α_i.

If $k_i >> 1$, the TCR of the ML film at arbitrary values of P_{ij}, Q_{ij} and α_i is given by Eq.(30), where functions $J_{di}, J_{di}^*, J_{\alpha i}, J_{\alpha i}^*$ and $J_{\tau i}$ are given as

$$J_{di} = f(\alpha_i),\ J_{di}^* = 0,\ J_{\alpha i} = f^*(\alpha_i) - \frac{2\alpha_i}{\pi\ k_i}\left\{2\ \left(1-P_{ij}\right)\ \Gamma_{3,i} - Q_{ji}\tau_{0j,i}\Gamma_{4,i}\right\}, \tag{40}$$

$$J_{\alpha i}^* = \frac{2\alpha_j}{\pi k_i}Q_{ji}\tau_{0j,i}\Gamma_{4,j},\ J_{\tau i} = \frac{3}{8\ k_i}Q_{ji}\tau_{0j,i}\Gamma_{2,i}. \tag{41}$$

Here (see Ref. [10])

$$\Gamma_{3,i}=1-\frac{9\pi}{4}\alpha_i-6\left(5-\left(3-5\alpha_i^2\right)I_i-\frac{1-\alpha_i^2 I_i}{4\left(1-\alpha_i^2\right)}\right)\alpha_i^2+15\pi\alpha_i^3, \tag{42}$$

$$\Gamma_{4,i}=1-\frac{3\pi}{4}\left(2\alpha_i+\alpha_j\right)+\frac{3\pi}{2}\left\{4\alpha_i^3+3\alpha_i^2\alpha_j+2\alpha_i\alpha_j^2+\alpha_j^3\right\}\left\{1-\frac{2}{\pi}\frac{1-\left(1-\alpha_i^2\right)I_i}{\alpha_i}\right\}-$$

$$-\frac{3\alpha_j^4}{\alpha_i\left(\alpha_i-\alpha_j\right)}\left\{1+\frac{\alpha_j\left(1-\alpha_i^2\right)I_i-\alpha_i\left(1-\alpha_j^2\right)I_j}{\alpha_i-\alpha_j}\right\}-\frac{3\alpha_i^3 I_i}{\alpha_i-\alpha_j}. \tag{43}$$

If considered ML film is consisted of the metal layers of a large-grain structure ($\alpha_i<<1$) or a fine-graine structure ($\alpha_i>>1$), we obtain that the TCR is given by Eq.(30) for arbitrary values of the thickness $d_{j,i}$. Moreover, functions M_i and M_i^* have the following form

$$M_i=1-\frac{3}{2}\alpha_i-\frac{3}{8\ k_i}\left\{\left(1-P_{i\,j}\right)\left(1-\frac{12}{\pi}\alpha_i\right)+\frac{3}{5\pi}Q_{j\,i}\tau_{0j,i}\alpha_i\right\}-$$

$$-\left\{\frac{3}{2}\alpha_i+\frac{3}{4\ k_i}\left[\left(1-P_{i\,j}\right)\left(1-\frac{9}{\pi}\alpha_i\right)-\frac{1}{2}Q_{j\,i}\tau_{0j,i}\left(1-\frac{16}{3\pi}\left(\alpha_j+\frac{1}{4}\alpha_i\right)\right)\right]\right\}\frac{\chi_{di}}{\beta_{0i}}-$$

$$-\frac{3}{2}\alpha_i\left\{1-\frac{3}{2\ \pi\ k_i}\left[\left(1-P_{i\,j}\right)\left(1-\frac{13}{2}\alpha_i\right)-\frac{1}{10}Q_{j\,i}\tau_{0j,i}\left(1-\frac{7\pi}{3}\left(\alpha_i+\alpha_j\right)\right)\right]\right\}\frac{\chi_{Li}}{\beta_{0i}},\ \alpha_i<<1, \tag{44}$$

$$M_i^*=\frac{3}{8\ k_i}Q_{j\,i}\tau_{0j,i}\left\{\left[1-\frac{32}{3\pi}\left(\alpha_j+\frac{1}{16}\alpha_i\right)\right]\left(1+\frac{\chi_{dj}}{\beta_{0j}}\right)-\frac{16}{3\pi}\alpha_j\left[1-\frac{3\pi}{4}\left(2\alpha_j+\frac{9}{25}\alpha_i\right)\right]\frac{\chi_{Lj}}{\beta_{0j}}\right\},\alpha_i<<1. \tag{45}$$

$$M_i=\frac{4}{5\alpha_i}-\frac{3}{8\ k_i\ \alpha_i^2}\left\{\left(1-P_{i\,j}\right)\left(1-\frac{8}{5\alpha_i}\right)-\frac{\pi}{100}Q_{j\,i}\tau_{0j,i}\frac{\alpha_i}{\alpha_j}\right\}-$$

$$-\left\{1-\frac{4}{5\alpha_i}+\frac{1}{2\ k_i\ \alpha_i}\left[\left(1-P_{i\,j}\right)\left(1-\frac{3}{5\alpha_i^2}\right)-Q_{j\,i}\tau_{0j,i}\frac{\alpha_i}{\alpha_j}\left(1-\frac{\pi}{4\alpha_j}+\frac{\pi}{5\alpha_i^2}\right)\right]\right\}\frac{\chi_{di}}{\beta_{0i}}-$$

$$-\left\{1-\frac{4}{5\alpha_i}-\frac{1}{2\ k_i\ \alpha_i}\left[\left(1-P_{i\,j}\right)\left(1-\frac{3}{2\alpha_i}\right)-\frac{\pi}{135\alpha_j}Q\tau_{0j,i}\right]\right\}\frac{\chi_{Li}}{\beta_{0i}},\ \alpha_i>>1, \tag{46}$$

$$M_i^*=\frac{Q_{j\,i}\tau_{0j,i}}{2\ k_i\ \alpha_j}\left\{\frac{\pi}{4\alpha_j}\left(1+\frac{4}{5\alpha_i}-\frac{5\left(2\alpha_i+\alpha_j\right)}{2\pi\alpha_i\alpha_j}\right)\left(1+\frac{\chi_{dj}}{\beta_{0j}}\right)-\left(1+\frac{4}{5\alpha_i}-\frac{\pi\left(2\alpha_i+\alpha_j\right)}{4\alpha_i\alpha_j}\right)\frac{\chi_{Lj}}{\beta_{0j}}\right\},\ \alpha_i>>1 \tag{47}$$

If multilayered film is formed by thin metal layers, viz. $k_i<<1$, we obtain the following expressions for the TCR at arbitrary ratio between metal layer thicknesses

$$\beta\cong\sum_{i\neq j}\frac{\beta_{0i}}{1+D_{j,i}}\left\{\left(1+\frac{2\chi_{di}}{\beta_{0i}}\right)\frac{1}{\ln\left(1/k_i\right)}-\frac{2\chi_{di}}{\beta_{0i}}\right\},\ \alpha_i\le k_i, \tag{48}$$

$$\beta\cong\sum_{i\neq j}\frac{\beta_{0i}}{1+D_{j,i}}\left\{\frac{1}{\ln\left(1/k_i\right)-\left(4/\pi\right)\alpha_i}\left[1+\frac{2\chi_{di}}{\beta_{0i}}-\left(1+\frac{\chi_{di}}{\beta_{0i}}+\frac{\chi_{L\,i}}{\beta_{0i}}\right)\frac{4}{\pi}\alpha_i\right]-\frac{2\chi_{di}}{\beta_{0i}}\right\},\ k_i<\alpha_i<<1 \tag{49}$$

$$\beta\cong-\sum_{i\neq j}\frac{\beta_{0i}}{1+D_{j,i}}\left\{\frac{1}{\ln\left(1/\alpha_i k_i\right)}\left[\left(1+\frac{\chi_{di}}{\beta_{0i}}+\frac{\chi_{Li}}{\beta_{0i}}\right)\frac{3}{4\alpha_i^2}+\frac{\chi_{Li}}{\beta_{0i}}-\frac{\chi_{di}}{\beta_{0i}}\right]+\frac{2\chi_{di}}{\beta_{0i}}\right\},1<\alpha_i<<\frac{1}{k_i}. \tag{50}$$

Consequently, the grain-boundaries scattering is the main electron scattering mechanism when the metal layers have a fine-grain structure (α_i>>1) and the TCR is negative in this case.

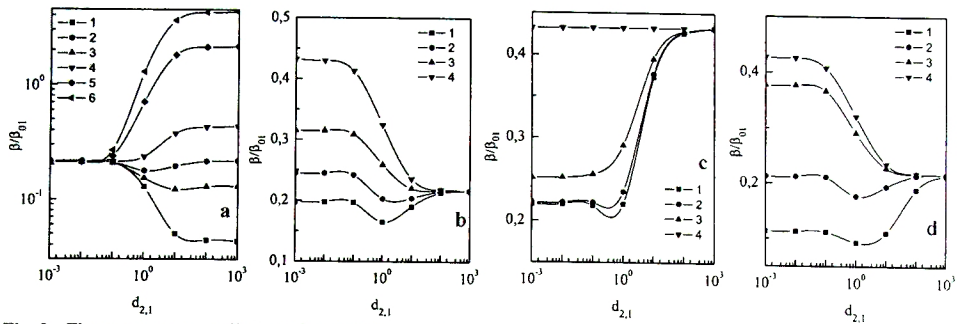

Fig. 3 The temperature coefficient of the resistance of a polycrystalline multilayered film vs. the layer-thickness ratio $d_{2,1}$ (figures 3a-e) and the grain-boundary parameter α_2 (figure 3f) for the following values of the parameters: **a)** P_{ij} = 0.1, Q_{ij}= 0.2, α_i=1, k_1=0.1; $\lambda_{1,2}$ = 1, χ_{di} = χ_{Li} = 0.0001, (1) $\beta_{2,1}$=0.1, (2) $\beta_{2,1}$=0.5, (3) $\beta_{2,1}$=0.3, (4) $\beta_{2,1}$=1, (5) $\beta_{2,1}$=5, (6) $\beta_{2,1}$=10; **b)** Q_{ij}=0.2, k_1=0.1, $\lambda_{1,2}$ = 1, χ_{di} = χ_{Li} = 0.0001, α_i=1, $\beta_{2,1}$=0.5, (1) P_{ij} = 0, (2) P_{ij} = 0.3, (3) P_{ij} = 0.6, (4) P_{ij} = 0.8; **c)** P_{ij} = 0.2, α_i=1, k_1=0.1, $\beta_{2,1}$=1, (1) Q_{ij}=0, (2) Q_{ij}=0.1, (3) Q_{ij}=0.4, (4) Q_{ij}=0.8; **d)** P_{ij} = 0.1, Q_{ij}= 0.2, $\lambda_{1,2}$ = 1, α_i=1.5, χ_{di} = χ_{Li} = 0.0001, $\beta_{2,1}$=0, α_i=1, (1) k_1=0.01, (2) k_1=0.1, (3) k_1=1, (4) k_1=10; **e)** P_{ij} = 0.1, Q_{ij}= 0.2, $\lambda_{1,2}$ = 1, α_1=1, k_1=0.1, χ_{di} = χ_{Li} = 0.0001, $\beta_{2,1}$=1, (1) α_2=0.1, (2) α_2=1, (3) α_2=10, (4) α_2=20; **f)** Q_{ij}= 0.2, P_{ij} = 0.1, α_1=1, k_1=0.1, $\lambda_{1,2}$ = 1, χ_{di} = χ_{Li} = 0.0001, $\beta_{2,1}$=1, (1) $d_{2,1}$=0.1, (2) $d_{2,1}$=1, (3) $d_{2,1}$=10, (4) $d_{2,1}$=20.

To calculate the TCR numerically it is convenient to rewrite the exact result (30) in the following form

$$\frac{\beta}{\beta_{0\,1}} = -\frac{1}{1+D_{2,1}}\sum_{i\neq j}\left(D_{i,j}\beta_{0\,j,i}\right)^{i-1}\left\{M_i+\beta_{0\,j,i}M_i^*\right\}. \tag{51}$$

If $D_{j,i}$<<1 we obtain the following approximated result

$$\beta = \beta_{0\,i}M_i + \beta_{0\,j}M_i^* - D_{j,i}\left\{\beta_{0\,i}\left(M_i - M_i^*\right) - \beta_{0\,j}\left(M_j - M_j^*\right)\right\}. \tag{52}$$

It follows that in the limiting cases of the layer thickness the TCR is given by

$$\frac{\beta}{\beta_{0\,1}} \cong \begin{cases} M_1 + \dfrac{\beta_{0\,2}}{\beta_{0\,1}}M_1^*, & d_{2,1}\to 0, \\ \dfrac{\beta_{0\,2}}{\beta_{0\,1}}M_2 + M_2^* \cong \dfrac{\beta_{0\,2}}{\beta_{0\,1}}\left\{1-\left\{1+\dfrac{\chi_{d\,2}}{\beta_{02}}+\dfrac{\chi_{L2}}{\beta_{02}}\right\}\dfrac{f^*(\alpha_2)}{f(\alpha_2)}\right\}, & d_{2,1}\to\infty. \end{cases} \tag{53}$$

The results of our numerical calculation according to the exact formula (51) are displayed in figures 3a-3f. The curves depict the layer-thickness ratio ($d_{2,1}$) dependence of the TCR of the ML film (in units of the TCR of the bulk layer with the index "1") at various values of the task parameters. Our results demonstrate the qualitatively different behavior of the TCR as a function of the layer-thickness ratio depending on the ratio between limiting values of the TCR at small and large values of $d_{2,1}$. When $d_{2,1}$<<1, the TCR of the ML film is determined by the TCR of the layer with the index "1". Meanwhile, for large values of the layer-thickness ratio, when $d_{2,1}$>>1, the value of the total TCR is determined by the ratio between the TCR of a bulk single-

crystal metal layers and the TCR of the bulk polycrystalline layer with the index "2" (see Eq. (53) and figure 3a). If $d_{2,1} \sim d_{2,1}$, the function $\beta(d_{2,1})$ has a minimum (see figure 3). The minimum is a result of the diffusive scattering at the interlayer interface. Note, the minimum disappears and the TCR monotonically increases (or decreases) with increasing of one of the following functions: (i) the probability of the specular scattering at the interlayer interface, (ii) the probability of tunneling between adjacent layers or (iii) the thickness of the layer with index "1" (see figure 3b, figure 3c and figure 3d, respectively). In figure 3e we display the function $\beta(d_{2,1})$ for various values of the parameter α_2 and function $\beta(\alpha_2)$ for various values of the parameter $d_{2,1}$.

4 Experiment and comparison with theory

The Cr, Cu and Co-based multilayered films were produced by condensing in a vacuum of 10^{-3}–10^{-4} Pa at the substrate temperature T_s = 300K by the method of Ref. [12]. The electrical resistance of the films was measured within the accuracy of ±0,01 Ω by an digital voltmeter using polished glass and sytal plates. The thermostabilization was carried out by annealing in a vacuum up to the temperature 670K followed by cooling down to 300K. The electrical properties of the ML film were stable after a triple "heating-cooling" cycle. The temperature was controlled by the digital voltmeter using the chromel-alumel thermocouple. The final thicknesses of the ML films were measured by an interferometer; during the process of condensation the layer thicknesses were controlled by a quartz resonator. The phase and element composition of the films were controlled by methods of the electromyography and mass-spectroscopy of secondary ions.

To compare theoretical and experimental values of the TCR we calculate Eq.(30) using simplified expressions for the functions M_i and M_i^* (Eqs.(44) and (45), respectively) and neglecting the temperature variations of average layer thickness and grain size. Note, we use the temperature coefficient of resistance $\beta_{\infty i}$ and the conductivity $\sigma_{\infty i}$ of the infinitely thick films instead of the β_{0i} and σ_{0i}; we assume also that $P_{ij} = P_{ji} = P$, and $Q_{ij} = Q_{ji} = Q$. Taking into account aforementioned, we may rewrite Eq.(30) in the following form

$$\beta = \sum_{i \neq j} \frac{\beta_{\infty i}}{1 + D_{j,i}} \left\{ 1 - \frac{3}{2}\alpha_i - \frac{3}{8}\frac{\lambda_i}{d_i}\left[(1-P)\left(1 - \frac{12}{\pi}\alpha_i\right) + \frac{3}{5\pi} Q \frac{\lambda_j}{\lambda_i}\alpha_i \right] + \frac{3}{8} Q \frac{\lambda_j}{d_i}\frac{\beta_{\infty j}}{\beta_{\infty i}}\left[1 - \frac{32}{3\pi}\left(\alpha_j + \frac{1}{16}\alpha_i \right) \right] \right\}. \quad (54)$$

The values of the parameter α_i were found experimentally using size-dependence of the TCR of the single-layer films. These values are 0.07, 0.5 and 0.55 for Cr, Cu and Co-based films, respectively. We assume that the layer interfaces and the grain boundaries have the same transmission coefficients as they are identical scattering objects.

Table 1 Experiment and theoretical results for the TCR of the multilayered films.

Multilayered films: (thickness, nm)	Experimental data	TCR, 10^{-3}K^{-1} Theoretical results calculated with the experimental values of α		the given values of α*	
		P=p	P=R	P=p	P=R
Cr(≈80)/Co(10)/ Cr(80)/Co(10)/S	0.42	0.40	0.34	0.39	0.32
Cr(80)/Co(10)/ Cr(≈80)/Co(10)/ Cr(80)/Co(10)/S	0.92	1.16	1.02	1.16	0.99
Cr(150)/Co(10)/ Cr(≈150)/Co(10)/ Cr(150)/Co(10)/ Cr(≈150)/Co(10)/S	0.91	1.08	0.95	1.06	0.91
Cu(22)/Cr(20)/ Cu(22)/Cr(20)/S	1.71	0.85	1.61	0.88	1.60
Cu(30)/Cr(30)/ Cu(30)/Cr(10)/S	1.80	1.38	1.92	1.29	1.80

S – substrate; *) α value enlarged by 10%

In table 1 we present theoretical and experimental results for the TCR of the multilayered films. Theoretical results were obtained using Eq.(54) and experimental data of the parameters of the electron transport in single-layer films. We turn now to an analysis of our results.

Firstly, the results of calculation are in a satisfactory agreement with the experimental data for both types of Cr, Cu and Co-based multilayered films. Moreover, the experiments on electron diffraction in Cr and Co-based

films have demonstrated that these systems are heterogeneous systems, and their phase composition is a compound of the bcc phase of Cr and the hcp and fcc phases of Co. The presence of a little content of the fcc phase of Co both in films and in bulk samples is well explained by the following reason. The fact is that defects of the hcp lattice packing are the nucleating centers for the fcc Co lattice. Additionally, rather intensive grain-boundaries diffusion processes are inherent to the both multilayered systems but the mutual atomic solubility both in Cr - Co and in Cr - Cu systems is limited. Nevertheless, it was demonstrated by mass-spectroscopy of secondary ions that each layer keeps its "individuality" (see, e.g. [13]). In this connection, much attention has been paid to the processes of the phase formation in multilayered films (see, e.g., Refs.[14, 15]).

Secondly, on the whole, we achieved a good agreement between theoretical and experimental results when assume that the probability of specular reflection of electrons from the interlayer interface (P) is equal to the grain-boundary scattering coefficient (R) but not to the probability of specular reflection of electrons from the interface between the upper layer and vacuum (p). Moreover, the better agreement may be achieved by variation of the coefficients p and R (see table 1). This fact can be explained by the grain-boundary diffusion of atoms which leads to changes of the scattering conditions at the grain boundaries.

Thirdly, a proper consideration of the electron scattering and tunneling processes at the interlayer interfaces is of fundamental importance (see, for example, Refs.[16,17]).

5 Summary

In conclusion, we have demonstrated that size dependencies of the conductivity and the temperature coefficient of the resistance of a polycrystalline multilayered film are differ essentially from the corresponding functions in a single-layer polycrystalline film. When layer thicknesses are differs substantially, i.e. $d_{2,1}<<1$, the behavior of these functions is determined by the characteristics of the electron interaction with the interlayer interface. If the electron scattering at the interlayer interface is dominating, the values of the σ and β decrease with increasing of the $d_{2,1}$ (this is due to the decreasing of the relative number of electrons which have small effective mean-free paths of the order of d_2). When d_1=*const.* the bulk values of the conductivity and the TCR plays more and more essential role at increasing of the repeat period of the ML film. Both functions have a minimum at $d_1 \sim d_2$. At the further increase of the $d_{2,1}$, both σ and β approaches asymptotically the bulk values in the second layer (see Eqs. (24) and (53), respectively). In the case of Cr, Cu and Co-based multilayered films we demonstrate a qualitative agreement between the theoretical and experimental values of the TCR.

Acknowledgments The research was partially supported by the Ministry of Education and Science of Ukraine (grants #0103 U 000773, 2003 – 05 and # M/18 – 2004).

References

[1] M. N. Baibich, J. M. Broto, and A. Fert, et al. Phys. Rev. Lett **61,** 2472 (1988).
[2] R. E. Camley and R. L. Stamps, J. Phys.: Cond. Matter. **5,** 3727 (1993).
[3] A. F. Mayadas and M. Shatzkes, Phys. Rev. B.: Cond. Matter. **1,** 1382 (1970).
[4] M. I. Kaganov and V. B. Fiks, Sov. Phys. JETF **46,** 393 (1977).
[5] V. V. Ustinov, Fiz. Met. Metalloved **49,** 31 (1980). (in russian).
[6] K. Fuchs, Proc. Cambr. Phil. Soc. **A34,** 100 (1938).
[7] A. B. Pippard, Proc. Roy. Soc. **224,** 273 (1954).
[8] L. V. Dekhtyaruk and Yu. A. Kolesnichenko, Ukr. Fiz. Zh. **42,** 1094 (1997). (in ukr.).
[9] V. I. Verchenko, V. I. Grishayev, L. V. Dekhtyaruk, et al., Sov. Phys. Met. Metall. **69,** 100 (1990). (in russian).
[10] L. V. Dekhtyaruk, S. I. Protcenko, A. M. Chornous, et al., Ucr. J. Phys. **49,** 587 (2004).
[11] C. R. Tellier and A. J. Tosser, Size effects in thin films, Amsterdam: Elsevier Scientific Publ. 310 (1982).
[12] I. Yu. Protsenko, L. V. Odnodvoretz, and A. M. Chornous, Metallofiz. Noveishie Tekhnol. **20,** 36 (1998). (in ukr.).
[13] L. V. Odnodvoretz, S. I. Protsenko, and A. I. Saltikova, Metallofiz. Noveishie Tekhnol. **21,** 71 (1999). (in ukr.).
[14] I. Yu. Protsenko, V. V. Tokman, A. M. Chornous, et al., Metallofiz. Noveishie Tekhnol. **25,** 319 (2003). (in ukr.).
[15] C. Blaas, P. Weinberger, L. Szunyogh, et al., Phys. Rev. B **60,** 492 (1999).
[16] W. E. Bailey, S. X. Wang, and E. Yu. Tsymbal, Phys. Rev. B. **61,** 1330 (2000).
[17] A. Shpiro and P. M. Levy, Phys. Rev. B. **63,** 014419-1 (2000).