Forced Coupled Motion of the Nanoparticle Magnetic Moment and the Whole Nanoparticle in a Viscous Fluid

T. V. Lyutyy[1], E. S. Denisova, A. V. Kvasnina
Electronic and Informational Technologies Faculty,
Sumy State University,
2 Rimsky-Korsakov St., UA-40007 Sumy, Ukraine
[1]lyutyy@oeph.sumdu.edu.ua

***Abstract*—We considered the coupled motion of the magnetic moment of a ferromagnetic uniaxial nanoparticle and its mechanical rotation about the center of mass. This particle is supposed to be placed into a viscous liquid and excited by an external alternating magnetic field. Two modes of motion were studied analytically in the noise-free approximation. Within the first mode, both the nanoparticle magnetic moment and easy axis perform small oscillations around the initial position. The oscillation amplitudes were obtained in the harmonic approximation. Just the magnetic moment demonstrates the resonant behaviour that causes the main features of the oscillation mode. Within the second mode, the nanoparticle magnetic moment and easy axis are rotated synchronously. In this case we obtained the system of algebraic equations, where the precession and lag angles of the magnetic moment and easy axis are the solutions of these equations. The results obtained allow to make an analysis of the mechanism of ferrofluids heating by alternating magnetic fields.**

***Keywords*—ferrofluid; finite anisotropy; coupled dynamics; rotational oscillation mode; precessional mode**

I. Introduction

The problem of the trajectory description for a nanoparticle placed into a fluid is associated directly with the aspects of the microscopic description of the ferrofluid [1] response to an alternating field. On the one hand, the individual motion of each particle defines the ensemble collective behaviour, and the solvable single-particle model is very important in this regard. On the other hand, the adequate microscopic description allows to develop more advanced numerical methods for the ferrofluids simulation. But, at the same time, the assigned task is nontrivial enough and, as a rule, is treated in different approximations.

Here we need to mention two approaches which are widely used: 1) the frozen magnetic moment approach [2], when the nanoparticle magnetic moment is locked in the nanoparticle crystal lattice, 2) the rigidly fixed nanoparticle approach [3], when a nanoparticle is immobilized because of the rigid bound with a media carrier. Despite the restrictions, both approaches are applied for the modeling of the ferrofluid response to an alternating field including the heating problem, which is closely related to the magnetic fluid hyperthermia for cancer therapy [4].

The coupled dynamics of a nanoparticle body and its magnetic moment cannot be described by a simple superposition of the above mentioned types of motion because of the essential changes in the equations of motion. Despite this coupled motion was firstly considered in [6], the discussion about the basic equations is continued till now [7]–[10]. It is important in a context of a ferrofluid heating by an alternating field, when both these types of motion produce a heat. The first successful attempt to describe the energy absorption was presented in [11]. There the power loss was obtained by linearization of the Lagrangian equation in some specific cases. But the equations of motion within this approach cannot be used. The power loss was also investigated in the recent studies [7], [8]. The explicit form of the utilized basic equations is controversial there that stimulates the further discussion [9], [10]. The progress in the description of energy of a viscously coupled nanoparticle with a finite anisotropy driven by a time-periodic field was achieved recently in [12]. But the viscous term was not taken into account that motivates our study given in [13], [14].

II. Model and Basic Equations

We consider a uniform spherical single-domain ferromagnetic nanoparticle of radius R, magnetization $\mathbf{M}$ ($|\mathbf{M}| = M = \mathrm{const}$), and density ρ. This particle performs the spherical motion (or motion with the fixed center of mass) with respect to a fluid of viscosity η. Then, we assume that the nanoparticle is driven by the external time-periodic field of the type

$$\mathbf{H}(t) = \mathbf{e}_x H \cos(\Omega t) + \mathbf{e}_y \sigma H \sin(\Omega t), \qquad (1)$$

where $\mathbf{e}_x$, $\mathbf{e}_y$ are the unit vectors of the Cartesian coordinates, H is the field amplitude, Ω is the field frequency, t is the time, and σ is the factor which determines the polarization type ($\sigma = \pm 1$ corresponds to the circularly-polarized field, $0 < |\sigma| < 1$ corresponds to the elliptically-polarized field, and $\sigma = 0$ corresponds to the linearly-polarized field).

As follows from [9], the coupled magnetic dynamics and mechanical motion in the deterministic case obey the following pair of coupled equations:

$$\begin{aligned} &\dot{\mathbf{n}} = \boldsymbol{\omega} \times \mathbf{n}, \\ &J\dot{\boldsymbol{\omega}} = \gamma^{-1} V \dot{\mathbf{M}} + V\mathbf{M} \times \mathbf{H} - 6\eta V \boldsymbol{\omega}, \end{aligned} \qquad (2)$$

where $\mathbf{n}$ is the unit vector defining the anisotropy axis direction, $\boldsymbol{\omega}$ is the angular velocity of the particle, $J(= 8\pi\rho R^5/15)$ is the moment of inertia of the particle, γ is the gyromagnetic ratio, V is the nanoparticle volume, and dots over symbols represent derivatives with respect to time.

In fact, the first equation in (2) is the condition of spherical motion for a rigid body, and the second one is the classical torque equation, where the first term constitutes the main difference from the same equation for the frozen magnetic moment model. This term is originated from the motion of magnetization inside the nanoparticle with respect to its crystal lattice. In turn, the magnetization dynamics is described by the modified Landau-Lifshitz-Gilbert equation

$$\dot{\mathbf{M}} = -\gamma \mathbf{M} \times \mathbf{H}_{eff} + \alpha M^{-1} \left(\mathbf{M} \times \dot{\mathbf{M}} - \boldsymbol{\omega} \times \mathbf{M} \right), \quad (3)$$

where α is the damping parameter, $\mathbf{H}_{eff}$ is the effective magnetic field, which takes into account the uniaxial anisotropy field of magnitude H_a

$$\mathbf{H}_{eff} = \mathbf{H} + H_a M^{-1} (\mathbf{Mn})\, \mathbf{n}. \quad (4)$$

The presented model equations (2), (3) allow both the analytical and numerical studies. The purpose of this paper is the description of the simplest types of coupled motion, namely, the oscillations, which can be realized for the fields of all polarization types, and the precessions, which can be realized for a circularly-polarized field.

III. Results and Discussion

A. Coupled Oscillations of the Easy Axis and the Magnetic Moment

The solution of the set of equations (2), (3) in the case of negligibly small moment of inertia can be found in the linear approximation for the small oscillations mode. In this mode, vectors $\mathbf{M}$ and $\mathbf{n}$ perform small rotational oscillations in a vicinity of the initial position, which in turn, is defined by the angles θ_0 and φ_0 (see Fig. 1). This takes place for small enough field amplitudes ($H \ll H_a$). To describe this type of motion let us introduce a new coordinate system $x'y'z'$ rotated with respect to the laboratory one by the angles θ_0 and φ_0 (Fig. 1). The transition to the new framework is performed using the following rotation matrix:

$$\mathbf{C} = \begin{pmatrix} \cos\theta_0 \cos\varphi_0 & \cos\theta_0 \sin\varphi_0 & -\sin\theta_0 \\ -\sin\varphi_0 & \cos\varphi_0 & 0 \\ \sin\theta_0 \cos\varphi_0 & \sin\theta_0 \sin\varphi_0 & \cos\theta_0 \end{pmatrix}. \quad (5)$$

According to the procedure, which was explained in detail in [14], [15], we obtained the solution of (2), (3) in the form

$$\begin{aligned} n_{x'} &= a_n \cos\Omega t + b_n \sin\Omega t, \\ n_{y'} &= c_n \cos\Omega t + d_n \sin\Omega t, \\ m_{x'} &= a_m \cos\Omega t + b_m \sin\Omega t, \\ m_{y'} &= c_m \cos\Omega t + d_m \sin\Omega t, \end{aligned} \quad (6)$$

where $\mathbf{m} = \mathbf{M}/M$, a_n, b_n, c_n, d_n, a_m, b_m, c_m, and d_m are the constant coefficients defined as follows:

$$\begin{aligned} a_m &= Z^{-1} \left[\tilde{\Omega}_1 D_m + \tilde{\Omega}_2 B_m + \tilde{\Omega}_3 C_m + \tilde{\Omega}_4 A_m \right], \\ b_m &= Z^{-1} \left[\tilde{\Omega}_1 C_m + \tilde{\Omega}_2 A_m - \tilde{\Omega}_3 D_m - \tilde{\Omega}_4 B_m \right], \\ c_m &= Z^{-1} \left[-\tilde{\Omega}_1 B_m + \tilde{\Omega}_2 D_m - \tilde{\Omega}_3 A_m + \tilde{\Omega}_4 C_m \right], \\ d_m &= Z^{-1} \left[-\tilde{\Omega}_1 A_m + \tilde{\Omega}_2 C_m + \tilde{\Omega}_3 B_m - \tilde{\Omega}_4 D_m \right], \end{aligned} \quad (7)$$

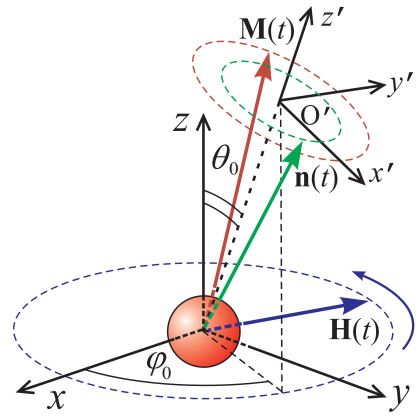

Fig. 1. Schematic representation of the behaviour of vectors $\mathbf{n}$ and $\mathbf{M}$ and the used coordinate systems for the rotational oscillations mode description

where

$$\begin{aligned} Z &= \tilde{\Omega}^4\alpha^4 + 2\tilde{\Omega}^4\alpha^2\beta^2 + \tilde{\Omega}^4\beta^4 + 4\tilde{\Omega}^4\alpha^2\beta + \\ &+ 4\tilde{\Omega}^4\beta^3 + 2\tilde{\Omega}^4\alpha^2 + 6\tilde{\Omega}^4\beta^2 - 2\tilde{\Omega}^2\alpha^2\delta^2 + \\ &+ 2\tilde{\Omega}^2\beta^2\delta^2 + 4\tilde{\Omega}^4\beta + 8\tilde{\Omega}^2\alpha\beta\delta + \\ &+ 4\tilde{\Omega}^2\beta\delta^2 + \tilde{\Omega}^4 + 2\tilde{\Omega}^2\alpha^2 + 8\tilde{\Omega}^2\alpha\delta - \\ &- 2\tilde{\Omega}^2\beta^2 + 2\tilde{\Omega}^2\delta^2 + \delta^4 - 4\tilde{\Omega}^2\beta - 2\tilde{\Omega}^2 + \\ &+ 2\delta^2 + 1, \end{aligned} \quad (8)$$

$$\begin{aligned} \tilde{\Omega}_1 &= -\tilde{\Omega}^2\alpha^2 - 2\tilde{\Omega}^2\alpha\beta\delta - 2\tilde{\Omega}^2\alpha\delta + \\ &+ \tilde{\Omega}^2\beta^2 + 2\tilde{\Omega}^2\beta + \tilde{\Omega}^2 - \delta^2 - 1, \\ \tilde{\Omega}_2 &= -\tilde{\Omega}^2\alpha^2\delta + 2\tilde{\Omega}^2\alpha\beta + 2\tilde{\Omega}^2\alpha + \\ &+ \tilde{\Omega}^2\beta^2\delta + 2\tilde{\Omega}^2\beta\delta + \tilde{\Omega}^2\delta + \delta^3 + \delta, \\ \tilde{\Omega}_3 &= \tilde{\Omega}^3\alpha^3 + \tilde{\Omega}^3\alpha\beta^2 + 2\tilde{\Omega}^3\alpha\beta + \\ &+ \tilde{\Omega}^3\alpha - \tilde{\Omega}\alpha\delta^2 + \tilde{\Omega}\alpha + 2\tilde{\Omega}\beta\delta + \\ &+ 2\tilde{\Omega}\delta, \\ \tilde{\Omega}_4 &= -\tilde{\Omega}^3\alpha^2\beta - \tilde{\Omega}^3\alpha^2 - \tilde{\Omega}^3\beta^3 - \\ &- 3\tilde{\Omega}^3\beta^2 - 3\tilde{\Omega}^3\beta - \tilde{\Omega}^3 - 2\tilde{\Omega}\alpha\delta - \\ &- \tilde{\Omega}\beta\delta^2 + \tilde{\Omega}\beta - \tilde{\Omega}\delta^2 + \tilde{\Omega}, \end{aligned}$$

$$\begin{aligned} A_m &= \sigma h(1+\beta)\cos\varphi_0 - \tilde{\Omega}^{-1} h \sin\varphi_0, \\ B_m &= -h(1+\beta)\sin\varphi_0 - \sigma\tilde{\Omega}^{-1} h \cos\varphi_0, \\ C_m &= -\sigma h(1+\beta)\cos\theta_0 \sin\varphi_0 - \tilde{\Omega}^{-1} h \cos\theta_0 \cos\varphi_0, \\ D_m &= -h(1+\beta)\cos\theta_0 \cos\varphi_0 + \sigma\tilde{\Omega}^{-1} h \cos\theta_0 \sin\varphi_0. \end{aligned}$$

$$\begin{aligned} a_n &= \delta c_m - \sigma\tilde{\Omega}^{-1} h \cos\theta_0 \sin\varphi_0, \\ b_n &= \delta d_m + \tilde{\Omega}^{-1} h \cos\theta_0 \cos\varphi_0, \\ c_n &= -\delta a_m - \sigma\tilde{\Omega}^{-1} h \cos\varphi_0, \\ d_n &= -\delta b_m - \tilde{\Omega}^{-1} h \sin\varphi_0. \end{aligned} \quad (9)$$

Here $h = H/H_a$, $\Omega_r = \gamma H_a$ is the ferromagnetic resonance frequency, $\tilde{\Omega} = \Omega/\Omega_r$, $\beta = \alpha M/6\gamma\eta$, and $\delta = \beta/\alpha$.

The expressions obtained allow us to analyze the features of the mechanical and magnetic dynamics of the nanoparticle, see Fig. 2. Firstly, since the small oscillations mode is not realized for very small frequencies ($\tilde{\Omega} \to 0$), the corresponding interval in the plot should not be considered. Secondly, mainly the nanoparticle magnetic moment demonstrates the resonant behaviour for the realistic system parameters. The amplitudes

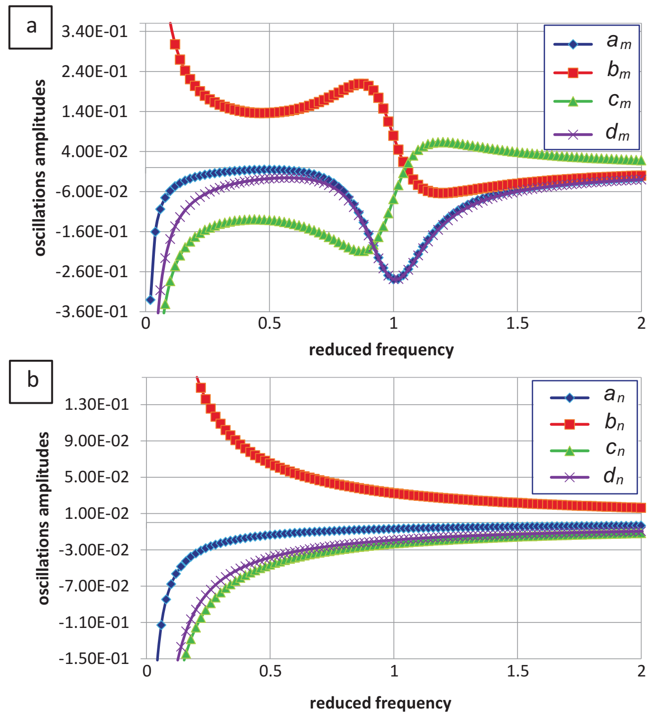

Fig. 2. (Color online) The dependencies of the amplitudes of coupled oscillations of the magnetic moment (7) and the easy axis (9) on the field frequency. The parameters used are $M = 338$ G, $H_a = 910$ Oe, $\eta = 0.006$ P, $\alpha = 0.15$ that corresponds to maghemite nanoparticles ($\gamma - \mathrm{Fe_2O_3}$) in water at the temperature of 42 °C, $\sigma = 0.5$, $h = 0.01$, $\theta_0 = 0.4\pi$, $\varphi_0 = 0.125\pi$.

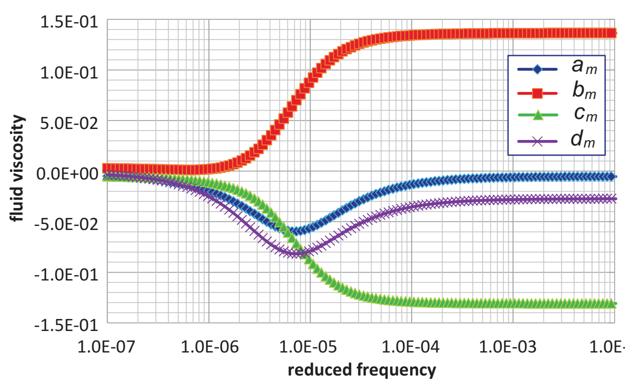

Fig. 3. (Color online) The dependencies of the amplitudes of coupled oscillations of the magnetic moment (7) and the easy axis (9) on the fluid viscosity. The parameters used are $M = 338$ G, $H_a = 910$ Oe, $\alpha = 0.15$, $\sigma = 0.5$, $\tilde{\Omega} = 0.5$, $h = 0.01$, $\theta_0 = 0.4\pi$, $\varphi_0 = 0.125\pi$.

of the nanoparticle easy axis decay monotonically with the field frequency for realistic viscosity and magnetization. Nevertheless, it is not a reason to neglect the viscous term in the second equation of (2), as it was done in [12]. Thus, it is obvious from Fig. 3, the viscosity value can impact considerably on the vector $\mathbf{M}$ amplitudes.

B. Coupled Rotation of the Easy Axis and the Magnetic Moment

In the case of synchronous precession of vectors $\mathbf{M}$ and $\mathbf{n}$ with the external circularly-polarized field (see Fig. 4), the

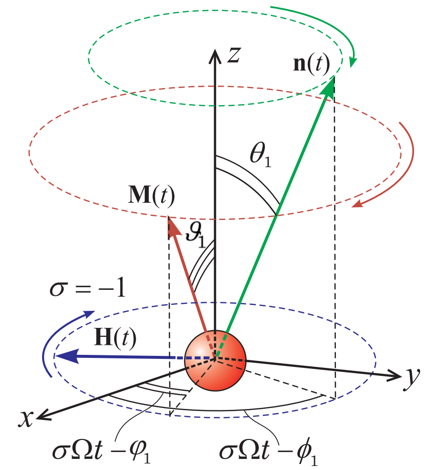

Fig. 4. Schematic representation of the behaviour of vectors $\mathbf{n}$, $\mathbf{M}$ and the used coordinate systems for the precessional mode description

stationary solution of the set of equations (2), (3) can be obtained in the form

$$\begin{array}{ll} \varphi = \sigma\Omega t - \varphi_1 & \vartheta = \vartheta_1, \\ \phi = \sigma\Omega t - \phi_1 & \theta = \theta_1, \end{array} \tag{10}$$

where φ, ϑ are the spherical coordinates of vector $\mathbf{M}$, ϕ, θ are the spherical coordinates of vector $\mathbf{n}$, φ_1, ϕ_1 are the lag angles and, finally, ϑ_1, θ_1 are the precession angles of vectors $\mathbf{M}$ and $\mathbf{n}$. To find the unknown constants φ_1, ϕ_1 and ϑ_1, θ_1, we used the condition of absence of the magnetic moment motion with respect to the nanoparticle crystal lattice

$$\dot{\mathbf{M}} - \boldsymbol{\omega} \times \mathbf{M} = 0. \tag{11}$$

After substitution of (11) into the second equation of (2), we derive

$$J\dot{\boldsymbol{\omega}} = -H_a M^{-1} V (\mathbf{M} \times \mathbf{n})(\mathbf{M} \cdot \mathbf{n}) - 6\eta V \boldsymbol{\omega}. \tag{12}$$

Then, let us introduce the double-primed coordinate system $x''y''z''$ which is rotated with the external field as follows from Fig. 4. In this new framework, the angular velocity has a very simple form

$$\boldsymbol{\omega}'' = (-\sigma\Omega \sin\theta_1, 0, 0)\,. \tag{13}$$

Since equation (3) cannot be easily represented in the double-primed system, we need to write the explicit form of all the vectors in the laboratory coordinate system. To perform the necessary transformations, we should use the rotation matrix

$$\mathbf{C}^{-1} = \begin{pmatrix} \cos\theta_1 \cos\Phi_1 & -\sin\Phi_1 & \sin\theta_1 \cos\Phi_1 \\ \cos\theta_1 \sin\Phi_1 & \cos\Phi_1 & \sin\theta_1 \sin\Phi_1 \\ -\sin\theta_1 & 0 & \cos\theta_1 \end{pmatrix}, \tag{14}$$

where $\Phi_1 = \sigma\Omega t - \phi_1$. Using the designation

$$F = M \sin\theta_1 \sin\vartheta_1 \cos(\phi_1 - \varphi_1) + \cos\theta_1 \cos\vartheta_1 \tag{15}$$

and representing vectors $\boldsymbol{\omega}$, $\mathbf{M}$, and $\mathbf{n}$ in the laboratory system, we straightforwardly obtain the set of algebraical equations

$$MV \sin\vartheta_1 \sin\varphi_1(\sigma\Omega\gamma^{-1} - H_{0z}) = \\ = -\Omega \sin\theta_1 \cos\theta_1 (J\Omega \sin\phi_1 + 6\sigma V \eta \cos\phi_1), \quad (16)$$

$$MH \sin\vartheta_1 \sin\varphi_1 = 6\eta\Omega^2 \sin^2\theta_1, \quad (17)$$

$$H_a F \sin(\vartheta_1 - \theta_1) \sin\varphi_1 = \sin\vartheta_1(\sigma\Omega\gamma^{-1} - H_{0z}), \quad (18)$$

$$H_a F \sin\theta_1 \sin(\varphi_1 - \phi_1) = H \sin\varphi_1, \quad (19)$$

where H_{0z} is the static field directed along the oz axis.

The exact solution of (19) can be found numerically, but some properties of the precessional mode are obvious from this algebraical system directly. Firstly, when $H_{0z} = 0$ and $\sigma = -1$, the relationships $\phi_1 > \varphi_1$, $\theta_1 > \vartheta_1$, and $\theta_1 > \pi/2$ hold. Therefore, the angle between $\mathbf{M}$ and $\mathbf{H}$ is always smaller than the angle between $\mathbf{n}$ and $\mathbf{H}$. Since the resulting energy dissipation is proportional to the scalar product of $\mathbf{M}$ and $\mathbf{H}$, its value decreases with decreasing anisotropy. Secondly, the value of $\sigma\Omega\gamma^{-1}$ can be associated with a some effective magnetic field, which can be comparable with H_a in the case of high frequencies.

At the same time, the questions about stability of precession, possible nonlinear modes of motion including chaotic and switching between them [15], [16] remain open. The further investigation can be performed numerically and this is purpose of our further research.

IV. Conclusions

The dynamics of a uniaxial ferromagnetic fine particle placed into a viscous environment and driven by a time-periodic external magnetic field was studied. Using the dynamical classical approach, which is based on the total momentum conservation law [9], the coupled motion of the nanoparticle magnetic moment and nanoparticle body was considered. In particular, the analytical solutions of the equations of motion in the case of small rotational oscillations and the case of uniform rotation of vectors $\mathbf{M}$ and $\mathbf{n}$ (the nanoparticle magnetization and unit vector coupled with the easy axis, respectively) have been obtained.

The first type of motion takes place when the amplitude of an external field is much smaller than the anisotropy field magnitude. We investigated this mode in the linear approximation, within which the exact expressions for the oscillations amplitudes have been derived. It was shown that the resonant character of the frequency dependence of the magnetic moment amplitudes is more pronounced, while the easy axis amplitudes decay monotonically for the realistic values of the system parameters. However, the viscous rotation of the whole nanoparticle impacts considerably on its magnetic dynamics, and such interference between $\mathbf{M}$ and $\mathbf{n}$ motion does not permit to separate the contributions of these two mechanisms into the energy dissipation. In this way we demonstrate the restriction of the simpler approximations, such as the frozen magnetic moment model or rigidly fixed nanoparticle.

For the precessional type of motion, the algebraic equations for precession and lag angles of $\mathbf{M}$ and $\mathbf{n}$ have been written. The methodological feature of our approach consists in the representation of the angular velocity vector in the frame rotating with the external circularly-polarized field followed by the transformation into the laboratory coordinate system. The main properties of the solution of the obtained algebraical equations are the following. Firstly, the nanoparticle magnetic moment always constitutes a smaller angle with an external field than the anisotropy axis, and, as a consequence, the energy losses will decrease with decreasing anisotropy field. Secondly, the precessional dynamics suggests the presence of a some effective field, which is perpendicular to the field polarization plane, depends on the filed frequency and the polarization direction. This effective field can be comparable with the anisotropy field in the case of high frequencies.

Acknowledgment

The authors are grateful to the Ministry of Education and Science of Ukraine for the financial support under Grant No. 0116U002622.

References

[1] R. E. Rosensweig, Ferrohydrodynamics, Cambridge University Press, 1985.

[2] M. I. Shliomis “Magnetic fluids,” Phys. Usp. +, vol. 17, no. 2, p. 153, 1974.

[3] W. F. Brown, “Thermal Fluctuations of a Single-Domain Particle,” Phys. Rev., vol. 130, pp. 1677-1686, 1963.

[4] A. Jordan, R. Scholz, P. Wust, H. Fahling, R. Felix, “Magnetic fuid hyperthermia (MFH): Cancer treatment with AC magnetic feld induced excitation of biocompatible superparamagnetic nanoparticles,” J. Magn. Magn. Mater., vol. 201, pp. 413-419, 1999.

[5] R. E. Rosensweig, “Heating magnetic fluid with alternating magnetic field,” J. Magn. Magn. Mater., vol. 252, pp. 370-374, 2002.

[6] A. O. Tsebers, “Simultaneous rotational diffusion of the magnetic moment and the solid matrix of a single-domain ferromagnetic particle,” Magnetohydrodynamics, vol. 11, pp. 273-278, 1975.

[7] H. Mamiya, and B. Jeyadevan, “Hyperthermic effects of dissipative structures of magnetic nanoparticles in large alternating magnetic fields,” Sci. Rep., vol. 1, pp. 157EP, 2011.

[8] N. A. Usov, and B. Ya. Liubimov, “Dynamics of magnetic nanoparticle in a viscous liquid: Application to magnetic nanoparticle hyperthermia,” J. Appl. Phys., vol. 112, pp. 023901, 2012.

[9] K. D. Usadel, and C. Usadel, “Dynamics of magnetic single domain particles embedded in a viscous liquid,” J. Appl. Phys., vol. 118, pp. 234303, 2015.

[10] N. A. Usov, and B. Ya. Liubimov, “Magnetic nanoparticle motion in external magnetic field,” J. Magn. Magn. Matter., vol. 385, pp. 339–346, 2015.

[11] Haiwen Xi, Kai-Zhong Gao, Yiming Shi, Song Xue, “Precessional dynamics of single-domain magnetic nanoparticles driven by small ac magnetic fields,” J. Phys. D: Appl. Phys., vol. 39, pp. 4746, 2006.

[12] H. Keshtgar, S. Streib, A. Kamra, Ya. M. Blanter, G. E. W. Bauer, “Magnetomechanical coupling and ferromagnetic resonance in magnetic nanoparticles,” Phys. Rev. B, vol. 95, pp. 134447, 2017.

[13] T. V. Lyutyy, O. M. Hryshko, A. A. Kovner, E. S. Denisova , “Precession of a Fine Magnetic Particle with Finite Anisotropy in a Viscous Fluid,” J. Nano- Electron. Phys., vol. 8, pp. 04086, 2016.

[14] T. V. Lyutyy, O. M. Hryshko, A. A. Kovner, “Power Loss for a Periodically Driven Ferromagnetic Nanoparticle in a Viscous Fluid: the Finite Anisotropy Aspects,” arXiv:1706.00777 [cond-mat.other], 2017 (submitted to J. Magn. Magn. Matter.).

[15] T. V. Lyutyy, S. I. Denisov, A. Yu. Peletskyi, C. Binns, “Energy dissipation in single-domain ferromagnetic nanoparticles: Dynamical approach,” Phys. Rev. B, vol. 91, pp. 054425, 2015.

[16] T. V. Lyutyy, A. Yu. Polyakov, A. V. Rot-Serov, C. Binns, “Switching properties of ferromagnetic nanoparticles driven by a circularly polarized magnetic field,” J. Phys.: Condens. Matter, vol. 21, pp. 396002, 2009.