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# New Nonrelativistic Three-Dimensional Spectroscopic Studies of NMGECSC Potential in Presence of External Electric

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In this work, we have investigated some aspects of the new more general exponential cosine screened Coulomb potential (NMGECSCP) in noncommutative three-dimensional real space-phase (NC: 3D-RSP) for one electron atoms through the generalized Bopp's shift method in the framework of four infinitesimal parameters $\Theta(\chi)$ and $\bar{\theta}(\bar{\sigma})$ due to (space-phase) noncommutativity, by means of the solution of the deformed Schrödinger equation (DSE). The perturbation property of the spin-orbital Hamiltonian operator $H_{so-ges}(r,\Theta,\bar{\theta})$ and new Zeeman Effect operator $H_{z-gec}(r,\chi,\bar{\sigma})$ are investigated and the corresponding energy eigenvalues $E_{ges-(d,\mu)}(p,j=l\pm1/2,l,s)$ and $E_{z-gec}(p,m)$ are easily calculated. The new eigenvalues reduce to known results in quantum mechanics if $(\Theta,\bar{\theta})\equiv(0,0)$. We have shown also that, the global quantum group (GQG) of (NC: 3D-RSP) reduce to new subgroup symmetry of NC three-dimensional real space (NC: 3D-RS) under three-dimensional NMGECSCP interactions.

**Keywords**: Three dimensional Schrödinger equation, More general exponential cosine screened Coulomb potential, Noncommutative space-phase, Star product and generalized Bopp's shift method.

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## 1. INTRODUCTION

The more general exponential cosine screened Coulomb potential (MGECSCP) are used to investigate important interactions in various fields of physics such as plasma physics, nuclear physics, condensed matter physics, and atomic physics [1-5]. In view of what has been mentioned, we would like to study the results of the interactions of this potentials in a large space of quantum mechanics, currently known by the noncommutative quantum mechanics or extended quantum mechanics, which known firstly by Heisenberg and was formalized by Snyder at 1947, suggest by the physical recent results in string theory [6]. Over the past few years, theoretical physicists have shown a great deal of interest in solving Schrödinger equation for various potentials in NC space-phase to obtaining profound interpretations at microscopic scale [7-11] and in particularly, our previously works in in (NC: 3D-RSP) [12-14]. It is well known that, the notions of noncommutativity of space and phase based essentially on.

Seiberg-Witten map, the Bopp's shift method and the star product, which modified the ordinary product $(fg)(x,p)$ to the new form $(f*g)(x,p)$ at first order of two infinitesimal antisymmetric constants tensors $2\left(\theta^{\mu\nu},\bar{\theta}^{\mu\nu}\right)\equiv\varepsilon^{k\mu\nu}\left(\theta_k,\bar{\theta}_k\right)$ as (Throughout this paper using atomic units $\hbar=e=m=1$ and $z=1$) [6-12]:

$$(f*g)(x,p)=\left(fg-\frac{i}{2}\theta^{\mu\nu}\partial_{\mu}^{x}f\,\partial_{\nu}^{x}g+\frac{i}{2}\bar{\theta}^{\mu\nu}\partial_{\mu}^{p}f\,\partial_{\nu}^{p}g\right)(x,p). \quad (1)$$

The above equation presents the noncommutativity effects of space and phase, allow us to obtaining the following new non nulls commutators for NC coordinate and momentum in GQG of (NC: 3D-RSP) symmetries as follows [11-14]:

$$[\hat{x}_{\mu},\hat{p}_{\mu}]_{*}=i,$$
$$[\hat{x}_{\mu},\hat{x}_{\nu}]_{*}=i\theta_{\mu\nu}, \quad (2)$$
$$[\hat{p}_{\mu},\hat{p}_{\nu}]_{*}=i\bar{\theta}_{\mu\nu}.$$

On the other hand, the studies of new more general exponential cosine screened Coulomb potential (NMGECSCP) for one electron atoms has attracted wide attention. Motivated by the studies of M. K. Bahar in ref. [5] and others in this paper, we find the new bound state solution of the time independent Schrodinger equation for NMGECSCP in (NC: 3D-RSP) model. However, the solutions of modified radial Schrodinger equation for any angular momentum quantum number $l$, with NMGECSCP, for one electron atoms, using generalized Bopp's shift method in (NC: 3D-RSP) which is the aim of this paper, has not yet been reported. The present paper consists of five sections. To make this present work self-contained, it is organized as follows: In the second section, we have briefly review the SE with 3D-MGECSCP. In the third section, we shall briefly give the fundamental concepts of the generalized Bopp's shift method, and then we derive the deformed potential and NC spin-orbital Hamiltonian operator for one-electron atoms with 3D-MGECSCP. In the next step, we apply the perturbation

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theory to find the modified spectrum $E_{gec-d}\left(p,j=l-1/2,l,s\right)$ and $E_{gec-u}\left(p,j=l+1/2,l,s\right)$ corresponding of one-electron atoms at first order of two parameters $\Theta$ and $\bar{\theta}$ for $n^{th}$ excited stats and then we end this section by deduce the spectrum $E_{z-gec}\left(n,r_e,D_e,m\right)$ produced automatically by the external magnetic field. In section four, we resume the global spectrum for NMGECSCP and we conclude the corresponding global NC Hamiltonian operator $\hat{H}_{nc-gec}$ in GQG of (NC: 3D-RSP) symmetries. Finally, section five is devoted to a brief summary and conclusion.

## 2. REVIEW THE SPECTRUM OF 3D-MGECSCP IN ORDINARY QUANTUM MECHANICS

The MGECSC potential in the presence of external electric field $F$ is considered for hydrogen like atoms [5]:

$$V_{gec}\left(r\right)=-\frac{Ze^2}{r}\left(1+br\right)e^{-r/\lambda}\cos\left(\frac{cr}{\lambda}\right)+eFr\cong-Ze^2\left\{\begin{array}{l}\left(b-\frac{1}{\lambda}\right)+\frac{1}{r}+\left(\frac{1-c^2-2b\lambda}{2\lambda^2}\right)r\\+\left(\frac{3c^2+3b\lambda-3bc^2\lambda-1}{6\lambda^2}\right)r^2\end{array}\right\}+eFr, \quad (3)$$

where $b$, $c$ and $\lambda$ are the screening parameters of MGECSC potential. The first part in eq. (3) is MGECSC potential while the second part is contribution of external electric field on system. The radial part $R_l^{(p)}\left(r\right)$ of the normalized wave functions $\Psi_{plm}\left(r,\theta,\phi\right)=\frac{R_l^{(p)}\left(r\right)}{r}Y_l^m\left(\theta,\phi\right)$ for 3D-SE satisfied the following equation for MGECSC potential in the presence of external electric field $F$ [5]:

$$\frac{d^2R_l\left(r\right)}{dr^2}+\left(\varepsilon-\frac{l\left(l+1\right)}{r^2}+\frac{2}{r}+\alpha_1 r+\alpha_2 r^2\right)R_l\left(r\right)=0, \quad (4)$$

where ansatzs are in the following form:

$$\varepsilon=2E_l+2b-\frac{2}{\lambda},\alpha_1=\frac{1-c^2-2b\lambda}{\lambda^2}+2\text{F and}\quad \alpha_2=\frac{3c^2+3b\lambda-3bc^2\lambda-1}{3\lambda^2} \quad (5)$$

With $E_l$ are the energy values. According to the references [5], the complete orthonormalized wave function $\Psi_{plm}\left(r,\theta,\phi\right)$ for MGECSC potential in the presence of external electric field $F$ in 3-dimensional spaces, is given by:

$$\Psi_{plm}\left(r,\theta,\phi\right)=\begin{cases}a_0\exp\left(\beta r+\frac{\gamma}{2}r^2\right)r^{\delta-1}Y_l^m\left(\theta,\phi\right) & \text{for} \quad p=0,\\ \left(a_0+a_1 r\right)\exp\left(\beta r+\frac{\gamma}{2}r^2\right)r^{\delta-1}Y_l^m\left(\theta,\phi\right) & \text{for} \quad p=1,\\ \left(a_0+a_1 r+.....a_p r^p\right)\exp\left(\beta r+\frac{\gamma}{2}r^2\right)r^{\delta-1}Y_l^m\left(\theta,\phi\right) & \text{for} \quad \text{any p}\end{cases} \quad (6)$$

with $\gamma^2=-\alpha_2$, $\beta=-\frac{1}{l+1}$ and $\delta=l+1$ while $\left(a_0,a_1,...a_p\right)$ can be calculated by using normalization condition.

## 3. THEORETICAL FRAMEWORK

### 3.1 Theoretical Overview of Generalized Bopp's Shift Method in 3-dimensional Spaces-phases for NMGECSCP

In order to obtain DSE in (NC: 3D-RSP) symmetries, we replace ordinary Hamiltonian operator $\hat{H}\left(p_i,x_i\right)$, ordinary complex function $\Psi_{plm}\left(r,\theta,\phi\right)$, ordinary energy $E_l$ and ordinary product by NC Hamiltonian operator $\hat{H}_{nc-gec}\left(\hat{p}_i,\hat{x}_i\right)$, new complex function $\hat{\Psi}\left(\vec{\hat{r}}\right)$, new energy $E_{nc-gec}$ and new star product $\left(*\right)$, respectively. Allow us to writing the new 3D-DSE for NMGECSCP as follows [12-14]:

$$\hat{H}_{nc-gec}\left(\hat{p}_i,\hat{x}_i\right)*\hat{\Psi}\left(\vec{\hat{r}}\right)=E_{nc-gec}\hat{\Psi}\left(\vec{\hat{r}}\right). \quad (7)$$

The new Hamiltonian operator $\hat{H}_{nc-gec}\left(\hat{p}_i,\hat{x}_i\right)$ acts on a suitable by $\left(*\right)$ on the wave new complex function $\hat{\Psi}\left(\vec{\hat{r}}\right)$ of the new system to give us the energy eigenvalues $E_{nc-gec}$ of the new system energy in (NC: 3D-RSP) symmetries. It is important to notice that, the new Hamiltonian operator $\hat{H}_{nc-gec}\left(\hat{p}_i,\hat{x}_i\right)$ can be expressed in three general varieties: both NC space and NC phase (NC: 3D-RSP), only NC space (NC: 3D-RS) and only NC phase (NC: 3D-RP) as, respectively:

$$\begin{aligned}
&\hat{H}_{nc-gec}\left(\hat{p}_i,\hat{x}_i\right)\equiv\hat{H}\left(\hat{p}_i=p_i+\frac{\bar{\theta}_{ij}}{2}x_j;\hat{x}_i=x_i-\frac{\theta_{ij}}{2}p_j\right) \quad \text{for (NC: 3D-RSP)},\\
&\hat{H}_{nc-gec}\left(\hat{p}_i,\hat{x}_i\right)\equiv\hat{H}\left(\hat{p}_i=p_i;\hat{x}_i=x_i-\frac{\theta_{ij}}{2}p_j\right) \quad \text{for (NC: 3D-RS)},\\
&\hat{H}_{nc-gec}\left(\hat{p}_i,\hat{x}_i\right)\equiv\hat{H}\left(\hat{p}_i=p_i+\frac{\bar{\theta}_{ij}}{2}x_j;\hat{x}_i=x_i\right) \quad \text{for (NC: 3D-RP)}.
\end{aligned} \tag{8}$$

To find the analytical solutions of the eq. (7) we must apply the generalized Bopp's shift method instead of solving the 3D-DSE for NMGECSCP directly with $(*)$; we treated by using directly the two commutators, in addition to usual commutators on quantum mechanics [11-14]:

$$\left[\hat{x}_\mu,\hat{x}_\nu\right]=i\theta_{\mu\nu} \text{ and } \left[\hat{p}_\mu,\hat{p}_\nu\right]=i\bar{\theta}_{\mu\nu}. \tag{9}$$

It is well known, that the two new operators ($\hat{x}_\mu$ and $\hat{p}_\mu$) are given by the following Darboux transformations [12-14]:

$$\hat{x}_\mu=x_\mu-\frac{\theta_{\mu\nu}}{2}p_\nu \text{ and } \hat{p}_\mu=p_\mu+\frac{\bar{\theta}_{\mu\nu}}{2}x_\nu. \tag{10}$$

The two variables $\left(x_\mu,p_\mu\right)$ satisfy the usual canonical commutation relations in ordinary quantum mechanics. In recently work, we are interest with the first variety in eq. (8). We may go a step further and consider the Bopp's method (modified by a shift), which allows us to reducing the above DSE to new ordinary form, in addition two fundamental translations of space and phase which are presenting in eq. (9):

$$H_{nc-gec}\left(\hat{p}_i,\hat{x}_i\right)\psi\left(\vec{r}\right)=E_{nc-gec}\psi\left(\vec{r}\right). \tag{11}$$

The new modified Hamiltonian $H_{nc-gec}\left(\hat{p}_i,\hat{x}_i\right)$ that appears above is given by:

$$H_{nc-gec}\left(\hat{p}_\mu,\hat{x}_\mu\right)=\frac{\hat{p}^2}{2}+V_{gec}\left(\hat{r}\right). \tag{12}$$

The new potential $V_{gec}\left(\hat{r}\right)$ in the GQG of (NC: 3D-RSP) can be written as:

$$V_{gec}\left(\hat{r}\right)=-\left\{\left(b-\frac{1}{\lambda}\right)+\frac{1}{\hat{r}}+\left(\frac{1-c^2-2b\lambda}{2\lambda^2}+F-2F\right)\hat{r}+\left(\frac{3c^2+3b\lambda-3bc^2\lambda-1}{6\lambda^2}\right)\hat{r}^2\right\}\equiv-\left\{\left(b-\frac{1}{\lambda}\right)+\frac{1}{\hat{r}}+\left(\alpha_1-2F\right)\hat{r}+\frac{\alpha_2}{2}\hat{r}^2\right\} \tag{13}$$

According to our references [12-14], we can write the two operators ($\hat{r}^2$ and $\hat{p}^2$) in GQG of (NC: 3D-RSP) symmetries as follows:

$$\begin{aligned}
&\hat{r}^2=r^2-\vec{\mathbf{L}}\overline{\overline{\boldsymbol{\theta}}}+O\left(\theta\right) \quad \text{and} \quad \hat{p}^2=p^2+\vec{\mathbf{L}}\overline{\overline{\boldsymbol{\theta}}}+O\left(\bar{\theta}\right) \quad \text{with}\\
&\mathbf{L}\Theta\equiv L_x\Theta_{12}+L_y\Theta_{23}+L_z\Theta_{13} \quad \text{and} \quad \vec{\mathbf{L}}\overline{\overline{\boldsymbol{\theta}}}\equiv L_x\bar{\theta}_{12}+L_y\bar{\theta}_{23}+L_z\bar{\theta}_{13}
\end{aligned} \tag{14}$$

After straightforward calculations one can obtains the important terms ($\frac{1}{\hat{r}}$, $\left(\alpha_1-2F\right)\hat{r}$ and $\frac{\alpha_2}{2}\hat{r}^2$), which will be used to determine the NMGECSCP $V_{gec}\left(\hat{r}\right)$ in GQG of (NC: 3D-RSP) symmetries as follows:

$$\begin{aligned}
&\frac{1}{\hat{r}}=\frac{1}{r}+\frac{\vec{\mathbf{L}}\overline{\overline{\boldsymbol{\theta}}}}{2r^3}+O\left(\theta\right),\\
&\left(\alpha_1-2F\right)\hat{r}=\left(\alpha_1-2F\right)r-\frac{\alpha_1-2F}{2r}\vec{\mathbf{L}}\overline{\overline{\boldsymbol{\theta}}}+O\left(\theta\right),\\
&\frac{\alpha_2}{2}\hat{r}^2=\frac{\alpha_2}{2}r^2-\frac{\alpha_2}{2}\vec{\mathbf{L}}\overline{\overline{\boldsymbol{\theta}}}+O\left(\theta\right).
\end{aligned} \tag{15}$$

Substituting, eq. (14) and eq. (15) (into eq. (13), one gets the NMGECSCP $V_{gec}\left(\hat{r}\right)$ in GQG of (NC: 3D-RSP) symmetries as follows:

$$V_{gec}\left(\hat{r}\right)=V_{gec}\left(r\right)-\left\{\frac{1}{2r^3}-\frac{\alpha_1-2F}{2r}-\frac{\alpha_2}{2}\right\}\vec{\mathbf{L}}\overline{\overline{\boldsymbol{\theta}}}. \tag{16}$$

It is clear that, the first term $V_{gec}\left(r\right)$ in above equation represent the ordinary 3D-MGECSCP while the rest terms, are produced by the deformations of noncommutativity of space-phase. Now simultaneously transforming $V_{gec}\left(\hat{r}\right)$ and $\vec{p}^2/2$ gives the global perturbative potential operators $H_{\text{pert-gec}}\left(r,\theta,\bar{\theta}\right)$ for 3D-MGECSCP in GQG of (NC: 3D-RSP) symmetries:

$$\begin{aligned}
&H_{pert-gec}\left(r,\theta,\bar{\theta}\right)=-\left(\frac{1}{2r^3}-\frac{\alpha_1-2F}{2r}-\frac{\alpha_2}{2}\right)\vec{\mathbf{L}}\overline{\overline{\boldsymbol{\theta}}}+\\
&+\frac{\vec{\mathbf{L}}\overline{\overline{\boldsymbol{\theta}}}}{2}+O\left(\theta,\bar{\theta}\right).
\end{aligned} \tag{17}$$

The above operator can be considering of the sum of $V_{pert-gec}\left(r,\theta,\bar{\theta}\right)$ and the couplings $\frac{\vec{\mathbf{L}}\overline{\overline{\boldsymbol{\theta}}}}{2}$. Since we are only interested in the corrections of order $\theta$ and $\bar{\theta}$, we can disregard the second term in the perturbative operator $H_{\text{pert-gec}}\left(r,\theta,\bar{\theta}\right)$.

### 3.2 3D-spin-orbital Hamiltonian Operators for NMGECSCP in GQG of (NC: 3D-RSP)

In this subsection, we apply the same strategy, which we have seen in our previously works [12-14], under such particular choice, one can easily reproduce both $\vec{\mathbf{L}}\bar{\bar{\boldsymbol{\theta}}}$ and $\vec{\mathbf{L}}\bar{\bar{\boldsymbol{\theta}}}$ to the new physical forms $\alpha\Theta\vec{S}\vec{L}$ and $\alpha\bar{\theta}\vec{S}\vec{L}$, respectively, to obtain the new forms of $H_{\text{pert-gec}}\left(r,\Theta,\bar{\theta}\right)$ for 3D-NMGECSCP as follows:

$$H_{\text{so-gec}}\left(r,\Theta,\bar{\theta}\right) \equiv -\alpha\left\{\left(\frac{1}{2r^3}-\frac{\alpha_1-2F}{2r}-\frac{\alpha_2}{2}\right)\Theta-\frac{\bar{\theta}}{2}\right\}\vec{\mathbf{L}}\vec{S}, \quad (18)$$

here $\vec{S}$ denote the spin of one electron atoms and $\alpha$ is real constant, which can be play the role of fine structure constant in the electromagnetic interactions, thus, the spin-orbital interactions $H_{\text{pert-gec}}\left(r,\Theta,\bar{\theta}\right)$ appear automatically because of the new properties of space-phase. Now, it is possible to rewrite the above equation as follows:

$$H_{\text{pert-gec}}\left(r,\theta,\bar{\theta}\right) = -\frac{\alpha}{2}\left\{\left(\frac{1}{2r^3}-\frac{\alpha_1-2F}{2r}-\frac{\alpha_2}{2}\right)\Theta-\frac{\bar{\theta}}{2\mu}\right\}\times \times\left(\vec{J}^2-\vec{L}^2-\vec{S}^2\right). \quad (19)$$

We have replaced the coupling $\vec{\mathbf{L}}\vec{S}$ by new physical values $\frac{1}{2}\left(\vec{J}^2-\vec{L}^2-\vec{S}^2\right)$. As it well known, the eigenvalues $j$ of the total operator $\vec{J}=\vec{L}+\vec{S}$ can be obtains from the interval: $|l-1/2| \le j \le |l+1/2|$, which allow us to obtaining the two eigenvalues of the operator $\left(\vec{J}^2-\vec{L}^2-\vec{S}^2\right)$ as follows:

$$k\left(j,l,s\right) \equiv j(j+1)+l(l+1)-s(s+1) = \begin{cases} k_-\left(j=l-1/2,l,s\right)=\left(l+1\right) & \text{for spin\_down,} \\ k_+\left(j=l+1/2,l,s\right)=-\dfrac{l+1}{2} & \text{for spin\_down.}\end{cases} \quad (20)$$

Then, one can form a diagonal matrix $H_{so-gec}\left(r,\Theta,\bar{\theta}\right)$ of order $\left(3\times 3\right)$, with diagonal elements $\left(H_{so-gec}\right)_{11}$, $\left(H_{so-gec}\right)_{22}$ and $\left(H_{so-gec}\right)_{33}$ as:

$$\begin{aligned}
&\left(H_{so-gec}\right)_{11} = -\alpha k_+\left\{\left(\frac{1}{2r^3}-\frac{\alpha_1-2F}{2r}-\frac{\alpha_2}{2}\right)\Theta-\frac{\bar{\theta}}{2}\right\} \text{if } j=l+\tfrac{1}{2},\\
&\left(H_{so-gec}\right)_{22} = -\alpha k_-\left\{\left(\frac{1}{2r^3}-\frac{\alpha_1-2F}{2r}-\frac{\alpha_2}{2}\right)\Theta-\frac{\bar{\theta}}{2}\right\} \text{if } j=l-\tfrac{1}{2}, \quad (21)\\
&\left(H_{so-gec}\right)_{33} = 0.
\end{aligned}$$

After straightforward calculation, one can show that, the radial function $R_l^{(p)}\left(r\right)$ satisfying the following differential equation, in GQG of (NC: 3D-RSP) symmetries for NMGECSCP:

$$\frac{d^2R_l^{(p)}\left(r\right)}{dr^2}+2\begin{pmatrix} E_{nc-gec}+\left\{\left(b-\dfrac{1}{\lambda}\right)+\dfrac{1}{r}+\left(\alpha_1-2F\right)r+\dfrac{\alpha_2}{2}r^2\right\}+ \\ \left(\dfrac{1}{2r^3}-\dfrac{\alpha_1-2F}{2r}-\dfrac{\alpha_2}{2}\right)\vec{\mathbf{L}}\bar{\bar{\boldsymbol{\theta}}}+\dfrac{\vec{\mathbf{L}}\bar{\bar{\boldsymbol{\theta}}}}{2}-\dfrac{l\left(l+1\right)}{2r^2}\end{pmatrix}R_l^{(p)}\left(r\right)=0\ . \quad (22)$$

In the next parts of this article we consider the term $H_{\text{pert-gec}}\left(r,\Theta,\bar{\theta}\right)$, as an infinitesimal part compared of the principal part of Hamiltonian operator $H_{gec}\left(p,x\right)$ for 3D- MGECSCP in ordinary quantum mechanics, this allows to apply standard perturbation theory to obtaining the nonrelativistic energy corrections $E_{gec-d}\left(p,j=l-1/2,l,s\right)$ and $E_{gec-u}\left(p,j=l+1/2,l,s\right)$ corresponding $\left(j=l-1/2\right)$ and $\left(j=l+1/2\right)$ of one electron atoms at first order of two parameters $\Theta$ and $\bar{\theta}$.

### 3.3 The Exact Spin-orbital Spectrum for NMGECSCP in GQG of (NC: 3D-RSP) Symmetries

In order to find the nonrelativistic energy corrections $E_{gec-d} \equiv E_{gec-d}\left(p,j=l-1/2,l,s\right)$ *and* $E_{gec-u} \equiv E_{gec-u}\left(p,j=l+1/2,l,s\right)$ *corresponding* $\left(j=l-1/2\right)$ and $\left(j=l+1/2\right)$ of one electron atoms at first order of two parameters $\Theta$ and $\bar{\theta}$ we apply standard perturbation theory and through the structure constants which specified the dimensionality of NMGECSCP for one electron atoms, thus, we have the following results, in two typical varieties $p=0$ (ground states) and $p=1$ (first excited states). For the ground states case of $p=0$ we have:

$$\begin{aligned}
\frac{E_{gec-d}(p{=}0)}{\left|a_0\right|^2k_-} &= \int_0^{+\infty} r^{2\delta}\exp\left(2\beta r+\gamma r^2\right)\left\{\left(\frac{1}{2r^3}-\frac{\alpha_1-2F}{2r}-\frac{\alpha_2}{2}\right)\Theta-\frac{\bar{\theta}}{2}\right\}dr,\\
\frac{E_{gec-u}(p{=}0)}{\left|a_0\right|^2k_+} &= \int_0^{+\infty} r^{2\delta}\exp\left(2\beta r+\gamma r^2\right)\left\{\left(\frac{1}{2r^3}-\frac{\alpha_1-2F}{2r}-\frac{\alpha_2}{2}\right)\Theta-\frac{\bar{\theta}}{2}\right\}dr.
\end{aligned} \quad (23)$$

Now, it is important to introduce the following four factors $T_i^0\left(i=\overline{1,4}\right)$ as follows:

$$\begin{aligned} T_1^0 &= \frac{1}{2}\int_0^{+\infty} r^{(2\delta-2)-1}\exp\left(2\beta r+\gamma r^2\right)dr, \\ T_2^0 &= -\left(\frac{\alpha_1-2F}{2}\right)\int_0^{+\infty} r^{2\delta-1}\exp\left(2\beta r+\gamma r^2\right)dr, \\ T_3^0 &= \alpha_2 T_4^0 = -\frac{\alpha_2}{2}\int_0^{+\infty} r^{(2\delta+1)-1}\exp\left(2\beta r+\gamma r^2\right)dr. \end{aligned} \tag{24}$$

On arranging eq. (23), we get our nonrelativistic energy levels $E_{gec-d}(p{=}0)$ and $E_{gec-u}(p{=}0)$ at first order of two parameters $\Theta$ and $\bar{\theta}$ for one-electron atoms as:

$$\begin{aligned} E_{gec-d}(p{=}0) &= \left|a_0\right|^2 k_-\left\{\Theta\sum_{i=1}^{3}T_i^0+\bar{\theta}T_4^0\right\}, \\ E_{gec-u}(p{=}0) &= \left|a_0\right|^2 k_+\left\{\Theta\sum_{i=1}^{3}T_i^0+\bar{\theta}T_4^0\right\}. \end{aligned} \tag{25}$$

It is very important to calculate the four terms $T_i^0\left(i=\overline{1,4}\right)$, to achieve this goal; we apply the following special integral [15]:

$$\int_0^{+\infty} x^{\nu-1}\exp\left(-\beta' x^2-\gamma' x\right)dx = \left(2\beta'\right)^{-\frac{\nu}{2}}\Gamma(\nu)\exp\left(\frac{\gamma'^2}{8\beta'}\right)D_{-\nu}\left(\frac{\gamma'}{\sqrt{2\beta'}}\right) \tag{26}$$

where $D_{-\nu}\left(\frac{\gamma'}{\sqrt{2\beta'}}\right)$ denote to the Parabolic cylinder functions function, $\Gamma(\nu)$ Gamma function $\mathrm{Re}l\left(\beta'\right)\rangle 0$ and $\mathrm{Re}l\left(\nu\rangle 0\right)$. After straightforward calculations, we can obtain the results:

$$\begin{aligned} T_1^0 &= \frac{1}{2}\left(-2\gamma\right)^{-l}\Gamma(2l)\exp\left(-\frac{\beta^2}{2\gamma}\right)D_{-(2l)}\left(\frac{-2\beta}{\sqrt{-2\gamma}}\right) \\ T_2^0 &= -\left(\frac{\alpha_1-2F}{2}\right)\left(-2\gamma\right)^{(l+1)}\Gamma(2l+2)\exp\left(-\frac{\beta^2}{2\gamma}\right)D_{-(2l+2)}\left(\frac{-2\beta}{\sqrt{-2\gamma}}\right) \\ T_3^0 &= \alpha_2 T_4^0 = -\frac{\alpha_2}{2}\left(-2\gamma\right)^{-\frac{2l+3}{2}}\Gamma(2l+3)\exp\left(-\frac{\beta^2}{2\gamma}\right)D_{-(2l+3)}\left(\frac{-2\beta}{\sqrt{-2\gamma}}\right) \end{aligned} \tag{27}$$

Further, the substitution of eq. (27) into eq. (25) enables us to obtain the first quantum correction $E_{gec-d}(p{=}0)$ and $E_{gec-u}(p{=}0)$ at first order of two parameters $\Theta$ and $\bar{\theta}$ for one-electron atoms as:

$$\begin{aligned} E_{gec-d}(p{=}0) &= \left|a_0\right|^2 k_-\left\{\Theta T_0(\gamma,l)+\bar{\theta}T_4^0(\gamma,l)\right\}, \\ E_{gec-u}(p{=}0) &= \left|a_0\right|^2 k_+\left\{\Theta T_0(\gamma,l)+\bar{\theta}T_4^0(\gamma,l)\right\}, \end{aligned} \tag{28}$$

with $T_0(\gamma,l)=\sum_{i=1}^{3}T_i^0$. Allow us, the following important physical results:

$$H_{\text{so-gec}}\left(r,\Theta,\bar{\theta}\right)\frac{R_l^{(0)}(r)}{r}Y_l^m(\theta,\phi) = \begin{cases} E_{gec-d}(p{=}0)\dfrac{R_l^{(0)}(r)}{r}Y_l^m(\theta,\phi) & \text{for} \quad j=l-1/2 \\ E_{gec-u}(p{=}0)\dfrac{R_l^{(0)}(r)}{r}Y_l^m(\theta,\phi) & \text{for} \quad j=l+1/2. \end{cases}, \tag{29}$$

For the first excited states case of $p=1$:
We have:

$$\begin{aligned} \frac{E_{gec-d}(p{=}1)}{k_-} &= \int_0^{+\infty}\left(a_0+a_1 r\right)^2 r^{2\delta}\exp\left(2\beta r+\gamma r^2\right)\left\{\left(\frac{1}{2r^3}-\frac{\alpha_1-2F}{2r}-\frac{\alpha_2}{2}\right)\Theta-\frac{\bar{\theta}}{2}\right\}dr, \\ \frac{E_{gec-u}(p{=}1)}{k_+} &= \int_0^{+\infty}\left(a_0+a_1 r\right)^2 r^{2\delta}\exp\left(2\beta r+\gamma r^2\right)\left\{\left(\frac{1}{2r^3}-\frac{\alpha_1-2F}{2r}-\frac{\alpha_2}{2}\right)\Theta-\frac{\bar{\theta}}{2}\right\}dr. \end{aligned} \tag{30}$$

Now, it is important to introduce the following 12-factors $T_i^2\left(i=\overline{1,12}\right)$ as follows:

$$\begin{aligned} T_1^1 &= \frac{a_0{}^2}{2}\int_0^{+\infty} r^{(2\delta-2)-1}\exp\left(2\beta r+\gamma r^2\right)dr, \\ T_2^1 &= \frac{2a_0a_1}{2}\int_0^{+\infty} r^{(2\delta-1)-1}\exp\left(2\beta r+\gamma r^2\right)dr, \\ T_3^1 &= \frac{a_1{}^2}{2}\int_0^{+\infty} r^{2\delta-1}\exp\left(2\beta r+\gamma r^2\right)dr, \end{aligned} \tag{31}$$

$$\begin{aligned} T_4^1 &= -\frac{a_0{}^2}{2}\left(\frac{\alpha_1-2F}{2}\right)\int_0^{+\infty} r^{(2\delta)-1}\exp\left(2\beta r+\gamma r^2\right)dr, \\ T_5^1 &= -\frac{2a_0a_1}{2}\left(\frac{\alpha_1-2F}{2}\right)\int_0^{+\infty} r^{(2\delta+1)-1}\exp\left(2\beta r+\gamma r^2\right)dr, \\ T_6^1 &= -\frac{a_1^2}{2}\left(\frac{\alpha_1-2F}{2}\right)\int_0^{+\infty} r^{(2\delta+2)-1}\exp\left(2\beta r+\gamma r^2\right)dr. \end{aligned} \tag{32}$$

$$T_7^1 = \alpha_2 T_{10}^1 = -\frac{\alpha_2 a_0^{\ 2}}{2} \int_0^{+\infty} r^{(2\delta+1)-1} \exp\left(2\beta r + \gamma r^2\right) dr,$$
$$T_8^1 = \alpha_2 T_{11}^1 = -a_0 a_1 \alpha_2 \int_0^{+\infty} r^{(2\delta+2)-1} \exp\left(2\beta r + \gamma r^2\right) dr, \quad (33)$$
$$T_9^1 = \alpha_2 T_{12}^1 = -\frac{a_1^{\ 2} \alpha_2}{2} \int_0^{+\infty} r^{(2\delta+3)-1} \exp\left(2\beta r + \gamma r^2\right) dr.$$

On arranging eq. (30), we get our nonrelativistic energy levels $E_{gec-d}(p{=}1)$ and $E_{gec-u}(p{=}1)$ at first order of two parameters $\Theta$ and $\bar{\theta}$ for one-electron atoms as:

$$E_{gec-d}(p{=}1) = k_- \left\{ \Theta \sum_{i=1}^{9} T_i^1 + \bar{\theta} \sum_{i=10}^{12} T_i^1 \right\},$$
$$E_{gec-u}(p{=}1) = k_+ \left\{ \Theta \sum_{i=1}^{9} T_i^1 + \bar{\theta} \sum_{i=10}^{12} T_i^1 \right\}. \quad (34)$$

To calculate the terms $T_i^1\left(i = \overline{1,12}\right)$, we apply the special integral, which presented in eq. (26) to obtain:

$$T_1^1 = \frac{a_0^{\ 2}}{2} (-2\gamma)^{(-l)} \Gamma(2l) \exp\left(-\frac{\beta^2}{2\gamma}\right) D_{-(2l)}\left(\frac{-2\beta}{\sqrt{-2\gamma}}\right)$$
$$T_2^1 = \frac{2a_0 a_1}{2} (-2\gamma)^{-\left(\frac{2l+1}{2}\right)} \Gamma(2l+1) \exp\left(-\frac{\beta^2}{2\gamma}\right) D_{-(2l+1)}\left(\frac{-2\beta}{\sqrt{-2\gamma}}\right) \quad (35)$$
$$T_3^1 = \frac{a_1^{\ 2}}{2} (-2\gamma)^{-(l+1)} \Gamma(2l+2) \exp\left(-\frac{\beta^2}{2\gamma}\right) D_{-(2l+2)}\left(\frac{-2\beta}{\sqrt{-2\gamma}}\right)$$

$$T_4^1 = -\frac{a_0^{\ 2}}{2} \left(\frac{\alpha_1 - 2F}{2}\right) (-2\gamma)^{-(l+1)} \Gamma(2l+2) \exp\left(-\frac{\beta^2}{2\gamma}\right) D_{-(2l+2)}\left(\frac{-2\beta}{\sqrt{-2\gamma}}\right)$$
$$T_5^1 = -\frac{2a_0 a_1}{2} \left(\frac{\alpha_1 - 2F}{2}\right) (-2\gamma)^{-\frac{2l+3}{2}} \Gamma(2l+3) \exp\left(-\frac{\beta^2}{2\gamma}\right) D_{-(2l+3)}\left(\frac{-2\beta}{\sqrt{-2\gamma}}\right) \quad (36)$$
$$T_6^1 = -\frac{a_1^{\ 2}}{2} \left(\frac{\alpha_1 - 2F}{2}\right) (-2\gamma)^{-(l+21)} \Gamma(2l+4) \exp\left(-\frac{\beta^2}{2\gamma}\right) D_{-(2l+4)}\left(\frac{-2\beta}{\sqrt{-2\gamma}}\right)$$

$$T_7^1 = \alpha_2 T_{10}^1 = -\frac{\alpha_2 a_0^{\ 2}}{2} (-2\gamma)^{-\frac{2l+3}{2}} \Gamma(2l+3) \exp\left(-\frac{\beta^2}{2\gamma}\right) D_{-(2l+3)}\left(\frac{-2\beta}{\sqrt{-2\gamma}}\right)$$
$$T_8^1 = \alpha_2 T_{11}^1 = -a_0 a_1 \alpha_2 (-2\gamma)^{-(\delta+1)} \Gamma(2\delta+2) \exp\left(-\frac{\beta^2}{2\gamma}\right) D_{-(2\delta+2)}\left(\frac{-2\beta}{\sqrt{-2\gamma}}\right) \quad (37)$$
$$T_9^1 = \alpha_2 T_{12}^1 = -\frac{a_1^{\ 2} \alpha_2}{2} (-2\gamma)^{-\frac{2l+5}{2}} \Gamma(2l+5) \exp\left(-\frac{\beta^2}{2\gamma}\right) D_{-(2l+5)}\left(\frac{-2\beta}{\sqrt{-2\gamma}}\right)$$

Further, the substitution of equations (35), (36) and (37) into eq. (34), enables us to obtain the first quantum correction $E_{gec-d}(p{=}1)$ and $E_{gec-u}(p{=}1)$ at first order of two parameters $\Theta$ and $\bar{\theta}$ for one-electron atoms as:

$$E_{gec-d}(p{=}1) = k_- \left\{ \Theta T_{1-s}(\gamma, l) + \bar{\theta} T_{1-p}(\gamma, l) \right\},$$
$$E_{gec-u}(p{=}1) = k_+ \left\{ \Theta T_{1-s}(\gamma, l) + \bar{\theta} T_{1-p}(\gamma, l) \right\} \quad (38)$$

with $T_{1-s}(\gamma, \delta, \beta) = \sum_{i=1}^{9} T_i^1$ and $T_{1-p}(\gamma, \delta, \beta) = \sum_{i=10}^{12} T_i^1$. Allow us, the following important physical results:

$$H_{\text{so-gec}}\left(r, \Theta, \bar{\theta}\right) \frac{R_l^1(r)}{r} Y_l^m(\theta, \phi) = \begin{cases} E_{gec-d}(p{=}1) \dfrac{R_l^1(r)}{r} Y_l^m(\theta, \phi) & \text{for} \quad j = l - 1/2, \\ E_{gec-u}(p{=}1) \dfrac{R_l^1(r)}{r} Y_l^m(\theta, \phi) & \text{for} \quad j = l + 1/2. \end{cases} \quad (39)$$

### 3.4 The Exact Magnetic Spectrum for NMGECSCP in GQG of (NC: 3D-RSP) Symmetries

On other hand, it's possible to found another automatically symmetry for NMGECSCP related to the influence of an external uniform magnetic field $\vec{\aleph}$, if we make the following transformations to ensure that previous calculations are not reputed:

$$\left(\theta, \bar{\theta}\right) \to \left(\chi, \bar{\sigma}\right) \aleph \quad (40)$$

Here $\chi$ and $\bar{\sigma}$ are two infinitesimal real proportional constants and further insight can be gained when we choose the magnetic field $\vec{\aleph} = \aleph \vec{k}$, then we can make the following translation:

$$\frac{\alpha}{2} \left\{ \theta \left( \frac{1}{2r^3} - \frac{\alpha_1 - 2F}{2r} - \frac{\alpha_2}{2} \right) + \frac{\bar{\theta}}{2} \right\} L_z \to \left( \chi \left( \frac{1}{2r^3} - \frac{\alpha_1 - 2F}{2r} - \frac{\alpha_2}{2} \right) + \frac{\bar{\sigma}}{2} \right) \aleph L_z. \quad (41)$$

Allow us to introduce the modified magnetic Hamiltonian operator $H_{\text{z-gec}}\left(r, \chi, \bar{\sigma}\right)$ for NMGECSCP P in global (NC: 3D-RSP) as:

$$H_{\text{z-gec}}\left(r, \chi, \bar{\sigma}\right) = \left( \chi \left( \frac{1}{2r^3} - \frac{\alpha_1 - 2F}{2r} - \frac{\alpha_2}{2} \right) + \frac{\bar{\sigma}}{2} \right) \left( \vec{\aleph} \vec{J} - \vec{S} \vec{\aleph} \right). \quad (42)$$

Here $\left(-\vec{S}\vec{\aleph}\right)$ denote to the ordinary Hamiltonian of Zeeman effect for 3D-NMGECSCP. To obtain the exact NC magnetic modifications of energy $E_{\text{z-gec}}(p = 0, m)$ and $E_{\text{z-gec}}(p = 1, m)$ corresponding ground states and first excited states for 3D-NMGECSCP, we replace both $k_+\left(j = l + 1/2, l, s\right)$ and $\Theta$ into equations (28) and (38) by the discreet quantum number $m(-l \le m \le +l)$ and new infinitesimal parameter $\chi$, respectively:

$$E_{\text{z-gec}}(p = 0, m) = \left|a_0\right|^2 m \left\{ \chi T_0(\gamma, l) + \bar{\sigma} T_4^0(\gamma, l) \right\} \aleph,$$
$$E_{\text{z-gec}}(p = 1, m) = m \left\{ \chi T_{1-s}(\gamma, l) + \bar{\sigma} T_{1-p}(\gamma, l) \right\} \aleph. \quad (43)$$

However, very little has been achieved in the solution of 3D-DSE for NMGECSCP. Thus, we can con conclude immediately:

$$\begin{aligned} H_{\text{z-gec}}\left(r,\chi,\bar{\sigma}\right)\Psi_{0lm}\left(r,\theta,\phi\right) &= \left|a_0\right|^2 m\left\{\chi T_0\left(\gamma,l\right)+\bar{\sigma}T_4^0\left(\gamma,l\right)\right\}\aleph\Psi_{olm}\left(r,\theta,\phi\right), \\ H_{\text{z-gec}}\left(r,\chi,\bar{\sigma}\right)\Psi_{1lm}\left(r,\theta,\phi\right) &= m\left\{\chi T_{1-s}\left(\gamma,l\right)+\bar{\sigma}T_{1-p}\left(\gamma,l\right)\right\}\aleph\Psi_{1lm}\left(r,\theta,\phi\right). \end{aligned} \quad (44)$$

## 4. RESULTS AND DISCUSSION OF GLOBAL SPECTRUM FOR NMGECSCP IN GLOBAL (NC: 3D-RSP) SYMMETRIES

We have solved the deformed radial Schrödinger equation and obtained the differences in the energy eigenvalues ($E_{gec-d}\left(p,j=l-1/2,l,s\right)$, $E_{gec-u}\left(p,j=l+1/2,l,s\right)$) and $E_{\text{z-gec}}\left(p,m\right)$ for the 3D-NMGECSCP in equations (29), (39) and (44) which are produced automatically by the effects of spin-orbital interaction $H_{\text{pert-gec}}\left(r,\theta,\bar{\theta}\right)$ and new Zeeman effect $H_{\text{z-gec}}\left(r,\chi,\bar{\sigma}\right)$, respectively. In the following, we summarize obtained results of the modified energy levels $E_{\text{nc-d}}\left(p,j,l,s,m\right)$ and $E_{\text{nc -u}}\left(p,j,l,s,m\right)$ of one electron atoms moving in 3D-NMGECSCP as provided in subsections (3.3) and (3.4), according to three equations (29), (39) and (44) the explicit bound state energies takes the form for ground states and first excited states for one electron atoms:

$$\begin{aligned} E_{\text{nc -d}}\left(p=0,j,l,s,m\right) &= E\left(p{=}0\right)+\left|a_0\right|^2 k_-\left\{\Theta T_0\left(\gamma,l\right)+\bar{\theta}T_4^0\left(\gamma,l\right)\right\}+\left|a_0\right|^2 m\left\{\chi T_0\left(\gamma,l\right)+\bar{\sigma}T_4^0\left(\gamma,l\right)\right\}\aleph, \\ E_{\text{nc -u}}\left(p=0,j,l,s,m\right) &= E\left(p{=}0\right)+\left|a_0\right|^2 k_+\left\{\Theta T_0\left(\gamma,l\right)+\bar{\theta}T_4^0\left(\gamma,l\right)\right\}+\left|a_0\right|^2 m\left\{\chi T_0\left(\gamma,l\right)+\bar{\sigma}T_4^0\left(\gamma,l\right)\right\}\aleph \end{aligned} \quad (45)$$

and

$$\begin{aligned} E_{\text{nc -d}}\left(p=1,j,l,s,m\right) &= E\left(p{=}1\right)+\left|a_0\right|^2 k_-\left\{\Theta T_0\left(\gamma,l\right)+\bar{\theta}T_4^0\left(\gamma,l\right)\right\}+\left|a_0\right|^2 m\left\{\chi T_0\left(\gamma,l\right)+\bar{\sigma}T_4^0\left(\gamma,l\right)\right\}\aleph, \\ E_{\text{nc -u}}\left(p=1,j,l,s,m\right) &= E\left(p{=}1\right)+\left|a_0\right|^2 k_+\left\{\Theta T_0\left(\gamma,l\right)+\bar{\theta}T_4^0\left(\gamma,l\right)\right\}+\left|a_0\right|^2 m\left\{\chi T_0\left(\gamma,l\right)+\bar{\sigma}T_4^0\left(\gamma,l\right)\right\}\aleph, \end{aligned} \quad (45)$$

where $E\left(p{=}0\right)$ and $E\left(p{=}1\right)$ are ordinary energy for ground state and first excited states in ordinary quantum mechanics which are determining numerically in the main reference [5]. On other hand, the total energy $E_{\text{nc-gec}}\left(p,j,l,s,m\right)$ is the sum of the principal part of energy $E\left(p\right)$ and the two corrections of energy ($E_{gec-d}\left(p,j=l-1/2,l,s\right)$, $E_{gec-u}\left(p,j=l+1/2,l,s\right)$), $E_{\text{z-gec}}\left(p,m\right)$ and $E_{\text{z-gec}}\left(p,m\right)$, this is one of the main motivations for the topic of this work. It is clear, that the obtained eigenvalues of energies are reals, which allow us to consider the NC diagonal Hamiltonian $\hat{H}_{nc-gec}$ as a Hermitian operator, $\left(\hat{H}_{nc-gec}=\left(\hat{H}_{nc-gec}\right)^+\right)$ and regarding the previous obtained results (eq. (21) and eq. (42)), the new Hamiltonian operator with NMGECSCP for studied diatomic molecules takes the form at first order in $\Theta$ and $\bar{\theta}$, as:

$$\begin{aligned} \hat{H}_{nc-gec} &= \left(-\frac{\Delta}{2}-\frac{Ze^2}{r}\left(1+br\right)e^{-r/\lambda}\cos\left(\frac{cr}{\lambda}\right)+eFr\right)+\alpha\left(\frac{1}{2r^3}-\frac{\alpha_1-2F}{2r}-\frac{\alpha_2}{2}+\frac{\bar{\theta}}{2}\right)\vec{\mathbf{L}}\vec{S}+ \\ &+\left(\chi\left(\frac{1}{2r^3}-\frac{\alpha_1-2F}{2r}-\frac{\alpha_2}{2}\right)+\frac{\bar{\sigma}}{2}\right)\left(\vec{\aleph}\vec{J}-\vec{S}\vec{\aleph}\right). \end{aligned} \quad (46)$$

This is the equation of one-electron atoms under the influence of NMGECSCP interactions. It should be pointed out that this treatment considers only first order terms in either $\Theta$ or $\bar{\theta}$. It's worth to note that the first part presents the Hamiltonian operator in the ordinary quantum mechanics for MGECSCP while the second and the third parts are respectively present the spin-orbital and new Zeeman Hamiltonians operators for NMGECSCP which are induced automatically by the NC properties of space and phase. Thus, the important result from this work is:

$$\begin{aligned} &\left\{H_{\text{gec}}\left(r\right)+H_{\text{so-gec}}\left(r,\theta,\bar{\theta}\right)+H_{\text{z-gec}}\left(r,\chi,\bar{\sigma}\right)\right\}\Psi_{plm}\left(r,\theta,\phi\right) \\ &=\begin{cases}\left\{E_1\left(p\right)+E_{\text{so-d}}\left(p,j,l,s\right)+E_{\text{z-gec}}\left(p,m\right)\right\}\Psi_{plm}\left(r,\theta,\phi\right) & for: j=l-1/2 \\ \left\{E_1\left(p\right)+E_{\text{so-u}}\left(p,j,l,s\right)+E_{\text{z-gec}}\left(p,m\right)\right\}\Psi_{plm}\left(r,\theta,\phi\right) & for: j=l+1/2\end{cases} \end{aligned} \quad (47)$$

## 5. CONCLUSION

In this paper, the energy levels $E_{\text{nc -d}}(p,j,l,s,m)$ and $E_{\text{nc -u}}(p,j,l,s,m)$ of one electron atoms have been examined analytically under 3D-NMGECSCP in the case of GQG of (NC: 3D-RSP) via the generalized Bopp's method and standard perturbation theory, we briefly summarize what has been achieved in this reach work and comment on the outlook on future work that can follow from this paper:

- We have reviewed the 3D nonrelativistic MGECSCP for one-electron atoms and the generalized Bopp's method.
- We have solved the 3D-DSE for its new bound states with 3D-NMGECSCP.
- We hope to get some interesting applications to this new potential in the study of different fields of matter sciences, because our results are not only interesting for the pure theoretical physicists but also for experimental physicists (various fields of physics such as plasma physics, nuclear physics, condensed matter physics, and atomic physics).
- Regarding, our obtained results for new Hamiltonian operator $\hat{H}_{nc-gec}$, which contain three important physical terms, and corresponding eigenvalues ($E_{\text{nc-d}}(p,j,l,s,m)$ and $E_{\text{nc -u}}(p,j,l,s,m)$), we can decelerate: the high precision measurements of eigenvalues in ordinary quantum mechanical systems may be able to reveal the noncommutativity of space and phase simultaneously.


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