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Analysis of the Methodology of Constructing a Production Function Using Quality Criteria

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Abstract. In the article to forecast the trends of development of the state's industry (for example, Ukraine), a method for evaluating the parameters included in the classical Cobb-Douglas formula is developed. On the basis of the computational experiment it was established that if the values of the production function Y are close to the numerical values and the deviation between them does not exceed 3.7 %, then for approximation of Y in the industry for small time periods, it makes no sense to complicate the set of its parameters and coefficients. For forecasting the values of Y_i ($Y_i \in Y$) we have evaluated the parameters that are included in the classical Cobb-Douglas formula. To improve the methodology for estimating the coefficients included in the Cobb-Douglas type formula, new variants of the multiplicative and additive quality criteria and for the Ukrainian industry are proposed, based on the analytical data for 2010–2017, the evaluation of the relevant criteria was carried out.

Keywords: production function, Cobb-Douglas function, quality criteria, industry, industrial products.

1 1 Introduction

Over the past decades, a special feature of the practice of analyzing manufacturing processes has been the active use of special methods of statistics modelling. An important role in this context belongs to production functions.

At the present stage of research [1, 2]:

1) the essence of the production function is that it enables to formulate concrete alternatives (variants) of the combination of factors of production to ensure a certain amount of production, that is, the possibility of one factor of production to replace others;

2) the production function is not only the prospect of one of the analytical methods for forecasting the development of the state's economy, but also an applied instrument used to assess and compare the effectiveness of national economies;

3) the macroeconomic production function has a wide range of applications , since its dynamic analysis allows to solve such key important tasks:

– study the dynamics of the efficiency of production factors (labor productivity, return on investments);

– identify factors of production growth;

– determine the contribution of each production factor to the overall increase in production.

For Ukraine in the conditions of European integration [3], where one of the pressing issues is today to identify the reserves of the growth of the national economy, the use of information about local rates of change in the production function can give impetus to the improvement of existing mechanisms for management and activation of internal factors of development.

2 Literature Review

In theoretical and applied analysis, the most widely used are the following 4 types of production functions:

1) linear [2, 4];

2) Cobb–Douglas function [2, 5–7];

3) function CES (with postmobile elastics) [2];

4) Leontief function [4].

Their advantages are small number of coefficients (parameters) [2], which greatly facilitates the statistical evaluation, as well as indicators of economic growth (efficiency, intensification), calculated on their basis, have a convenient analytical form.

Among the most famous functions is the classic production function of Cobb-Douglas, which has the form [5–7]:

$$Y = AL^{\alpha} K^{\beta}, \quad (1)$$

where A – technological coefficient (or coefficient characterizing production efficiency); L – work resources; K – volume of fixed investments; α, β – coefficients of elasticity for labor and investments, respectively.

Partial elasticity factor of the product on the funds:

$$E_K = \frac{\partial Y}{\partial K}\frac{K}{Y} = A\beta L^{\alpha} K^{\beta-1} \frac{K}{AL^{\alpha} K^{\beta}} = \beta. \quad (2)$$

Elasticity of the product by labor:

$$E_L = \frac{\partial Y}{\partial L}\frac{L}{Y} = A\alpha L^{\alpha-1} K^{\beta} \frac{L}{AL^{\alpha} K^{\beta}} = \alpha. \quad (3)$$

These coefficients of elasticity reflect the percentage of output growth while increasing resource costs by 1 %.

These coefficients of elasticity reflect the percentage of output growth while increasing resource costs by α, β are constant and independent of the factor K, L.

In practice, the use of production functions, verification the sum of the coefficients α and β on one equality is very important, since it determines the type of economic growth [2]:

1) $\alpha + \beta > 1$ (production function with increasing return on a scale) corresponds to intensive economic growth, and in the case of $\alpha > \beta$ there is an intensive economic growth of labor; at $\alpha < \beta$ intensive economic growth of funds;

2) $\alpha + \beta < 1$ (production function with decreasing returns to scale) means that output is growing slower than the growth of the factors K and L, that is, there is no economic growth (or other important factors remain outside consideration);

3) $\alpha + \beta = 1$ there is an extensive type of economic growth (a production function with constant returns on a scale).

Based on research results [8–11] and taking into account the information in the paper [12], one has to agree with the author's opinion [2], that among the important factors not taken into account in the production function of type (1), it is worth noting the elements of scientific and technological progress, in particular, the place and role of information technology. The impact of scientific and technological progress is manifested in the growth or aggregate efficiency of resources, or the effectiveness of an individual resource.

At the same time, the special importance of modern information technologies for economic growth is presented in the work of American economists S. D. Oliner and D. E Sichel [11], who carried out the evaluation of the parameters of the production function, in which the indicators of information technology were included as independent factors of production. The authors [11] built a model that assessed the impact on the economic growth of the following three factors:

1) investments in the software;
2) investments in communications;
3) other costs.

Here the labor quality index was included as labor costs. Quality was taken into account by means of indicators of changes in levels of education, qualifications and structure (by level of education and sex) employed – [12].

In the context of the analysis of various important factors of development, one should note the important feature of the apparatus of production functions, the use of which makes it possible to compare the trends of economic development of different countries [6, 13–15].

An analysis of the methodology for constructing a production function using quality criteria, which is the purpose of this article, can be considered as one of the steps for the active use of the production function in practice. At the same time, it will enable to activate promising directions of research of the Ukrainian economy.

The achievement of the above goal led to the following tasks:

1) to introduce the mathematical tools of the methodology of refining the production function using an expanded set of constant coefficients.

2) to propose the main elements of the methodology for building a production function, using quality criteria in practice.

3 Results and Discussion

3.1 The mathematical tools of the methodology for refining the production function using an expanded set of coefficients

In the paper [6] we propose a method for constructing a function Y with variable elasticity coefficients in the form:

$$Y(L,K,A,\alpha,\beta) = A \cdot L^{f(L,K,\varepsilon)} \cdot K^{g(L,K,\beta)}, \quad (4)$$

$$f(L,K,\alpha) = \sum_{i=0}^{M}\sum_{m=0}^{N} a_{im}\varphi_i(L)\varphi_m(K), \quad (5)$$

$$f(L,K,\beta) = \sum_{i=0}^{M}\sum_{m=0}^{N} b_{im}\varphi_i(L)\varphi_m(K), \quad (6)$$

where a_{im}, b_{im} – constant coefficients; $\varphi_i(L)$, $\varphi_m(K)$ – transcendental functions.

Ukraine is characterized by an unstable development of the economy. Output data for Ukrainian industry is given in Table 1. These data are generated by analogy in accordance with the data given in the papers [5, 6].

To data Table 1, we will apply the valuation technique and take into account the inflation index (Table 2).

Considering the relative index (Table 2), the obtained (calculated) refined data for the parameters Y, L, K and are shown in the Table 3.

Table 1 – Output (general) data on Ukrainian industry [16]

Year	Employed population L in industry, thousand people	Volume of sold industrial products (goods, services) Y, million UAH	Investments K in industry, million UAH
2010	3 462	1 043 111	55 384
2011	3 353	1 305 308	78 726
2012	3 237	1 367 926	91 598
2013	3 170	1 322 408	97 574
2014	2 898	1 428 839	86 242
2015	2 574	1 776 604	87 656
2016	2 495	2 158 030	117 754
2017	2 441	2 625 863	143 300

Table 2 – Inflation indicators in Ukraine [16]

Year	Inflation index, %	The value of the inflation index Inf relative to 2010 year
2010	109.1	1.000
2011	104.6	1.046
2012	99.8	1.044
2013	100.5	1.049
2014	124.9	1.310
2015	143.3	1.878
2016	112.4	2.111
2017	113.7	2.400

Table 3 – Data on Ukrainian industry considering the inflation index, million UAH

Year	Volume of sold industrial products (goods, services) Y	Investments K
2010	1 043 112	55 384
2011	1 248 000	75 263
2012	1 310 200	87 738
2013	1 260 600	93 016
2014	1 090 400	65 813
2015	946 000	46 683
2016	1 022 500	55 792
2017	1 094 100	59 708

For data Table 3 received a production function of a classical type in the form:

$$Y_0 = 0.298 L^{0.75} K^{0.25}. \qquad (7)$$

For function (7) according to Table 3, square deviation is established $\sigma_0 = 0.108$.

Using the approach of constructing the function of Y with variable coefficients, similar to that in [6], Y_1 was given a more complex form

$$Y_1 = A_1 L^{\alpha_1+\alpha_2 L+\alpha_3 K} K^{\beta_1+\beta_2 L+\beta_3 K}, \qquad (8)$$

for which the mean-square deviation $\sigma_1 = 0.093$, that is less comparing with σ_0 by 14 % ($\Delta\sigma/\sigma_i \sim 0.14$). Here A_1, α_1, α_2, α_3, β_1, β_2, β_3 – are constant coefficients for a given dataset.

To find the refined value σ_1, the classical econometric and mathematical method of exponential smoothing is used to clarify the errors of the results of econometric studies for the Ukrainian industry which do not exceed $\Delta_1 = 8$ %.

In the paper [6], the deviations of the σ_i, Δ_i for agricultural products in Ukraine do not exceed 37 % ($\Delta\sigma/\sigma_i \sim 0{,}37$).

The ratio (8) shows 7 coefficients A_1, α_1, α_2, α_3, β_1, β_2 and β_3, which can be considered conditionally constant for a given set of data, in particular for industry. If we choose data for a period of 7 years, then we will determine the coefficients of this type with a high accuracy that does not exceed the accuracy of the relevant statistics Δ_1. Relevant constraints will be derived from the principles of statistics and metrology for a particular category of research, in particular for the parameters Y, L, K, which characterize the Ukrainian industry.

Since the analysis of the effectiveness of using the production function with variable coefficients in the tasks of the study of manufacturing problems, there is a need in the first approximation to test the advanced method of constructing production functions [6].

We draw attention to a plurality of data in the Table 3. Data from Table 3 for Y_i (2010, 2014, 2016, 2017) are similar and their errors (deviations from the mean value) do not exceed $\Delta_2 = 3.7$ %, that is, significantly less error systems of the type Δ_1 ($\Delta_2 < \Delta_1$). In this case, the task of determining the coefficients A_1, α_1, α_2, α_3, β_1, β_2 and β_3 will be mathematically incorrect and as a result, as established on the basis of a computational experiment, these coefficients will be estimated with a large error of $\Delta_3 \sim 20$–35 %.

On the basis of the data analysis Table 3 observe the unstable development of Ukrainian industry. Therefore, there is no point in forecasting trends in the development of industry in Ukraine to apply the ratio (8). Here it is expedient to use the ratio of type (7) for certain small intervals of time, for example, for 2010–2012 and 2012–2015. For these small periods of time, it is advisable to estimate the rate of change in the coefficients type A, α, β, and information about them to use to predict trends in the volume of sales of industrial products (goods, services) Y.

3.2 Elements of the methodology of constructing a production function using quality criteria

Like in the works [17, 18] we use the product to evaluate the quality of products (industry) $k_P = \{k_1 \cdot k_2 \cdot k_3\}$, where k_1 – coefficient of commercial gain, k_2 – coefficient of product competitiveness level – [19], k_3 – coefficient of product reliability.

Similarly to [17, 18] and taking into account the information given in the Table 3 and [20–23], the multiplicative qualimetric quality criterion will be presented as:

$$Z_1 = \prod_{i=1}^{6} k_i \Rightarrow \max, \qquad (9)$$

where k_4 – coefficient of the level of quality (reliability) of the parameters of the type of production function Y_i; k_5 – coefficient of quality (reliability) of type parameters L, α_1, α_2, α_3; k_6 – coefficient of quality (reliability) of type parameters K, β_1, β_2, β_3.

Let's mention the quality criterion of Z_2 in the additive form in the same way as in scientific work [17]:

$$Z_2 = \sum_{i=1}^{6} a_i k_i, \qquad (10)$$

where a_j (j = 1, 2, …, 6) – weight coefficients.

In the first approximation we choose a_j = 1/6, a $k_j = 1/\Delta_i$. In the first approximation, as an example, let it be $k_1 = k_2 = k_3 = 1/\Delta_1 \approx 12.5$.

According to the Table 3, it is evaluated: k_4= 2.83, k_5 = 2.90, and k_6 = 5.13.

As a result, Z_1 = 214 402, Z_2 = 17.11. (11)

If the errors of the parameters corresponding to the methodology for assessing the quality criteria of the corresponding methodology can be reduced, then the integral parameters of type Z_1 and Z_2 will be increased.

4 Conclusions

According to the results of the research, a method for evaluating the parameters included in the classical Cobb-Douglas formula is developed. On the basis of the computational experiment it was established that if the values of the production function Y are close to the numerical values and the deviation between them does not exceed 3.7 %, then for approximation of Y in the industry for the period 2010–2017, it makes no sense to complicate its appearance. To predict Y_i values, we evaluate the parameters that are included in the classical Cobb-Douglas type formula. To improve the methodology for estimating the coefficients included in the Cobb-Douglas type formula, new variants of the multiplicative and additive quality criteria and for the Ukrainian industry are proposed, based on the analytical data for 2010–2017, the evaluation of the relevant criteria was carried out. The above example has an analytical orientation and illustrates the possibility of using a Cobb-Douglas formula for analyzing the coefficients of a given model with sufficient reliability. Research focused on controlling the development of industry in Ukraine by means of quality criteria.

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Аналіз методології побудови виробничої функції з використанням критеріїв якості

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Анотація. У статті на прикладі України розроблено методику оцінювання параметрів класичної формули типу Кобба–Дугласа для прогнозування тенденцій розвитку промисловості держави. На основі обчислювального експерименту встановлено, що для значень виробничої функції Y, близьких за числовими значеннями і відхиленнями до 3,7 %, апроксимування Y у сфері промисловості для невеликих часових періодів є неможливим з огляду на ускладнення множини її параметрів та коефіцієнтів. Для прогнозування значень Y_i ($Y_i \in Y$) проведено оцінювання параметрів, що входять до класичної формули типу Кобба–Дугласа. Для удосконалення методики оцінювання коефіцієнтів, що входять до цієї формули, запропоновано нові варіанти мультиплікативного та адитивного критеріїв якості. Виходячи із аналітичних даних за 2010–2017 рр., проведено оцінювання відповідних критеріїв для промисловості України.

Ключові слова: виробнича функція, функція Кобба–Дугласа, критерії якості, промисловість, промислова продукція.