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Interaction of Two-Dimensional Extremely Short Optical Pulses in a Zig-Zag Carbon Nanotubes in the Presence of a High-Frequency Electric Field

N.N. Konobeeva[1], D.S. Skvortsov[1], M.B. Belonenko[1,2]

[1] *Volgograd State University, 400062 Volgograd, Russia*
[2] *Volgograd Institute of Business, Volgograd, Russia*

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The paper addresses a two-dimensional problem of propagation and interaction of extremely short optical pulses in the array of carbon nanotubes in the presence of external high-frequency electric field. In particular, the effects governed by the external electric field are studied and the dependence of pulses propagation on the initial distance between the centers of the pulses is investigated.

Keywords: Extremely short optical pulse, High-frequency electric field, Carbon nanotubes.

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1. INTRODUCTION

Carbon nanotubes are novel graphene structures, which demonstrate unique properties and attract strong attention of different research groups. The discovery in 1991 by Dr. Sumio Iijima [1] caused great interest among researchers engaged in the creation of materials with unusual physicochemical properties. In particular, such nanotubes have high electron mobility, high strength, dependence of conductivity on nanotube geometry and different non-linear effects [2-6], which can have attractive practical applications.

Last years it was found that, in addition to the other non-linear effects, carbon nanotubes allow an existence of electromagnetic solitary waves, which propagate through the material. At the same time, there is a very important question related to the study of the propagation dynamics of multi-dimensional optical pulse localized in all coordinates, also called “light bullets” [7]. For example, in [8] one observed a stable extremely short optical pulses in Bragg medium with carbon nanotubes. The authors of Ref. [9] showed the possibility of a stable propagation of two-dimensional pulses without taking into account the external high-frequency electric field, which can have a significant effect on the extremely short pulses propagation in the system.

In the paper, we consider the properties of two 2D extremely short electromagnetic pulses and study a dependence of the pulses propagation through the array of carbon nanotubes on the parameters of the system. Duration of the considered pulses is about 10^{-12} sec and an amplitude of the external electromagnetic field is 10^6 V/m.

At the same time, the problem, which associated with the external magnetic field, re-mains unsolved.

2. BASIC EQUATIONS

Let us consider the propagation of two extremely short electromagnetic pulses in a medium of 2D zig-zag array of carbon nanotubes. The geometry of problem is shown in Fig. 1.

In the figure, the electric field is spread in the array of carbon nanotubes and external high-frequency electric field is governed by harmonic function: $E_{ex} = E_0 \sin w_0 t$.

The dispersion law for zig-zag carbon nanotubes is written as follows:

$$\varepsilon_s(p) = \pm\gamma\sqrt{1+4\cos(ap_z)\cos(\pi s/n)+4\cos^2(\pi s/n)}\ , \qquad (1)$$

where $\gamma \approx 2.7$ eV, $a = 3b/2\hbar$ with the distance between adjacent carbon atoms $b = 0.142$ nm, and (p_z,s) stands for a quasi-momentum with the momentum component along carbon nanotube axis p_z and $s = 0,..,n$. Different signs in the formula (1) specify valence and conductance zones, correspondently.

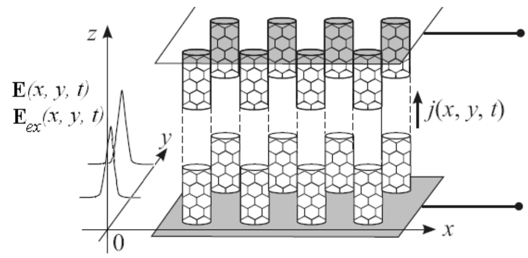

Fig. 1 – Geometry of the problem

From the geometry of the system and the dispersion law it follows that the Hamiltonian of electron system in the presence of an external alternating electric field (and $\vec{E} = -\partial\vec{A}/c\cdot\partial t$) has the following form:

$$H = \sum_{ps}\varepsilon_s\Big(p-\frac{e}{c}A(t)-\frac{eE_0\cos w_0 t}{w_0}\Big)a^+_{ps}a_{ps}\ , \qquad (2)$$

where a^+_{ps}, a_{ps} are operators of creation and annihilation of electrons, correspondently, $\vec{A} = (0,0,A(x,y,t))$ is a vector-potential, $\varepsilon_s(p)$ is the dispersion law of electrons (1), in which an electron interaction in one unit is taken into account, and E_0 is an amplitude of external electromagnetic field.

Maxwell equations in two-dimensional case are specified as follows:

$$\Delta A - \frac{1}{c^2}\frac{\partial^2 A}{\partial t^2} + \frac{4\pi}{c}j_0 = 0\ . \qquad (3)$$

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Notice that according to the considered problem, the diffraction spread of the laser beam in z direction is neglected.

Finally, a standard expression for the current density is following:

$$j_0 = \frac{eN}{\pi\hbar}\sum_{ps} \upsilon_s(p - \frac{e}{c}A(t) - \frac{eE_0 \cos w_0 t}{w_0})\left\langle a^+_{ps} a_{ps} \right\rangle, \quad (4)$$

where $\upsilon_s(p) = \partial\varepsilon_s(p)/\partial p$, N is the concentration of electrons in nanotubes ($N \sim 10^{12}$ sm^{-2}) [10], and the brackets $\langle\ \rangle$ stand for the average value with the non-equilibrium density matrix $\rho(t)$, i.e. $\langle B \rangle = Sp(B(0)\rho(t))$.

Let us decompose the carriers speed $\upsilon_s(p)$ into Fourier series. Then, since distribution function $\rho(0)$ is even, one obtains:

$$\upsilon_s(p - \frac{e}{c}A(t) - \frac{eE_0 \cos w_0 t}{w_0}) = -\sum_k A_{ks}\cos(kp)\sin(\frac{ke}{c}A(t) + \frac{keE_0 \cos w_0 t}{w_0}) =$$
$$= -\sum_k A_{ks}\cos(kp)\left\{\sin(\frac{ke}{c}A(t))\cos(\frac{keE_0 \cos w_0 t}{w_0}) + \cos(\frac{ke}{c}A(t))\sin(\frac{keE_0 \cos w_0 t}{w_0})\right\} \quad (5)$$

Consider the pulses by the use of effective medium approximation [11, 12]. Since spatial size of the pulse is essentially greater than the size of nanotubes and distances between them, we vary the electromagnetic field at the points such that the distances between the points have the same order as the size of nanotubes. Consequently, the rate of the electric current through the system is represented by a sum of the current rates over the nanotubes.

The considered extremely short pulses are characterized by a duration τ_{imp}; hence:

$$\frac{2\pi}{w_0} << \tau_{imp} << \tau_{rel}, \quad (6)$$

where τ_{rel} is a relaxation time of the electromagnetic field in the system of carbon nanotubes. In the case of extremely short pulses, the values, which appear in the Eq. (6), are the following [10]: pulse duration τ_{imp} is approximately equivalent to 10^{-12} sec, and the frequency of external field is $w_0 \approx 2\pi \cdot 10^{14} - 2\pi \cdot 10^{15}$ sec^{-1}.

According to the above-presented approximation, let us calculate an average speed (5) of the carries over the oscillation period of the electromagnetic field, and then substitute the result into the formula (4). Then, summation of the obtained value over s and p results in the following:

$$j_0 = -\frac{eN}{\pi\hbar}\sum_k R_k \sin(\frac{ke}{c}A(t))$$
$$R_k = C_k(E_0)\sum_{s=1}^{m}\int_{-\pi/a}^{\pi/a} dp A_{ks}\cos(kp)\frac{\exp(-\beta\varepsilon_s(p))}{1+\exp(-\beta\varepsilon_s(p))}$$
$$C_k(E_0) = \frac{w_0}{2\pi}\int_0^{2\pi/w_0} dt \cos(\frac{keE_0 \cos w_0 t}{w_0})$$

Finally, Maxwell equation (3) in the dimensionless form with respect to the obtained values is written as follows:

$$\frac{\partial^2 B}{\partial x'^2} + \frac{\partial^2 B}{\partial y'^2} - \frac{1}{c^2}\frac{\partial^2 B}{\partial t'^2} + sgn(R_1)sin(B) + \sum_{k=2}^{\infty} R_k sin(kB)/|R_1| = 0 \quad (7)$$
$$B = \frac{eaA}{c}; x = x'a; y = y'a; t = t'$$

Notice that in the sum, which appears in Eq. (7), coefficients R_k are decreasing while k is increasing. Thus, it enough to consider the first non-vanishing terms in the sum, and the equation obtains the form of double sine-Gordon equation [13]. In the theory of sine-Gordon equation, the possibility of the pulses amplification in presence of external alternating field has been demonstrated. In particular, the pulses with small amplitude are unstable and tend to increase their area up to π with the changing of $\mathrm{sgn}(R_1)$, while such a changing follows the theory of self-induced transparency applied to the inverted medium. Below, we consider the influence of the sign of R1 on the propagation and interaction of coupled 2D pulses.

3. NUMERICAL RESULTS

The equations were solved using the cross-type difference scheme [14]. The initial pro-file of each pulse had a Gaussian form.

Dynamics of two pulses and its dependence on the amplitude of external field is E_0 were studied in the case of two-dimensional array of zig-zag carbon nanotubes (11,0) without impurities. The initial size of both pulses was 0.3 × 1.3 μm.

The evolution of the waves intensity E^2 (x,y,t) is presented in Fig. 2.

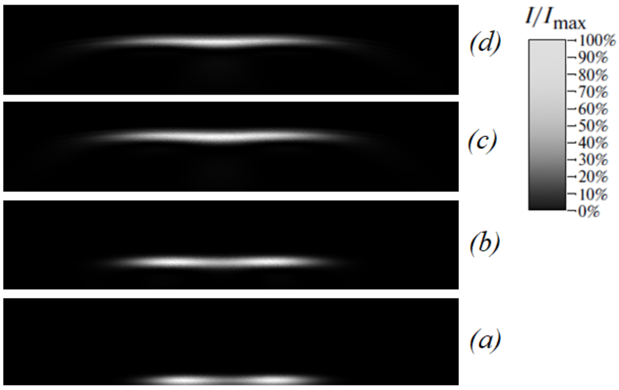

Fig. 2 – Intensity of two electromagnetic pulses at different time moments, while the amplitude of external field is $E_0 = 0.5 \cdot 10^7$ V/m: initial pulse form, where distance between the centers of pulses is $r = 1.008$ μm; (*a*) $t = 0.7 \cdot 10^{-13}$ s; (*b*) $t = 2.0 \cdot 10^{-13}$ s; (*c*) $2.7 \cdot 10^{-13}$ s; (*d*) $3.0 \cdot 10^{-13}$ s

From Fig. 2 it follows that the dynamics of a couple of pulses essentially differs from the dynamics of a single pulse. In the case of a single pulse spreading, the pulse splits into two pulses with substantially different amplitudes, however, in the case of two pulses, all energy is focused at the area of interaction between the pulses, and a diffraction spreading over a couple of extremely short optical pulses has not been observed. Two pulses are joining, and the maximal intensity is moving from the centers of the pulses to their common central area. A comparison of Fig. 2c and Fig. 2d shows that in the presence of an external high-frequency field, the couple of pulses reach its equilibrium, and then the pulses begin spreading with saving their form.

As indicated above, the resulting evolution of the electromagnetic pulses is determined by the sign of the following value:

$$R_1(E_0) = \frac{w_0}{2\pi}\int_0^{2\pi/w_0} dt\cos\left(\frac{eE_0\cos w_0 t}{w_0}\right)$$

A comparison of two cases depends on $R_1(E_0)$ is presented in Fig. 3.

From Fig. 3 it follows that in the case of $sgn(R_1(E_0)) < 0$ the curvature of the pulse is greater than the one in the case of $sgn(R_1(E_0)) > 0$ In addition, notice that the diffraction spreading at the edges of the pulse becomes stronger.

The nature of the interaction between two extremely short optical pulses depends on the initial distance r between the centers of the pulses. In contrast to the previous figures, where $r = 1.008$ μm was applied, evolution of the waves intensity with $r = 1.890$ μm is shown in Fig. 4.

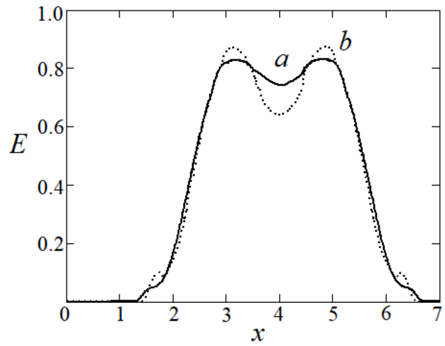

Fig. 3 – Cross-section of electric field of two electromagnetic pulses at the time moment $t = 3.0\cdot10^{-13}$ s: (*a*) $E_0 = 0.5\cdot10^7$ V/m ($sgn(R_1(E_0)) > 0$); (*b*) $E_0 = 2.5\cdot10^7$ V/m ($sgn(R_1(E_0)) < 0$). 1 r.u. along x-axis corresponds to 370 nm, 1 r.u. along E-axis corresponds to 10^7 V/m

Note, that for Fig. 4A the curve (b) is shifted to the left for clarity. The figure shows that one pulse with smaller amplitude spreads after the main pulses. The diffraction increases at the boundary of the interaction pulse and leads to the pulse profile distortion. However, because of the pulse merger at the next time moments, diffraction weakens. In the case of $sgn(R_1(E_0)) < 0$ (see Fig. 4B), diffraction spreading is more pronounced as for the main pulses, and at their boundary interactions.

In general, the observed propagation of 2D pulses is steady, and their curvature can be controlled by varying the amplitude E_0 of the external electromagnetic field. By such a control, the resulting diffraction is either compensated or amplified by effective nonlinearity, which depends on the sign of R_1, and so – on the amplitude of the field E_0.

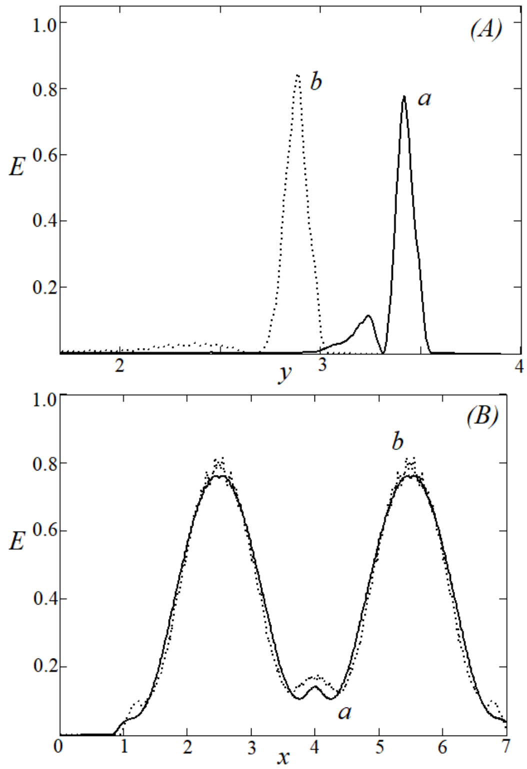

Fig.4 – Electric field of two electromagnetic pulses at the time moment: $t = 3.0\cdot10^{-13}$ s: (A) longitudinal section ($E_0 = 0.5\cdot10^7$ V/m): (a) $r = 1.008$ μm; (b) $r = 1.890$ μm; (B) cross-section ($r = 1.890$ μm): (a) $E_0 = 0.5\cdot10^7$ V/m): (b) $E_0 = 2.5\cdot10^7$ V/m. 1 r.u. along x-axis corresponds to 370 nm, 1 r.u. along y-axis – 300 nm, 1 r.u. along E-axis – 10^7 V/m

4. CONCLUSIONS

In the paper, we studied the propagation of 2D electromagnetic extremely short pulses through the zig-zag array of carbon nanotubes in presence of external electromagnetic field. We obtained the appropriate equations, which govern the pulses dynamics, and conducted numerical experiments, which demonstrated the evolution the system.

As a result, the following two main observations regarding the considered pulses ("light bullets") were obtained:

1. While initial distance between the centers is small, the pulses lead to join, so that the pulse with smaller amplitude is not observed. However, while this distance is relatively large, each "light bullet" is divided into two "bullets" with different amplitudes, along with large diffraction spreading.
2. The propagation of 2D electromagnetic pulse in the carbon nanotubes is stable, the pulse is localized in both two coordinates.

The obtained properties of 2D electromagnetic pulses in carbon nanotubes can be useful for fabrication graphene structures and devices for spot transmission and amplification of extremely short optical pulses.

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