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Modeling of Photon Crystals of Microwave Range Using Interference Matrixes

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The ability to control the properties of photonic crystals by changing the parameters of the layers allows to create unique optoelectronic devices. Properties of such environments are due to the formation of permitted and forbidden areas for electromagnetic radiation. The behavior of forbidden and permitted zones (high reflection and high-transmission areas) is well described by the theory of multilayer coatings. Photonic crystals may be considered using multilayer interference structures. Interference systems consisting of alternating films of the required optical thickness with high and low refractive indexes allow reducing the reflection of light in a narrow or wide spectral region, to increase reflection of incident light at different sections of spectral width, to separate narrow spectral region of monochromatic light. Theoretical studies can be carried out using both matrix methods and analytical formulas developed for multilayer. The simulation of a heterogeneous layer is carried out by replacing the smooth distribution of the refractive index with a stepped profile. Each layer is described using matrices of interference. The developed calculation programs using the matrix method make it possible to obtain the given optical characteristics (reflection, transmission, etc.) for any multilayer coatings. We simulated an interference mirror with quarter-wave optical thicknesses of alternating layers and normal incidence of light. The graphs presented show a remarkable coincidence of the results of optical characteristics of multilayer coatings obtained using the matrix method with experimental and numerical FDTD (Finite-Difference Time-Domain) method. A multilayer system with a defective layer has a bandwidth in the band gap; it is a conventional interference filter. Thus, this confirms that photonic crystals can be modeled using multilayer interference structures and calculations using matrix methods.

Keywords: Photonic crystals, Interference of light, Multi-layered system, Matrix method.

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1. INTRODUCTION

Photonic crystals are known to be artificially created multilayer structures in which geometric sizes and dielectric constant vary with a period that can be compared with the wavelength of electromagnetic radiation.

Changes in the layer parameters allow us to control the properties of photonic crystals. Properties of such environments are due to the formation of permitted and forbidden areas for electromagnetic radiation.

Disturbance of periodicity of photonic crystals leads to the appearance of "defective modes". This mode represents a wave, whose electromagnetic field is localized near the defective layer.

The results of theoretical studies of defective modes known today in photonic crystals do not give a complete description of this effect.

However, the behavior of forbidden and permitted zones (high reflection and high-transmission areas) is well described by the theory of multilayer coatings. Interference systems consisting of alternating films of the required optical thickness with high and low refractive indexes allow to reduce the reflection of light in a narrow or wide spectral region (illuminating coatings), to increase reflection of incident light at different sections of spectral width (a mirror), to separate narrow spectral region of monochromatic light (interference light filters).

Modern calculations of multilayer interference systems are based on matrix methods [1]. The simulation of a heterogeneous layer is carried out by replacing the smooth distribution of the refractive index with a stepped profile. Each layer is described using matrices of interference. The reflection and transmission coefficients for direct and reverse transmissions give the possibility to view the plane boundary between the two media as a "black box", the input of which acts incident and reflected waves, and at the exit – the broken wave.

A flat monochromatic wave in a layered structure is characterized by a unitary 2×2 matrix m (the so-called characteristic or matrix of interference).

For a single layer of thickness t_j with refractive index n_j, the matrix has the form:

$$m_j = \begin{pmatrix} \cos(\beta_j) & \dfrac{i}{u_j}\sin(\beta_j) \\ iu_j \sin(\beta_j) & \cos(\beta_j) \end{pmatrix} \qquad (1)$$

where $\beta_j = 0.5\pi \nu g_j c_j$ is the phase layer thickness, $g_j = 4n_j t_j/\lambda_0$ is the optical layer thickness expressed in units of quarter wavelength λ_0, $c_j = \cos(\theta_j)$.

The refraction angles θ_j are interconnected with each other and with the angle of incidence by Snellius law $n_j \sin(\theta_j) = \text{const}$ in all media.

The effective refractive indices u_j are equal to $n_j c_j$ for S- and n_j/c_j for P-component of the incident light polarization. Frequency $\nu = \lambda_0/\lambda$, where λ is the current wavelength.

The characteristic matrix in the case of a multilayer coating consisting of K thin homogeneous layers will have a form:

$$A_{i,k} = \prod_{j=1}^{K} m_j \ . \qquad (2)$$

We can calculate the reflection and transmission coefficients using the following matrices:

$$r = \frac{\left(A_{11}u_0 - A_{22}u\right) + i\left(u_0 u A_{12} - A_{21}\right)}{\left(A_{11}u_0 + A_{22}u\right) + i\left(u_0 u A_{12} + A_{21}\right)}, \qquad (3)$$

$$t = \frac{2u_0}{\left(A_{11}u_0 + A_{22}u\right) + i\left(u_0 u A_{12} + A_{21}\right)} \qquad (4)$$

and the corresponding energy coefficients:

$$R = |r|^2, \qquad (5)$$

$$T = \frac{u}{u_0} \,|\, t \,|^2 \qquad (6)$$

The optical characteristics of multilayer coatings were calculated in [2] with the help of the developed programs using matrix methods.

The authors of work [3] conducted experimental studies in the microwave range 22-40 GHz. The experimental installation had the form of two horn antennas, which act as the emitter and receiver of electromagnetic waves with a wave front, close to the plane. The research structure – anisotropic wire metamaterial (AWM) – was located between these antennas. The structure was made of 8 rows of wires and 7 intervals between them with period *b*. That is, from a two-dimensional (2D) periodic array.

The elementary cell of such array is a rectangle with the dimensions of the sides $a \neq b$. The geometric parameters of the elementary cell were calculated in such way that the wavelength was about the period of the structure. In this case, the frequency transmission spectrum takes the band character inherent to the photonic crystal (FC).

It was observed the first forbidden zone FZ-1, which was limited by the allowed zones AZ on the frequency spectrum of the passage of the investigated AWM with the properties of the FC. FZ-1 was limited by the frequencies 24-34 GHz.

The presence of a "defective" layer (which differs from the properties of layers of metamaterial by changing the thickness *b*) led to the formation of the defective modes. The transmission peak was formed in forbidden zone when such defect was introduced into the AWM. This peak shifted in the considered frequency range within the FZ-1 and varied in amplitude depending on the size of the defect.

The transmission peak shifted to the low-frequency region of the spectrum with the rising of the defective layer thickness.

The authors of [3] constructed a set of transmission spectra for the investigated wire metamaterial in order to conduct a comparative analysis with the help of numerical simulation. The theoretical calculation is performed using the numerical FDTD method (Finite-Difference Time-Domain, a method of finite difference in the time domain) implemented in the CST STUDIO SUITE – Student Edition.

Typical graphs of the frequency dependence of the transmission coefficients for the AWM case without spatial defects and for the case of AWM with a defective layer are characterized by the presence of a transmission peak in the forbidden band. The experimental results and those obtained using the theoretical models are superimposed on each other, demonstrating a good qualitative coincidence.

2. THE RESULTS OF MODELING

The purpose of this work was to simulate transmission and reflection spectra for photonic crystals in the microwave range with the help of multilayer interference structures and to conduct the theoretical studies using matrix methods.

The grid of the wires was modeled as a uniform layer having an optical thickness $\lambda/4$ and a refractive index n_H. There were 8 layers of wire, 7 layers of air between them with refractive index n_L, 15 layers together.

It is known that the width of the high reflection region depends on the magnitude of the refractive indices n_H and n_L and is determined by the ratio:

$$\Delta\nu = \frac{4}{\pi} \boldsymbol{arcsin}\left(\frac{n_H - n_L}{n_H + n_L}\right), \qquad (7)$$

where $\Delta\nu = \lambda_0/\lambda_2 - \lambda_0/\lambda_1$.

The refractive index n_H was selected in a such a way to obtain a band gap width of 24 to 34 GHz as in [3]. It was approximately $n_H = 1.64$.

The magnitude of the coefficient of reflection of a multilayered structure at the maximum is determined by the number of layers at given values of the refractive indices n_H and n_L: the reflection coefficient increases with an increase of the number of the layers. The coefficient of reflection of a multilayer structure in the spectral region $\Delta(\nu)$ is practically constant and is equal to the reflection coefficient for $\nu = \lambda_0/\lambda = 1$. The coefficient of reflection of a multilayered structure with an odd number of layers for $\nu = 1$ can be calculated as

$$R_{2K+1} = \left[\frac{n_0 - \dfrac{n_H^2}{n}\left(\dfrac{n_H}{n_L}\right)^{2K}}{n_0 + \dfrac{n_H^2}{n}\left(\dfrac{n_H}{n_L}\right)^{2K}}\right]^2. \qquad (8)$$

Consider the results of our calculations. Fig. 1 shows the frequency dependences of the transmission and reflection spectra of an interference system consisting of 15 layers, $n_H = 1.64$, $n_L = 1$ for media $N_0 = 1$, $N_s = 1$ without a defective layer. This is a mirror within the 24-34 GHz frequency range. Absorption taken leads to a slight decrease in the transmission coefficient outside this frequency range (Fig. 2).

The mirror is transformed into a light filter at a frequency of about 28 GHz when the interference system (photonic crystal) has a defective layer whose optical thickness is equal to $\lambda/2$. The transmission and reflection spectra of a multilayered interference system $n_H = 1.64$, $n_L = 1$ for media $N_0 = 1$, $N_s = 1$ with a defective layer of optical thickness $\lambda/2$ are shown in Fig. 3. It can be seen that the transmission coefficient on the resonant frequency is equal to 1.

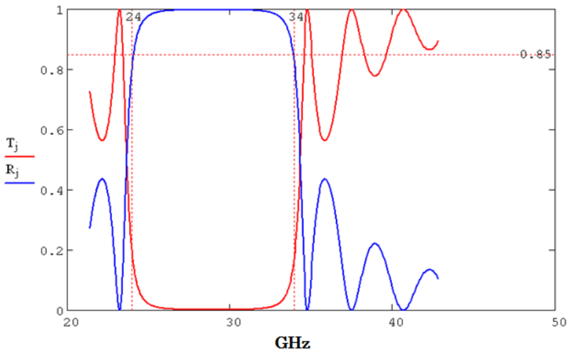

Fig. 1 – The transmission (red) and reflection (blue) spectra of the multilayered interference system $n_H = 1.64$, $n_L = 1$ for the media $N_0 = 1$, $N_s = 1$ without a defective layer

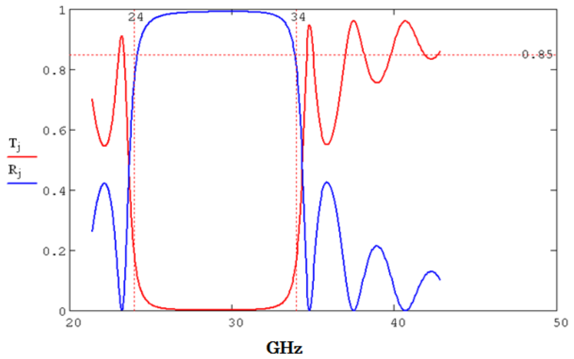

Fig. 2 – Transmission (red) and reflection (blue) spectra of a multilayer interference system $n_H = 1.64$, $n_L = 1$ for media $N_0 = 1$, $N_s = 1$ without a defective layer, taking into account absorption $K_1 = 0.0018$ layers H

The mirror is transformed into a light filter at a frequency of about 28 GHz when the interference system (photonic crystal) has a defective layer whose optical thickness is equal to $\lambda/2$. The transmission and reflection spectra of a multilayered interference system $n_H = 1.64$, $n_L = 1$ for media $N_0 = 1$, $N_s = 1$ with a defective layer of optical thickness $\lambda/2$ are shown in Fig. 3. It can be seen that the transmission coefficient on the resonant frequency is equal to 1.

The transmission spectrum also has a maximum in the band gap at the same frequency of 28 GHz if compared with the studies and calculations of the 2D periodic array with a defective layer carried out in [3]. The theoretical maximum transmittance of less than 1 indicates the presence of losses of a multilayer system.

We have adopted the absorption $K_1 = 0.0018$ for the H layers to take into account the energy losses of electromagnetic radiation during calculations using matrix methods. The results are shown on the graph (Fig. 4, red) for an interference system with a defective layer of optical thickness $\lambda/2$. Taking into account the absorption allows to obtain a transmission coefficient 0.85, almost as in the work [3].

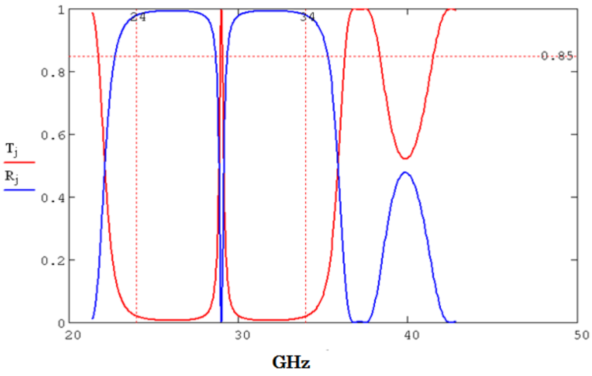

Fig. 3 – Transmission and reflection spectra of a multilayered interference system $n_H = 1.64$, $n_L = 1$ for media $N_0 = 1$, $N_s = 1$ with a defective layer of optical thickness $\lambda/2$

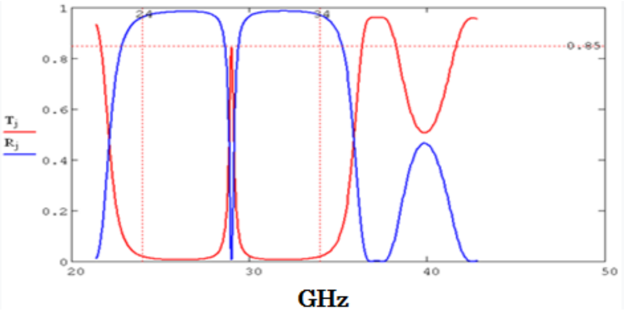

Fig. 3 – Transmission and reflection spectra of a multilayered interference system $n_H = 1.64$, $n_L = 1$ for media $N_0 = 1$, $N_s = 1$ with a defective layer of optical thickness $\lambda/2$ taking into account absorption $K_1 = 0.0018$ layers H

The presence of weak absorption has the greatest influence in the region of the resonance frequency.

The graphs presented show a remarkable coincidence of the results of optical characteristics of multilayer coatings obtained using the matrix method with experimental and numerical FDTD (Finite-Difference Time-Domain) method realized in the paper [3]. A multilayer system with a defective layer has a bandwidth in the band gap, that is, a conventional interference filter. Thus, this confirms that photonic crystals can be modeled using multilayer interference structures and calculated using matrix methods.

3. CONCLUSIONS

The obtained results are in good agreement with the experimental and theoretical ones presented in [3]. A defective layer system is a common interference filter. Photonic crystals can be modeled using multilayer interference structures. We can conduct theoretical studies using both matrix methods and analytical formulas developed for multilayers. The results of modeling of photon crystals of microwave range using interference matrixes are useful for the development of various telecommunication devices.

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Моделювання фотонних кристалів мікрохвильового діапазону з використанням матриць інтерференції

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Можливість управління властивостями фотонних кристалів шляхом зміни параметрів шарів дозволяє створювати унікальні оптоелектронні пристрої. Властивості таких середовищ зумовлені утворенням дозволених та заборонених ділянок для електромагнітного випромінювання. Поведінка заборонених та дозволених зон (зони з високим відбиттям та високою прохідністю) добре описана теорією багатошарових покриттів. Фотонні кристали можна моделювати за допомогою багатошарових інтерференційних структур. Інтерференційні системи, що складаються із змінних плівок необхідної оптичної товщини з високими та низькими показниками заломлення, дозволяють зменшити відбиття світла у вузькій чи широкій спектральній області, збільшити відбиття падаючого світла на різних ділянках спектральної ширини, відокремити вузьку спектральну область монохроматичного світла. Теоретичні дослідження можна проводити, використовуючи як матричні методи, так і аналітичні формули, які розроблені для багатошарових структур. Моделювання гетерогенного шару здійснюється шляхом заміни плавного розподілу показника заломлення на ступінчастий профіль. Кожен шар описується за допомогою матриць інтерференції. Розроблені розрахункові програми з використанням матричного методу дозволяють отримати задані оптичні характеристики (відбиття, пропускання) для будь-яких багатошарових покриттів. Ми моделювали інтерференційне дзеркало з четверть хвильовими оптичними товщинами шарів, що чергуються, та нормальним падінням світла. Наведені графіки демонструють неабиякий збіг результатів оптичних характеристик багатошарових покриттів, отриманих за допомогою матричного методу з експериментальним та числовим FDTD (Finite-Difference Time-Domain) методом. Багатошарова система з дефектним шаром має пік пропускання у забороненій зоні, тобто це звичайний інтерференційний фільтр. Таким чином фотонні кристали можна моделювати за допомогою багатошарових інтерференційних структур та обчислювати за допомогою матричних методів.

Ключові слова: Фотонні кристали, Інтерференція світла, Багатошарова система, Матричний метод.