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Ensuring the Vibration Reliability of Rotors Connected by Spline Joints

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Abstract. This article is devoted to the development of refined numerical mathematical models of rotor dynamics of high-performance turbomachines having a spline connection. These models consider the dependence of the critical frequencies of the shaft on the angular stiffness of the spline connection, as well as the procedure of virtual balancing. As a result of the complex application of this approach, the methods of calculation of vibration characteristics taking into ac-count variable values of angular rigidity of splined connection are offered. In addition, the method of evaluating the system of initial imbalances with the corresponding displacements of the rotor axis in the correction and calculation sections has also been improved. The proposed approaches, based on the integrated application of CAE software and computational intelligent systems, allow for modal and harmonic analysis and implement virtual balancing with a significant reduction in preparation and machine time without loss of relative accuracy. In addition, the developed mathematical model of free and forced vibrations of rotor systems have been implemented in the program code operational files "Critical Frequencies of the Rotor" and "Forced Oscillations of the Rotor" of the computer algebra system MathCAD that allows improving the dynamic balancing procedure for evaluating primary imbalances. The high accuracy of the proposed approach is confirmed by checking the dynamic deviations of the rotor axis by the system of residual imbalances in accordance with the standards of vibration stability.

Keywords: turbomachine, spline connection, angular stiffness, virtual balancing, modal analysis, harmonic analysis.

1 Introduction

Nowadays, due to the increasing demand for the use of such power equipment as rotary machines, in particular, turbo-pump turbochargers and units, the question of their vibration reliability become more and more urgent. The main source of vibration of any pump unit is an unbalanced rotor. But in addition to an unbalanced rotor, there are also many other factors that create in their totality a complex effect on the vibrational state of the unit. This can be the body stiffness, stiffness of the bearing seals and various types, the compression ratio of the axial rotor, the type and technical condition of the connection of the rotors of the unit and the engine, and others. One type of shaft connection with the engine is a connection that wears out with the operation process and, as a consequence, loses angular stiffness.

The article is devoted to the study of the influence on the vibration state of turbopump units with angular stiffness of splined joints based on the finite element method with the use authors' files "Critical frequencies of the rotor" and "Forced oscillations of the rotor" of the computer algebra system Mathcad.

2 Literature Review

The problem posed above can be solved after the study of special scientific literature on the topic of extreme research in computer modeling of rotor dynamics. For example, in work [1] dynamics of rotor systems of turbopump units with the present splined connection, and also the calculation of vibrational characteristics of a rotor taking into account its preliminary axial preload is considered. The research work [2] is devoted to methods of research of dynamics of a rotor on ball bearings with the combined application of spatial and beam settlement models. In the research work [3], the influence of different classes of bearings on vibrations is considered.

In research papers [4–7], examples of the introduction of neural network technology in computer calculation in solving problems of increasing vibration reliability are described for various rotary machines. Methods of nonlinear identification of stiffness characteristics of bearing supports were also developed in the works [8, 9, 10]. Upto-date trends in the field of rotor dynamics analysis are discovered in the research papers [11, 12, 13].

3 Research Methodology

Evaluation of eigenfrequencies 3.1

The object of the study was the shaft line of a liquidpropellant engine, which consists of an oxidizer rotor and a fuel pump rotor connected to each other by means of a splined connection. During the operation of the unit, the phenomenon of wear of the slots is inevitable, as a result of which the angular stiffness of this connection changes, which in turn leads to corresponding changes in the natural frequency spectrum.

The computer program "Critical Frequencies of the Rotor", which is based on the finite element method, allows determining the natural frequencies of the shaft, the scheme of which is presented in Figure 1.

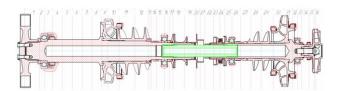


Figure 1 – The design scheme of the rotor

To further study the dynamics of the rotor, it is necessary to create a computational model. In the program "Critical frequencies of the rotor" was created beam finite element model (Figure 2).

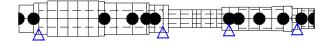


Figure 2 – The finite element model of the rotor dynamics with spline joint using the file "Critical Frequencies of the Rotor"

In this paper, the critical frequencies of the shaft line are for two variants: "Hinge" and "Rigid connection". For the first option, the values of the stiffness of the bearings were taken the minimum, wherein the spline connection for the shafts was taken hinged. For the second option, the maximum stiffness for the bearings was adopted, and the splined connection was taken according to the scheme of rigid fastening. The connection of both parts of the rotors in the shaft through the slot forms a weak dynamic connection between them.

3.2 Simulation of forced oscillations

The program "Forced oscillations of the rotor", allows one to calculate the forced oscillations of the rotor at a given speed of rotation under the influence of certain imbalances. The result of the calculation is the amplitude and shape of the rotor deflection when operating at a certain operating frequency.

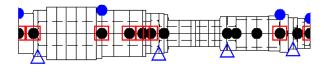


Figure 3 – The finite element model of the rotor dynamics with a splined joint for calculation of forced oscillations using the file "Forced Oscillations of the Rotor"

The calculation of forced oscillations in the program is performed using the already known beam finite element model and the same mathematical algorithms that are laid down to solve the problems of finding eigenfrequencies and critical frequencies, but the right side of the matrix equation when-van rotating rotor, divided into finite elements of the beam type-other than zero:

$$\bar{M}\ddot{U} + \bar{K}\bar{U} = \bar{E}\bar{M}_d\omega^2,\tag{1}$$

where \bar{E} – column-vector of the eccentricity of the unbalanced local masses; M_d – column-vector of the unbalanced local masses; ω – operating frequency.

To solve the problem of forced oscillations requires data such as density and modulus of elasticity of the material, the length of each section, external and internal diameter sections, mass details, support stiffness, the working speed of the rotor, the modulus and phase of point imbalances.

Theoretically, the unbalance of a flexible rotor is characterized by a certain spatial continuous curve, which is a hodograph of a continuous set of imbalance vectors normal to the rotor axis. Therefore, since this method of setting is quite time-consuming, then in further calculations it is sufficient to limit the task to a finite number of point imbalances located in the planes of the impellers and in the plane of the wheel since these elements are the most influential on the overall imbalance.

All the calculations and mode shapes of forced oscillations were performed using the MathCAD software.

3.3 The performance of the virtual rotor balancing

Due to the above-mentioned results, the first critical frequency of the oxidizer's rotor is close to the first critical frequency of the turbopump unit and not significantly increased the maximum rotor speed. According to the practice of designing and exploitation of pumping units, this relatively small deviation from the resonance (about 10 %) determines the dynamic characteristics of a rotor as a flexible one. In this case, the rotary system needs balancing both at operating and critical frequencies. Moreover, the total number of correction planes should be not less than three.

A system of imbalances $\overline{D}_{i\delta}$ $(i=\overline{1,l})$ has been introduced to balancing the rotor at all operating frequencies. Then the deflection $\overline{Y}_k^{(v)}$ at each measurement point at

any operating frequency ω_v ($v = \overline{1,n}$) must be equal to zero. Therefore, for each measurement frequency $\omega_1, \omega_2, \ldots, \omega_n$ can be written:

$$\begin{cases} \overline{Y}_{1}^{(v)} = \overline{Y}_{10}^{(v)} + \overline{W}_{11}^{(v)} \overline{D}_{1\delta} + \overline{W}_{12}^{(v)} \overline{D}_{2\delta} + ... + \overline{W}_{1l} \overline{D}_{l\delta} = 0 \\ \overline{Y}_{2}^{(v)} = \overline{Y}_{20}^{(v)} + \overline{W}_{21}^{(v)} \overline{D}_{1\delta} + \overline{W}_{22}^{(v)} \overline{D}_{2\delta} + ... + \overline{W}_{2l} \overline{D}_{l\delta} = 0 \end{cases} (2)$$

$$\vdots$$

$$\overline{Y}_{n}^{(v)} = \overline{Y}_{kn}^{(v)} + \overline{W}_{k1}^{(v)} \overline{D}_{1\delta} + \overline{W}_{k2}^{(v)} \overline{D}_{2\delta} + ... + \overline{W}_{kl} \overline{D}_{l\delta} = 0$$

As a result, the system $n \times k \times v$ a linear equation with complex coefficients is achieved to determine l complex balancing weights $\overline{D}_{1\delta}, \dots, \overline{D}_{l\delta}$. Dynamic complex weights are defined by the following equation:

$$\overline{W}_{ai}^{(v)} = \frac{\overline{Y}_{ai}^{(v)} - \overline{Y}_{a0}^{(v)}}{D_{t}},$$
 (3)

where a – number of points (a = 1, 2, ..., k); i – initial number of measured points (i = 1, 2, ..., l); l – number of correction planes; $\overline{Y}_{a,i}^{(\mu)}$ – deflection at the measurement point a when setting the test imbalance in the correction plane i at the rotor speed μ ; D_t – trial imbalance. The system of equations (3) can be rewritten in a matrix form:

$$\bar{\bar{Y}} = \bar{\bar{W}}\bar{\bar{D}},\tag{4}$$

where $\overline{\overline{Y}} = (-\overline{Y_{10}^{(v)}}, \dots, -\overline{Y_{k0}^{(v)}})^T$, (v = 1..n) — the column vector of complex amplitudes, measured at the "zero" start; $\overline{\overline{D}} = (\overline{\overline{D_{1b}}}, \dots, \overline{\overline{D_{lb}}})^T$ — a column vector of the estimated complex balancing of masses; $\overline{\overline{W}}$ — a rectangular matrix $k \times l$ of complex mass coefficients $\overline{W}_{ai}(a = \overline{1,k}, i = \overline{1,l})$.

The linear regression formula cannot be used directly to solve the matrix equation (4). It is necessary to form relations between the two vectors with the corresponding real and imaginary components. Vector $\overline{\overline{Y}}$ and k of complex amplitudes correspond to the vector 2k real and imaginary components \overline{Y} :

$$\overline{Y} = (Y_1^{(r)}, Y_1^{(i)}, Y_2^{(r)}, Y_2^{(i)}, \dots, Y_k^{(r)}, Y_k^{(i)})^T.$$
 (5)

Accordingly, an elongated vector with 2l real and imaginary imbalances can be introduced:

$$\overline{D} = (D_{1\mathcal{E}}^{(r)}, D_{1\mathcal{E}}^{(i)}, D_{2\mathcal{E}}^{(r)}, D_{2\mathcal{E}}^{(i)}, \dots, D_{l\mathcal{E}}^{(r)}, D_{l\mathcal{E}}^{(i)})^T \quad (6)$$

So, the matrix of mass coefficients \overline{W} $2k \times 2l$ can be presented in the following form:

where the arbitrary coefficient of the matrix W is defined by the formula:

$$\overline{W}_{ai} = W_{ai}^{(r)} + iW_{ai}^{(i)} \left(a = \overline{1, k}; i = \overline{1, l}\right).$$
 (8)

The complex matrix equation (4) corresponds to the real matrix equation:

$$\bar{\bar{Y}} = \bar{W}\bar{\bar{D}} \tag{9}$$

where the columns of vectors Y and D are determined by equations (5), (6). The linear regression formula can be implemented in the resulting equation (8). In this case, the system of imbalances D is estimated by the vector of measured displacements \overline{Y}^* by the following formula:

$$\widehat{\overline{D}} = \left[\overline{W}^T \overline{W} \right]^{-1} \overline{W}^T \overline{Y}^* \tag{10}$$

If the balancing procedure is provided by the measure at multiple frequencies $\omega_v(v=\overline{1,n})$, the vector $\overline{\overline{Y}}$ has size $k \times n$:

$$\overline{\overline{Y}} = (\overline{Y_1}^{(1)}, \dots, \overline{Y_k}^{(1)}; \dots; \overline{Y_1}^{(n)}, \dots, \overline{Y_k}^{(n)})^T, \quad (11)$$

and the elongated vector Y has size $2 \times k \times n$

$$\overline{Y} = (Y_{11}^{(r)}, Y_{11}^{(j)}, \dots, Y_{k1}^{(r)}, Y_{k1}^{(j)}; \dots; Y_{1n}^{(r)}, Y_{1n}^{(j)}, \dots, Y_{kn}^{(r)}, Y_{kn}^{(j)})^T$$
(12)

The matrix W will be expanded n times vertically and have size $2k \times n \times l$.

Each matrix $\overline{W}_1(v = \overline{1, n};)$ is similar to the matrix \overline{W} . To assess the system of imbalances

$$\widehat{\overline{D}}_{i} = D_{i}^{(r)} + iD_{i}^{(i)} \tag{13}$$

the balancing process is implemented by establishing a mass balancing system $D_{\delta i} = |\overline{D}_i|$ in the planes of correction under the phase angles:

$$\varphi_{\delta i} = \arg(\overline{D_i}), (i = \overline{1, l})$$
 (14)

On the basis of a series of virtual experiments, it is shown that the turbopump rotor (which initially has a system of imbalances) gives an unsatisfactory level of vibration at the rotor operating speed and cannot be balanced at low rotor speed in two correction planes. Therefore, it is necessary to balance the rotor as flexible at operating frequencies.

3.4 Assessment of the quality of rotor balancing

Residual imbalances are estimated by the algorithms described above, namely by the formulas:

The equation of the dependence of the deflection amplitudes on the imbalance is as follows:

$$\overline{Y_0} = \overline{WD} , \qquad (15)$$

where $\overline{Y_0}$ – a column vector of the deflection amplitudes; \overline{W} – matrix of influence coefficients; \overline{D} – vector-column of estimated imbalances.

If we introduce the concept of the i-th vector of trial imbalances, as a certain set of imbalances consisting of a trial imbalance D set in the i-th plane of correction, then or the i-th start of the rotor (i = 1, 2, ..., 8) you can write the equation:

$$\overline{Y_i} = \overline{W} \left(\overline{D} + \overline{D_{i_{np}}} \right) \tag{16}$$

or

$$\overline{Y_i} = \overline{W} \left(\overline{D} + D_{np} \cdot \overline{E} \right), \tag{17}$$

where $D = 0.01 \text{ kg} \cdot \text{m}$ – the scalar value of trial imbalance; \overline{E} - identity matrix.

Subtracting (16) from (17), one can define a matrix of influence coefficients, each element of which is equal to

$$\overline{W_{a,i}} = \frac{\overline{Y_{a,i}} - \overline{Y_{0a}}}{D_{no}},$$
(18)

where k = 8 – number of nodes with estimated imbalances; l = 8 – number of runs excluding zero.

The value of the trial imbalance is assumed $D = 0.01 \text{ kg} \cdot \text{m}$.

Amplitudes of deflections in correction planes 1, 3, 10, 11, 29–31, 36 as components of the column-vector \overline{V}_0 .

4 Results

4.1 Critical frequencies of the rotor

In Table 1, data of calculations of critical frequencies of rotors and a shaft of the turbopump unit which have turned out by means of beam models result.

The lower and upper border corresponds to both "joint" and "solid shaft" connections. From the results of calculations, it can be seen that the first two critical frequencies of the rotor of the oxidizing turbopump are almost the same as the first two critical frequencies of the continuous shaft line. Therefore, we see that the forms of free oscillations also coincide for the first and second.

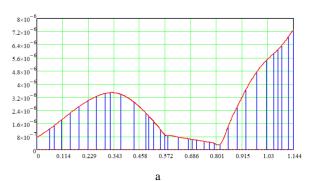
Therefore, to assess the displacement of the rotor system of the turbopump unit from the resonant modes, it is possible to consider only the dynamics of the rotor to the oxidizer turbopump unit.

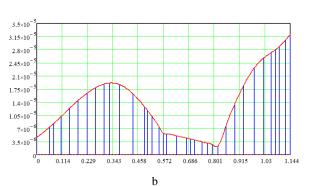
Table 1 – Critical frequencies of rotor systems found in the computer program "Critical Frequencies of the Rotor", rad/s

System	Number of critical frequency		
	1	2	3
Oxidizer	2623-3101	3251-6513	5429-10100
Fuel pump	3974-5270	5675-13420	7369-14310
Turbopump	2600–3363	3252-6211	3832–6615

4.2 Results of calculation of forced oscillations of the shaft line

Calculations and forms of forced oscillations that are performed using the Mathcad program are presented below.





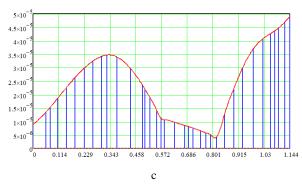


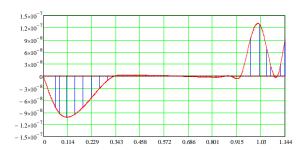
Figure 4 – Mode shapes of forced oscillations before balancing at 1100~rad/s (a), 1963~rad/s (b), and 2215~rad/s (c), m

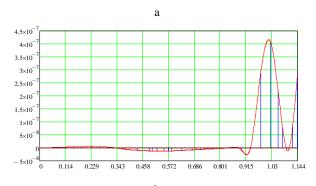
4.3 Balancing quality assessment

After the procedure of virtual balancing in the program "Forced oscillations of the rotor", forced oscillations of the shaft under the influence of residual imbalances were simulated. The characteristics of such forced oscillations are shown in the figures below.

Table 2 – Deflections in the correction planes found in the computer program "Critical frequencies of the rotor", μm

Node	Rotor speed, rad/s		
number	1100	1963	2215
1	0.77	4.65	8.79
3	1.49	8.38	15.35
10	3.49	19.16	34.65
11	3.39	18.75	33.99
29	1.45	7.81	12.97
30	2.64	13.56	22.11
31	3.71	18.58	29.99
36	6.38	28.88	44.67





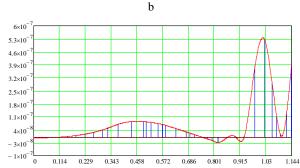


Figure 5 – Mode shapes of forced oscillations after balancing at 1100 rad/s (a), 1963 rad/s (b), and 2215 rad/s (c), m

Hence, the maximum amplitude after balancing has been decreased by two orders.

5 Conclusions

As a result of the work, free and forced oscillations of the shaft pipeline for two variations "joint" and "solid shaft" were obtained. We considered the existing methods of taking into account the stiffness of the splined joint at the critical frequencies of the rotor of the turbopump unit.

The Mathcad and ANSYS programs performed calculations and obtained the results of the maximum permissible critical frequencies of the oxidizer rotor for the turbopump, the fuel pump rotor and for the solid shaft line.

The rotor of the oxidizer turbopump and the fuel pump is a rigid rotor design in fact, that the value of the maximum operating frequency lies below the first critical frequency of both the entire shaft design and both rotors separately.

The values of the first critical frequency obtained from the calculation are 2600 rad/s (option "joint") and 3363 rad/s (option "solid shaft").

The next step was to obtain deflections in the nodes belonging to the correction planes in the initial state of the rotor. Determined that the maximum deflection is 45 μm , this result is unacceptable. Therefore, it is necessary to balance the rotor.

In order to reduce the vibration of the shaft, it was decided to conduct a virtual balancing of the rotor. The corresponding algorithm for calculating the vibration state of the rotor is implemented using the working file "Forced Oscillations of the Rotor" of the computer program MathCAD, followed by its improvement by a virtual balance to obtain results that meet the standards of GOST ISO 1940-1-2007 "Vibrations. Requirements to Quality of Balancing of Rigid rotors" and GOST ISO 11342-95 "Methods and Criteria of Balancing of Flexible Rotors".

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References

- 1. Pavlenko, I. V., Simonovskiy, V. I., Pitel', J., Demianenko, M. (2018). *Dynamic Analysis of Centrifugal Machines Rotors with Combined Using 3D and 2D Finite Element Models*. Sumy State University, Sumy, Ukraine.
- Pavlenko, I. V., Simonovskiy, V. I., Demianenko, M. M. (2017). Dynamic analysis of centrifugal machines rotors supported on ball bearings by combined application of 3D and beam finite element models. *IOP Conference Series: Materials Science and Engineering*, Vol. 233(1), article number 012053, doi: 10.1088/1757-899X/233/1/012053.

- 3. Yashchenko, A. S., Rudenko, A. A., Simonovskiy, V. I., Kozlov, O. M. (2017). Effect of bearing housings on centrifugal pump rotor dynamics. *IOP Conference Series: Materials Science and Engineering*, Vol. 233(1), article number 012054, doi: 10.1088/1757-899X/233/1/012054.
- 4. Pavlenko, I., Ivanov, V., Kuric, I., Gusak, O., Liaposhchenko, O. (2019). Ensuring vibration reliability of turbopump units using artificial neural networks. *Advances in Manufacturing II Volume 1. Lecture Notes in Mechanical Engineering. Springer, Cham*, pp. 165–175, 2019, doi: 10.1007/978-3-030-18715-6_14.
- Pavlenko, I., Simonovskiy, V., Ivanov, V., Zajac, J., Pitel, J. (2019). Application of artificial neural network for identification of bearing stiffness characteristics in rotor dynamics analysis. *Advances in Design, Simulation and Manufacturing, DSMIE 2018, Lecture Notes in Mechanical Engineering, Springer*, pp. 325–335, doi: 10.1007/978-3-319-93587-4_34.
- 6. Ding, F., Wang, Z., Qin, F. (2015). Two kinds of neural network fusion of aero-engine rotor vibration signal fault diagnosis. *4th International Conference on Mechatronics, Materials, Chemistry and Computer Engineering*, pp. 1546–1552.
- 7. Tanoh, A., Konan, D. K., Koffi, M., Yeo, Z., Kouacou, M. A., Koffi, B. K., N'guessan, K. R. (2008). A neural network application for diagnosis of the asynchronous machine. *Journal of Applied Sciences*, Vol. 8, pp. 3528–3531, doi: 10.3923/jas.2008.3528.3531.
- 8. Pavlenko, I., Neamtu, C., Verbovyi, A., Pitel, J., Ivanov, V., Pop, G. (2019). Using computer modeling and artificial neural networks for ensuring the vibration reliability of rotors. *CEUR Workshop Proceedings*, Vol. 2353, pp. 702–716.
- 9. Pavlenko, I., Trojanowska, J., Gusak, O., Ivanov, V., Pitel, J., Pavlenko, V. (2019). Estimation of the reliability of automatic axial-balancing devices for multistage centrifugal pumps. *Periodica Polytechnica Mechanical Engineering*, Vol. 63(1), pp. 277–281, doi: 10.3311/PPme.12801.
- 10. Kim, Y. W., Jeong, W. B. (2018). Reliability evaluation technique of compressor using pressure pulsation and vibration signals. *Journal of Physics: Conference Series*, Vol. 1075, article number 012076, doi: 10.1088/1742-6596/1075/1/012076.
- 11. Ben Rahmoune, M., Hafaifa, A., Guemana, M. (2015). Neural network monitoring system used for the frequency vibration prediction in gas turbine. *3rd International Conference on Control, Engineering and Information Technology*, article number 15418537, doi: 10.1109/CEIT.2015.7233185.
- 12. Pavlenko, I., Trojanowska, J., Ivanov, V., Liaposhchenko, O.: Scientific and methodological approach for the identification of mathematical models of mechanical systems by using artificial neural networks. *3rd Conference on Innovation, Engineering and Entrepreneurship, Regional HELIX 2018, Lecture Notes in Electrical Engineering, Springer*, Vol. 505, pp. 299–306, doi: 10.1007/978-3-319-91334-6_41.
- 13. Manjurul, M. M., Kim, I.-M. (2018). Motor bearing fault diagnosis using deep convolutional neural networks with 2D analysis of vibration signal. *Lecture Notes in Computer Science*, Vol. 10832, pp. 144–155, doi: 10.1007/978-3-319-89656-4_12.

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Підвищення вібраційної надійності роторів, з'єднаних шліцьовим з'єднанням Вербовий А. Є. ¹, Неамцу К. ², Сєрик М. Л. ¹, Вашист Б. В. ¹, Павленко В. В. ³, Симоновський В. І. ¹, Павленко І. В. ¹

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Анотація. Стаття присвячена розробленню уточнених математичних моделей динаміки роторних систем енергоємних турбомашин, що мають шліцьове з'єднання, та числових методів дослідження їх вільних і вимушених коливань. Запропоновані моделі враховують залежності критичних частот валопровода від кутової жорсткості шліцьового з'єднання, а також реалізують процедуру його віртуального балансування. У результаті комплексного застосування такого підходу запропоновано методи розрахунку вібраційних характеристик турбомашин з урахуванням можливої зміни кутової жорсткості шліцьового з'єднання. Крім цього, було вдосконалено методику оцінювання системи початкових дисбалансів за даними зміщень осі ротора у площинах колекції та розрахункових площинах. Запропоновані підходи, засновані на комплексному застосуванні програмного забезпечення на основі методу скінченних елементів та обчислювальних інтелектуальних систем, дозволяють проводити модальний і гармонічний аналіз та реалізовувати віртуальне балансування зі значним зменшенням підготовчого і машинного часу без втрати відносної точності. Крім того, розроблені математичні моделі вільних і вимушених коливань роторних систем були реалізовані у вигляді програмних кодів робочих файлів "Critical Frequencies of the Rotor" та "Forced Oscillations of the Rotor" системи комп'ютерної алгебри MathCAD, що дозволяє удосконалити процедуру динамічного балансування для оцінкювання системи початкових дисбалансів. Висока точність запропонованого підходу підтверджується перевіркою динамічних відхилень осі ротора у результаті дії системи залишкових дисбалансів відповідно до міжнародних стандартів вібраційної надійності.

Ключові слова: турбомашина, шліцьове з'єднання, кутова жорсткість, віртуальне балансування, модальний аналіз, гармонічний аналіз.