MINISTRY OF EDUCATION AND SCIENCE OF UKRAINE

SUMY STATE UNIVERSITY

DEPARTMENT OF COMPUTER SCIENCE

BACHELOR THESIS ON THE TOPIC:

COMPUTER SYSTEM FOR OPERATIONAL RECOGNITION OF THE PURITY OF RADIOACTIVE ISOTOPES

STUDENT: KAMA ONYEDIKACHI

SUPERVISOR: VIKTOR V. AVRAMENKO

STUDENT GROUP: IN-65AH

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Chairman of department Dovbysh A.S.

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Task of Bachelor Work

Fourth-year student, group IN-65 specialty "Informatics" Kama O. I.

On the topic: "Computer system for operational recognition of the purity of radioactive isotopes"

Approved by order of the Sumy State University

№________ of _________ 2020

Content of Explanatory Note: 1) An analytical review of the literature; 2) Statement of the problem; 3) The choice of methods for solving the problem; 4) Develop models 5) Conclusions.

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Supervisor of Bachelor work ______________ Avramenko V. V.

Task adopted to be implemented by ____________ Kama O. I.

ABSTRACT

Note: 41 pages, 6 figures, 5 tables, 24 sources literature, 1 app.

Object of study – Purity of radioactive isotope.

Purpose – To detect the presence of impurity in a radioisotope.

Research method – disproportionality function.

Results – A computer system which detects the presence of impurity in a radioactive isotope sample was created.

KEYWORDS – Radioactive, isotope, radionuclide, radioisotope, purity, impurity, chemical, element, gamma-ray spectroscopy, disproportionality, n-order derivative disproportionality function.

CONTENTS

INTRODUCTION

Radioactive isotopes have been very important in different spheres of life. This is because they are used in diverse sections such as the agricultural sector, medicine, pharmacy and industry. Medically, radioactive isotopes are used in detecting and killing cancer cells. For example, Iodine-131 is used in treating thyroid cancer, Strontium-89 is used to treat cancer in the bone and Radium-223 is used to treat prostate cancer. They could also be used as tracers in humans, plants and animals to identify irregular body sequences or processes. In Agriculture, they are used to increase the amount of crops yielded and also in reducing the amount of crops lost by proper management of pest control. Phosphorus-32 is one of the most famous radioactive isotopes used in agriculture to track how plants take up water and other nutrients. In industry, radioactive isotopes are used for picking out spots with structural damage or defects which enables fixing them easier replacing very big and expensive X-ray machines. Plutonium-238 is used as a source of electrical power by converting the heat dissipated during decay to electrical energy. I could go on and on listing all the uses and practical applications of radioactive isotopes because of how important it is to our daily lives from what we eat to electricity and treatment in the hospitals.

What then happens when there are impurities in a radioactive isotope and how do we control the purity of the isotopes? The same way water is not suitable for drinking in the presence of impurity, that’s the same way radioactive isotopes become harmful and their decay process is altered which could cost the life of someone undergoing cancer treatment. This is why creating a computer system which recognizes the presence of impurity in a radioisotope is essential.

1. LITERATURE REVIEW

So far, there has been great advancement in radioactive isotopes and it application in our day to day life which has been very essential in the daily lives of people and also a big boost to the economy of countries around the world. Before we go any further, we need to look at the basics and foundation of what has been researched upon already in the area of radionuclides.

1.1 WHAT IS A RADIOACTIVE ISOTOPE?

The proper way to go about it is to first of all split the words and look at them individually before we look at it as a whole. By doing this, we get a better understanding of what this research is all about.

Radioactivity is the emission of radiation spontaneously which occurs when an atomic nucleus is unstable because of the imbalance between the number of protons and neutrons. It starts losing energy so that it gets to a stable form or state [1].

An isotope is any atom from a group of two or more atoms which have exactly the same atomic number but differ in the number of neutrons [2].

Radioactive isotopes which could be called in different forms such as radionuclides, radioisotopes or even radioactive nuclides are simply different forms of a chemical element which vary in masses having unstable nuclei and lose energy by spontaneous emission of radiation which could be in the form of alpha rays, beta rays and gamma rays.

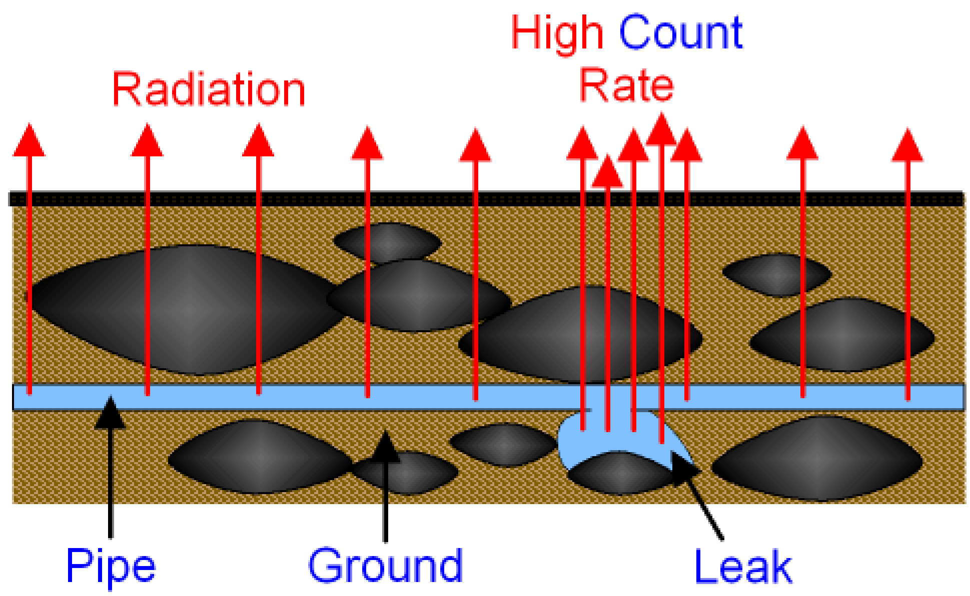
Radioactive isotopes are not meant for a particular set of chemical elements which means all chemical elements have radioactive isotopes starting right from the element with the least mass which is hydrogen with an atomic number of 1 to the heaviest element which is oganesson with an atomic number of 118.

1.2 USES AND APPLICATION OF RADIOACTIVE ISOTOPES

Radioactive isotopes have a wide variety of uses and application in different areas starting from agriculture, food, tracing, medical treatments and much more because we can harness the energy which they emit. Step by step, we will look at a few areas where radioisotopes have been used to great effect and how scientists have carefully harnessed the energy emitted from them.

1. TRACING

Detecting radioactivity has proven to be easy to detect which makes radionuclides very good tracers because they can be easily located and traced. What then is a tracer? Tracers are substances which are used to map a structure. For example, let's assume a pipe is leaking in a complex structure of pipes which are fixed together, it could take ages to locate the exact location where one or more pipes are leaking from hundreds or thousands of pipes connected together. This is where tracers come in. In lots of cases, tritium is poured into the water and allowed to circulate across all the pipes then special equipment such as the famous Geiger counter are used to detect any form of radioactive activity [3].



Picture 1.2.1 – Use of tracers in the field

2. RADIOACTIVE DATING

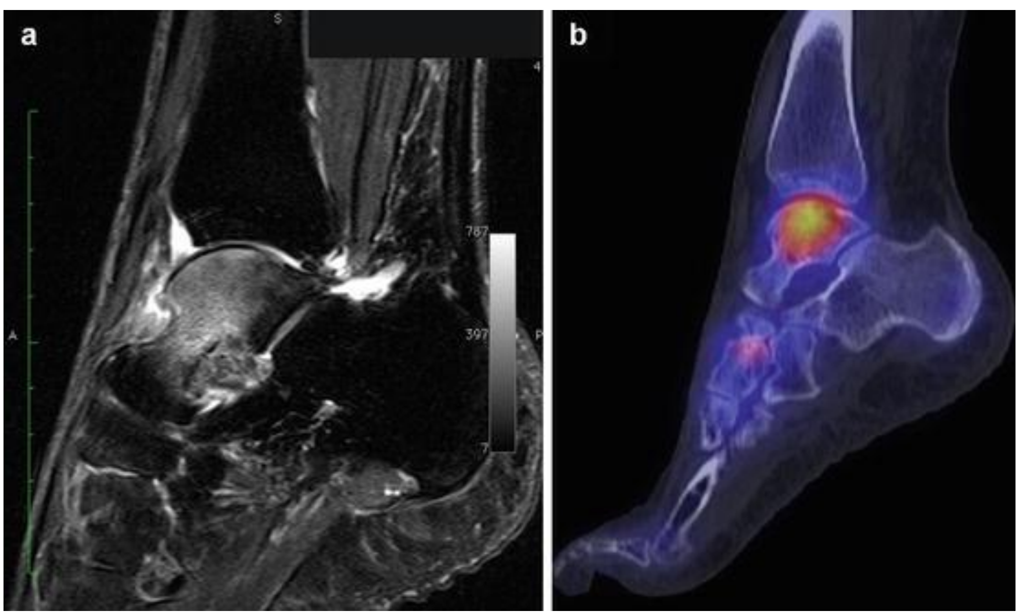
Have you ever wondered how scientists are able to tell how old a skull that was found in North Africa is? Have you wondered how original paintings are tested to know when the piece of art was exactly done? Radioactive dating. Unlike a typical clock which can affected by water, sunlight, moisture, human and animal factors, the half-life or radioactive isotopes is the ‘clock’ which cannot be affected by any of these. So the next time you see an artifact was made 2 million years ago, all you need to know is that the half-life of the radioisotope found was used to determine the age of the artifact in question [4].

3. FOOD IRRADIATION

There are quite a number of effective ways to preserve food such as pasteurization, use of pressure, drying to remove moisture and so on. Irradiation too is among one of the most popular and effective ways of elongating the shelf life or preserving food. The food is exposed to small amount of ionizing radiation which kills bacteria that hasten the decay of food. This is how different types of food is preserved without the risk of being radioactive itself [4].

4. MEDICAL USE

This is one of the biggest areas where radioactive isotopes are used. Radionuclides have been essential to the health of human beings for years. It doesn’t just help in diagnosing what the sickness is but also in treating the diseases found. We’ve discussed earlier about the use of radioisotopes as very good tracers and we will go further to outline how different organs and systems are mapped out using radioisotopes. Organs such as the thyroid gland, liver and even images of the bones can be gotten on screen and printed on paper [4].



Picture 1.2.2 – Nuclear images using radioactive isotopes

5. AGRICULTURAL APPLICATION

Radioisotopes are also greatly used in the field of agriculture and it has proven to be very crucial and important over the years. Plants largely depend on phosphorus and their rate of uptake is very important in figuring out potentially high yielding crops. This can be done by adding phosphorus-32 to manure or fertilizer and then the rate of intake is monitored by how much radioactivity is found on the leaves and how much time it took for the whole process to occur. It is also used in monitoring the process of photosynthesis in plants [4].

Below is a table which shows a list of some radioactive isotopes and their uses.

Radioactive Isotope	**Use/Application**
Americum-241	1. Used to detect smoke. 2. Detecting lead that is toxic. 3. Uniformity in the production os paper and steel.
Cadmium-109	Used to sort metallic alloys and to analyze them.
Calcium-47	1. Research in formation of the bones. 2. Research in cell functions.
Californium-252	1. Explosive detection. 2. Measuring the amount of moisture.

Carbon-14	1. Metabolism of new drugs. 2. Research and study of new drugs. 3. Control of pollution.
Cesuim-137	1. Cancer treatment. 2. Controlling flow rate of liquids in pipes.
Chromium-51	Red blood cell research.
Cobalt-57	Diagnosing anemia.
Cobalt-60	1. Cancer treatment. 2. Sterilization of medical and surgical tools. 3. Irradiation of food.
Copper-67	Used in medicine to fight medicine.
Curium-244	Used in mining.
Gallium-67	Diagnosis in medicine.
Iodine-123	1. Research in biomedical industry. 2. Diagnosing thyroid diseases.
Iodine-125	1. Research in biomedical industry. 2. Diagnosing thyroid diseases.
Iodine-129	1. Research in biomedical industry. 2. Diagnosing thyroid diseases.
Iodine-131	Treatment of Grave's disease (thyroid related diseases.
Iridium-192	1. Irradiation of tumor. 2. Pipeline welding.
Iron-55	1. Research in metabolism.

	2. Sulphur detection.
Krypton-85	1. Used in appliances as indicator lights. 2. To calculate the thickness of materials. 3. Used for purity test to check dust and pollution levels. 4. Detection of explosives. 5. Used in electronics for surge protectors. 6. Used in gas chromatographs.
Nickel-63	1. Detection of explosives. 2. Used in electronics for surge protectors. 3. Used in gas chromatographs.
Phosphorus-32	Used for research in biology and genetics.
Phosphorus-33	Used for research in biology and genetics.
Plutonium-238	Used in powering space crafts.
Polonium-210	Reduction of static charge.
Promethium-147	Calculating the thickness of thin materials such as paper, metal sheets and rubber.
Radium-226	In making better lighting rods.
Selenium-75	Used in research on proteins.
Sodium-24	In pinpointing leakages in pipes.
Strontium-85	1. Study of metabolism. 2. Study of the formation of the bone.

Strontium-90	1. Used in medical treatment. 2. In production of sensors.
Sulphur-35	Genetics research.
Technetium-99m	Used in nuclear imaging.
Thallium-201	1. Used to detect tumor. 2. Also used in nuclear cardiology.
Thallium-204	1. Used for purity test on filter paper. 2. To calculate thickness of materials.
Thoriated Tungsten	Used by welders in construction to make curves.
Thorium-229	Increases the longevity of fluorescent lights.
Thorium-230	Used as coloring in glass.
Tritium	1. Metabolism of drugs. 2. Making products that glow such as exit signs and wristwatches.
Uranium-234	Dentists use it for teeth whitening and making fixtures such as crowns.
Uranium-235	1. Used as fuel in nuclear powered plants. 2. Production of glass materials such as fluorescents and in making tiles.
Xenon-133	1. Tracing the flow of blood in the body of plants and animals. 2. Proper ventilation of the lungs

Table 1.2.1 – The table shows a list of radioactive isotopes and their uses [5].

2. PURITY OF RADIOISOTOPES

In this study, you will come to learn that purity is relative and this is because of different requirements from different sphere and operations. In one scenario, 99 percent of a radioactive isotope could be considered to be "pure enough" but in a different scenario under different circumstances and varying needs, that would be nowhere close enough to be considered as pure. Purity could be defined as a state of being free from fault, defect or imperfection but a state of total purity is sometimes hard to achieve when it comes to radionuclides [6]. What then one may ask is purity of radioactive isotopes?

Purity of radioactive isotopes or radionuclidic purity is defined as the percentage or ratio of the radioactivity of the radioactive isotope in question compared to that of the source [7].

2.1 IMPORTANCE OF PURITY

Purity is an important factor in all areas where radionuclides are used but one area where the purity of radioactive isotope is very crucial and important is in radiopharmacy because any impurities found could alter the amount of radiation the patient is exposed to which could be counterproductive because the patient's condition may become worse rather than get better [8]. Any form of impurity could pose a great threat to the health and wellbeing of a human life which is why it is very important that all acceptable limits are ensured before the use of any radioisotope, especially in the medical field where lives are the priority [9].

Also, the integrity of nuclear images could be altered vastly because the presence of impurity is degrades or brings about poor quality images [10]. This is particularly important because when the nuclear images are wrong or faulty, it affects diagnosis which results in the patient getting the wrong treatment because of the wrong diagnosis which all started as a result of having impurities found in the radioactive isotope which was crucial [11]. Impurities could be the difference between a life being saved and a life being lost so therefore the importance of having pure radionuclides cannot be overemphasized [12].

That being said, in practice there is no sample of radioactive isotope which is a hundred percent (100%) pure because either in production or decay of the radioactive isotope, there is contamination. This means limits have to be set and monitored closely which vary depending on what radioactive isotope is present [13].

2.2 GAMMA-RAY SPECTROSCOPY

Gamma-ray spectroscopy is a very efficient, quick and common way to identify all the radioisotopes in a given sample. In gamma-ray spectroscopy, detectors are used to measure the energy from gamma-rays. After the detection procedure, the energy emitted is then compared to the already known energy standard of gamma-rays the radioactive isotopes emit or produce [14].

Radionuclide		Energy (keV)	Emission probability ε (%)	Interferences
^{7}Be	Direct	477.61	10.3	
^{40}K	Direct	1460.8	10.67	

	Direct	186.2	3.5	
	^{214}Pb	295.21	18.2	^{235}U (185.72 keV;
	^{214}Pb	351.92	35.8	57.2%)
	^{214}Bi	609.32	44.6	^{211}Bi
	^{214}Bi	1120.3	14.8	(351.06 keV;
^{226}Ra	^{214}Bi	1764.5	15.4	12.91%)
	^{228}Ac			
	^{228}Ac	338.32	11.3	
	^{228}Ac	911.21	26.6	
^{228}Ra		968.97	15.8	
	^{224}Ra			
	^{212}Pb	240.99	4.0	
	^{212}Pb	238.63	43.3	^{214}Pb (241.98;
	^{208}Tl	300.09	3.3	7.12%)
	^{208}Tl	277.36	2.3	^{228}Ac
	^{208}Tl	583.17	30.5	(583.41 keV;
^{228}Th		860.56	4.5	0.114%)
	^{227}Th			
	^{227}Th	235.97	12.1, 11.2	
	^{223}Ra	256.5	7.0	
^{227}Ac		269.5	13.7	
		143.76	10.96	^{226}Ra (186.1;
^{235}U	Direct	163.33	5.08	3.51%)

		185.72 205.31	57.2 5.01	^{228}Ac (204.10; 0.171%)
^{238}U	^{234}Th ^{234}Th ^{234}Th ^{234M}Pa ^{234M}Pa	63.28 92.37 92.79 766.37 1001.03	4.3 2.5 2.4 0.21 0.84	^{232}Th (63.81; 0.267%) Weak line Weak line
^{210}Pb	Direct	46.54	4.2	Weak line
^{223}Ra	Direct ^{211}Bi ^{219}Rn ^{219}Rn	154.21 269.46 351.07 271.23 401.81	5.6 13.7 12.9 10.5 6.5	
^{231}Pa	Direct	300.07 302.67	2.5 2.2	

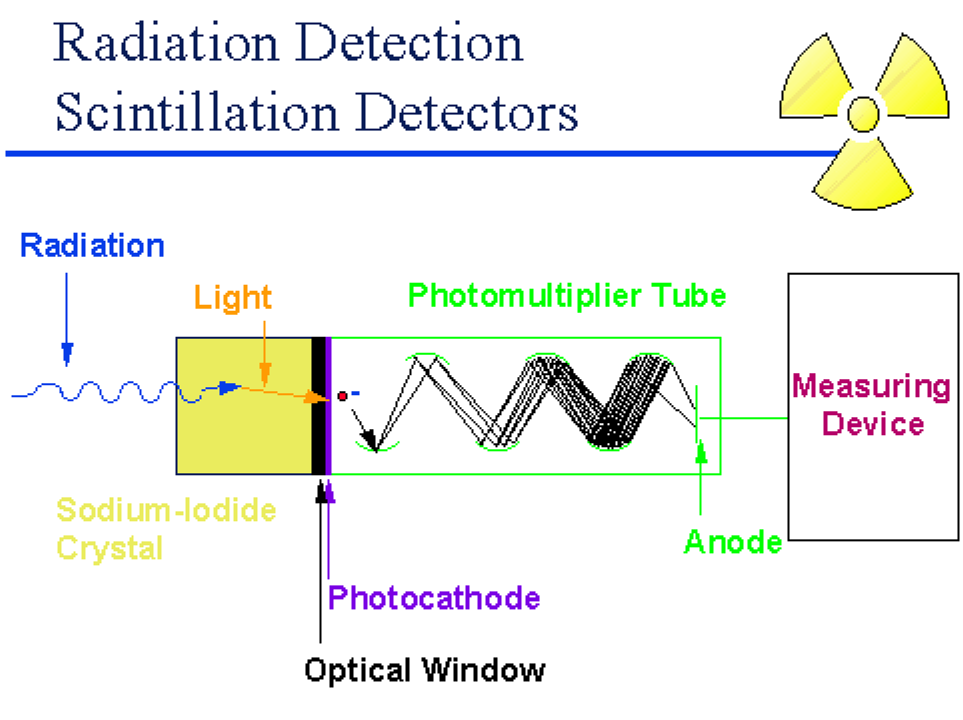
Table 2.2.1 – Mainstream emission data used in detecting inartificial radioactive isotopes using gamma-ray spectroscopy [15].

2.3 DETECTORS

So far, quite a number of detectors have been used to not just track gamma-ray but also the energy it emits. The energy spectrum from a given sample is measured coupled with the amount of radiation it emits. We will look at two highly efficient and commonly used detectors [16].

2.3.1 SCINTILLATION DETECTOR

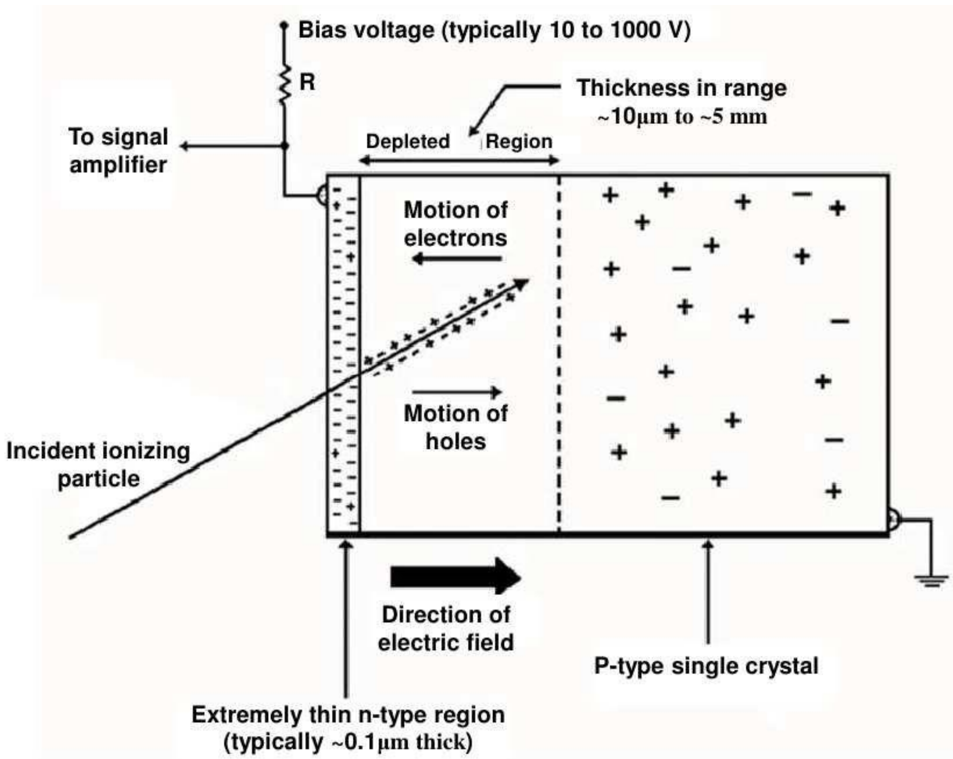
Ionizing radiation could be detected using different methods and a scintillation detector is one of them. What then is scintillation? Suppose we have any material in any form such as liquid form, solid form or gaseous form which emits light as a form of response when impacted upon by ionizing radiation [17]. This response process is called scintillation. The light which is emitted is then transformed into an electrical signal which a computer detects using photoelectric effect [18].



Picture 2.2.3.1 – Scintillation detector set up [19].

2.3.2 SEMICONDUCTOR DETECTOR

Similar to the scintillation detector, the semiconductor detector does the same job by converting gamma-rays into electrical signals but it uses a different procedure. When it comes to semiconductor detectors, HPGe (high purity germanium) are one of the most common [20].



Picture 2.3.2.1 – Formation and working process of HPGe detectors

COMPARING AND CONTRASTING

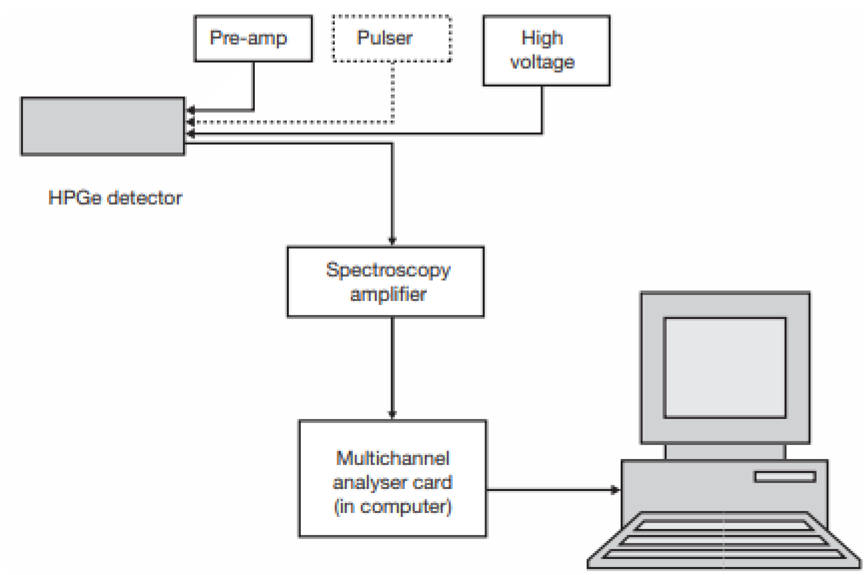
	Scintillation Detectors	Semiconductor Detectors
1	Uses sodium iodide with droplets of thallium – NaI(Tl).	Uses germanium with droplets of lithium – Ge(Li).
2	Cheaper	More expensive
3	Simple	Complex
4	Durable	Fragile
5	Lesser resolution	Higher resolution

6	Photopeaks are not easily visible	Photopeaks are easily visible
7	Does not require preserving in very low temperatures	Must be preserved in very low temperatures

Table 2.3.2.1 – Differences between scintillation detectors and semiconductor detectors

HPGe DETECTORS

HPGe detectors which means high purity germanium detectors are touted as the most efficient when it comes to gamma-ray precision. This is because compared to silicon used in silicon detectors, germanium has proven to be much more effective simply because it has a higher atomic number compared to that of silicon [21].



Picture 2.3.2.2 – HPGe gamma ray system set up [22].

The major problem with high purity germanium detectors is that they have to be kept and used in very cold temperature like that of nitrogen to prevent the detectors resolution from destroying [22].

When it comes to determining the main radioactive isotope and presence of impurities in a sample, there has to be a proper calibration in reference to efficiency and energy. It is very important to follow all the guidelines specified in the manual which comes from the manufacturer [23].

2.4 DISPROPORTIONALITY FUNCTION

In determining new characteristics of numerical equations or functions, we will look at the case of almost stationary objects which have unchanged characteristics at a particular time with the mathematical equation below:

$$y = k(t)x \tag{1}$$

x, y – input and output parameters respectively;

t – time

k(t) – performs a gradual change in respect to x(t).

These objects could range from sensors, converters, tracking systems and much more.

The unchanged character in a general overview becomes:

$$y = k(x,t)x + b(t), \tag{2}$$

Where $\mathrm{b(t)} \neq 0$.

Comparing the values of (1) and (2) will be ideal but k(t) in (1) changes continuously because we are dealing with almost stationary objects which makes it much more difficult.

In a case where x from (2) is not proportional to y, we do not compare the values because k(t) is unknown [24].

2.4.1 STATEMENT OF PROBLEM

Let X be a set of real numbers and T be a set of real ordered numbers. The set Y which is a set of real numerical functions be defined as;

$$y = f(x,t), \qquad (3)$$

Where $x \in X, t \in T$.

For given values $x \in X$ and $t \in T$, the functional on the set Y must be zero if x and y=f(x, t) is proportional [24].

2.4.2 PROBLEM SOLUTION

Firstly, solving for one variable is how to begin;

Having;

$$y = f(x), \; x \in X \; .$$

Derivative of y = f(x) is $\frac{dy}{dx}$

Limit of ratio $\frac{\Delta y}{\Delta x}$ for $\Delta x \to 0.$

Hence;

$$\frac{y}{x} = \frac{dy}{dx}, \qquad (4)$$

Is used for any given $x \in X$, and based on the relationship between x and y which is proportional, y = cx, with c = const, then (4) is satisfied [24].

Case 1

The disproportionality in respect to the 1st order derivative in the case of the function y = f(x) is the difference between $\frac{y}{x}$ and $\frac{dy}{dx}$.

Therfore,

$$@d_x^{(1)} y = \frac{y}{x} - \frac{dy}{dx} \qquad (5)$$

@ - disproportionality determining operation

d – derivative

Logic

y = cx, c = const, is the only case where condition (4) is performed

Looking at the first order derivative where, we see that from y = f(x, t), where $x \in X, t \in T$;

$$@d_x^{(1)} y_t = \frac{f(x,t)}{x} - f_x'(x,t), \qquad (6)$$

2.4.3 N-ORDER DERIVATIVE DISPROPORTIONALITY

The n-order derivative must be equal to zero so that;

$$y = cx^n \qquad (8)$$

For (8), we have;

$$\frac{y}{x^n} = \frac{1}{n!}\frac{d^n y}{dx^n} \qquad (9)$$

Case 2

The derivative of y = f(x) with respect to x;

$$@\, d_x^{(n)} y = \frac{y}{x^n} - \frac{1}{n!}\frac{d^n x}{dx^n}, \qquad (10)$$

The derivative of y = f(x, t) with respect to x;

$$@\, d_x^{(n)} y_t = \frac{f(x,t)}{x^n} - \frac{1}{n!}\frac{\partial^n x}{\partial x^n}, \qquad (11)$$

n – an integer greater than 0 (zero).

When the origin is transferred for fixed t to the point M(x0, y0, t), (11) changes to;

$$@\, d_{x-x_0}^{(n)} (y - y_0) = \frac{f(x,t) - y_0}{(x - x_0)^n} - \frac{1}{n!}\frac{\partial^n f(x,t)}{\partial x^n} \qquad (12)$$

In the case $y = f(x_1, x_2, x_3, \ldots, x_p, t)$;

$$@\, \partial_{x_i}^{(n)} y_t = \frac{y}{x_i^n} - \frac{1}{n!}\frac{\partial^n y}{\partial x_i^n}$$

2.4.3 DERIVATIVE DISPROPORTIONALITY WHEN A FUNCTION IS SPECIFIED IN FORM OF PARAMETERS

Let $x = \varphi(t), \quad y = \psi(t),$

Where,

$t \in [T_1, T_2]$ and inverse function $t = \Phi(x)$ exists.

For $y = \Psi[\Phi(x)]$, applying $\frac{d^n y}{dx^n}$ in the case of parametric dependence of y on x.

For n = 1;

$$@ d_x^{(1)} y = @_{\phi(t)}^{(1)} \psi(t) = \frac{y}{x} - \frac{y_t'}{x_t'} = \frac{\psi(t)}{\phi(t)} - \frac{\psi_t'(t)}{\phi_t'(t)} \qquad (15)$$

Is equal to zero for $\psi(t) = c\phi(t)$, where c = const.

2.4.4 VALUE DISPROPORTIONALITY FUNCTIONS

Case 3

The n-order value of disproportionality with respect to x^n equates (10) multiplied by x^n.

It is showcased as $@ v_x^{(n)} y$

$$@ v_x^{(n)} y = y - \frac{x^n}{n!} \frac{d^n y}{dx^n} \qquad (16)$$

For n = 1,

$$@ v_x^{(1)} y = y - x \frac{dy}{dx} \qquad (17)$$

For $y = f(x,t)$, $x \in X, t \in T$ the value of disproportionality from x^n.

n – integer greater than zero (0), which is solved for fixed t;

$$@v_x^{(n)} y_t = y_t - \frac{x^n}{n!} \frac{d^n y_t}{dx^n} \qquad (18)$$

If they are in parametric form according to (15), then

$$@v_x^{(1)} y = @v_{\phi(t)}^{(1)} \psi(t) = \psi(t) - \phi_t(t) \frac{\psi_t'(t)}{\phi_t'(t)} \qquad (19)\ [24]$$

2.4.5 PROPERTIES OF DISPROPORTIONALITIES FUNCTIONS

Case

Given the function y = f(x, t),

$x \in X, t \in T$. Let $@d_x^{(n)} y_t \neq 0$ but $@(n)@d_x^{(1)} y_t = 0$. Proving that y = f(x, t) takes the form;

$$y = k_n(t)x^n + k_{n-1}(t)x^{n-1} + \ldots + k_1(t)x, \qquad (20)$$

Logic

We need to prove that the disproportionality with respect to the n-order isn't zero (0).

From (11)

$$@d_x^{(n)} y_t = \frac{1}{x^n}[k_{n-1}(t)x^{n-1} + k_{n-2}(t)x^{n-2} .. + k_1(t)x]. \qquad (21)$$

From this, we know that one of $k_{n-1}(t), k_{n-2}, \ldots, k_1(t)$ doesn't equate to zero.

Looking at the sequence where $@(n)@d_x^{(1)} y_t$ of n first-order (6) for (20).

For easy comprehension, Z signifies each disproportionality in the sequence.

$$Z_1 = @d_x^{(1)} y_t = -[(n-1)k_n(t)x^{n-1} + (n-2)k_{n-1}(t)x^{n-2} + \ldots + 2k_3(t)x^2 + k_2(t)x]$$

$$Z_2 = @d_x^{(1)} Z_1 = [(n-1)(n-2)k_n(t)x^{n-2} + (n-2)(n-3)k_{n-1}(t)x^{n-3} + .. + 6k_4(t)x^2 + 2k_3(t)x] \ldots$$

$$Z_i = @d_x^{(1)} Z_{i-1} = (-1)^i \sum_{j=i}^{n} k_j(t) x^{j-i} \prod_{m=1}^{i} (j-m) \ldots$$

$$Z_{n-1} = @d_x^{(1)} Z_{n-1} = (-1)^{n-1}(n-1)! k_n(t)x$$

$$Z_n = @d_x^{(1)} Z_{n-1} = 0$$

This was the expected result needed [24].

3. FORMULATION OF THE PROBLEM

It is necessary to develop an algorithm and a computer program for simulating the operation of a system for the operative recognition of impurities in a radioactive isotope. Consider an example of the presence of ***strontium*** in a radioactive isotope of ***iron***.

3.1 MATHEMATICAL FORMULATION OF THE PROBLEM

The decay of a radioactive substance is described by the expression:

$$\boldsymbol{m(t) = m_o exp(-\lambda t)}, \quad \textbf{(1)}$$

Here:

t – time;

m – mass of substance;

m_o – initial mass;

λ – constant;

$\lambda = \frac{\ln 2}{T}$; **(2)**

T = isotope half-life.

3.2 CHOICE OF METHOD FOR SOLVING THE PROBLEM

The question at hand is a very complex case of detecting or recognizing the presence of an impurity in a given sample of a radioactive isotope which could cause a change in the amount of energy it emits.

To solve this dilemma, disproportionality function is used by working with the first order disproportion function of an exponential function.

3.3 PROCESS OF SOLVING THE PROBLEM

The first order disproportion function of an exponential function with respect to its first derivative is equal to zero. The function (1) is exponential. So therefore, if there is only one isotope in considered substance, it is necessary to calculate this disproportion for $\boldsymbol{m(t)}$ and a result to compare with zero.

The disproportion $\boldsymbol{Z_1(t)}$ is:

$$\boldsymbol{Z_1(t) = @d^{(1)}{}_{m'(t)}m(t) = \frac{m(t)}{m'(t)} - \frac{m'(t)}{m''(t)},} \quad \textbf{(3)}$$

Substitute (1) into (3):

$$\boldsymbol{Z_1(t) = \frac{m_o e^{(-\lambda_1 t)}}{-\lambda_1 m_o e^{(-\lambda_1 t)}} - \frac{-\lambda_1 m_o e^{(-\lambda_1 t)}}{-\lambda_1{}^2 m_o e^{(-\lambda_1 t)}} = 0,} \quad \textbf{(4)}$$

But if the considered substance includes two different isotopes, it is described as:

$$\boldsymbol{m(t) = m_{01} e^{(-\lambda_1 t)} + m_{02} e^{(-\lambda_2 t)},} \quad \textbf{(5)}$$

In this case, the disproportion (3) is defined as:

$$\boldsymbol{Z_2(t) = @d^{(1)}{}_{m'(t)}m(t) = \frac{m_{01} e^{(-\lambda_1 t)} + m_{02} e^{(-\lambda_2 t)},}{-\lambda_1 m_{01} e^{(-\lambda_1 t)} - \lambda_2 m_{02} e^{(-\lambda_2 t)}} - \frac{-\lambda_1 m_{01} e^{(-\lambda_1 t)} - \lambda_2 m_{02} e^{(-\lambda_2 t)}}{\lambda_1{}^2 m_{01} e^{(-\lambda_1 t)} + \lambda_2 m_{02} e^{(-\lambda_2 t)}},} \quad \textbf{(6)}$$

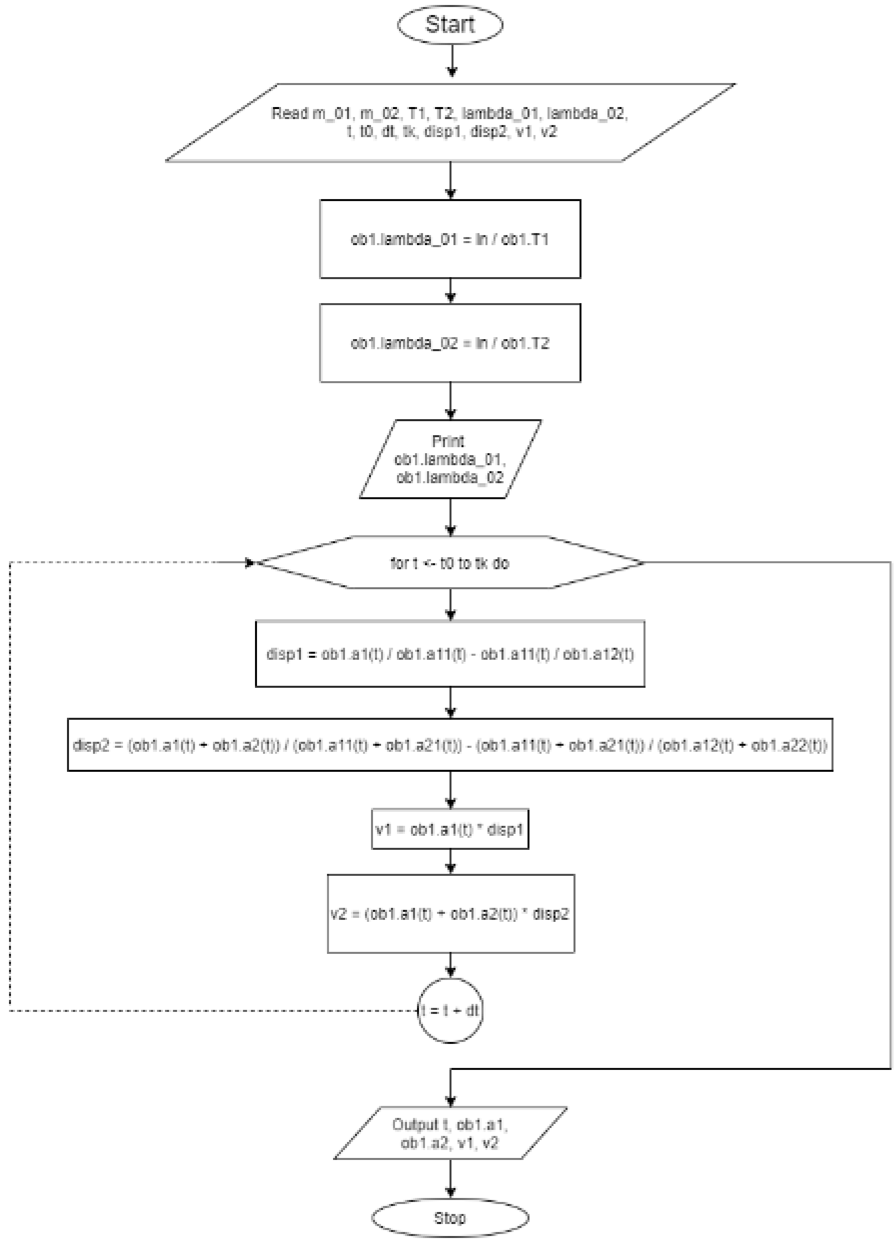
Obviously this expression isn't equal to zero if $\boldsymbol{\lambda_1 \neq \lambda_2}$ and$\boldsymbol{m_{01} \neq m_{02}}$. Thus to solve the problem, it is necessary to calculate the current values of disproportion (6) and compare the result with zero.

If $\boldsymbol{Z_2(t) = 0}$, then at this moment only one isotope is in the test substance. It is often convenient for analysis to use estimates that are measured in the same units as the monitored indicator. In this case, it is suitable for the result to be in mass units, so therefore, a first order value disproportion function is used [24].

This disproportion function is defined as:

$$\boldsymbol{V(t) = @V^{(1)}{}_{m'(t)}m(t) = m(t) - \frac{m'(t)}{m''(t)} m'(t)} \quad \textbf{(7)}$$

3.4 ALGORITHM FOR SOLVING THE PROBLEM



The table below shows the use of identifiers used in the algorithm

Table 3.3.1

VARIABLES	IDENTIFIERS	EXPLANATION
$m_1(t)$, $m_2(t)$	ob1.a1(t), ob1.a2(t)	$m_1(t)$ – Mass of iron over time. $m_2(t)$ – Mass of strontium over time
$V_1(t), V_2(t)$	v1, v2	V1 – Values of disproportion function for absence of impurity in radioactive isotope. V2 – Values of disproportion function for presence of impurity in radioactive isotope.
t, t0, dt, tk, T1, T2	t, t0, dt, tk, T1, T2	Refers to time
λ	lambda_01, lambda_02	constant
log	ln	Natural logarithm

4. MANUAL

The program was written using C# programming language so therefore an IDE built for running the program is needed.

For the test case, the program SharpDevelop was used for the best result.

Size of program [Bachelors.cs] - 2,141 bytes

Size on disk - 4,096 bytes

After running the program, there is an output on the screen which prints the two lamda values and a text file is created where all the results are written to.

File name – result.txt

File size - 4,096 bytes.

5. TEST CASE

A series of task have been tested using our custom built system for the operational recognition of the purity of radioactive isotopes and outlined below is the data for one of those test cases.

$\boldsymbol{m_{01} = 200\ grams,}$

$\boldsymbol{T_1 = 45\ days}$,

$\boldsymbol{t_0 = 0}$,

$\boldsymbol{t_k = 320\ days}$,

$\boldsymbol{m_{02} = 1\ gram,}$

$\boldsymbol{T_2 = 50.6\ days}$,

$\boldsymbol{dt = 4\ days}$

These are the variable representations used the table below.

$\boldsymbol{m_1}(\boldsymbol{t})$ – Mass of iron over time.

$\boldsymbol{m_2}(\boldsymbol{t})$ – Mass of strontium over time

$\boldsymbol{V_1}(\boldsymbol{t})$ **–** Values of disproportion function for absence of impurity in radioactive isotope.

$V_2(t)$ – Values of disproportion function for presence of impurity in radioactive isotope.

t	$m_1(t)$	$m_2(t)$	$V_1(t)$	$V_2(t)$
0.000	200.000000	1.000000	0.000000	-0.792478
4.000	188.049323	0.946680	0.000000	-0.750206
8.000	176.812739	0.896203	0.000000	-0.710188
12.000	166.247579	0.848417	0.000000	-0.672305
16.000	156.313723	0.803180	0.000000	-0.636442
20.000	146.973449	0.760354	0.000000	-0.602493
24.000	138.191288	0.719812	0.000000	-0.570354
28.000	129.933891	0.681431	0.000000	-0.539930
32.000	122.169901	0.645097	0.000000	-0.511128
36.000	114.869835	0.610701	0.000000	-0.483863
40.000	108.005974	0.578138	0.000000	-0.458052
44.000	101.552251	0.547312	0.000000	-0.433618
48.000	95.484160	0.518129	0.000000	-0.410487
52.000	89.778659	0.490502	0.000000	-0.388590
56.000	84.414080	0.464349	0.000000	-0.367861
60.000	79.370053	0.439590	0.000000	-0.348238
64.000	74.627423	0.416151	0.000000	-0.329661
68.000	70.168182	0.393961	0.000000	-0.312076
72.000	65.975396	0.372955	0.000000	-0.295428
76.000	62.033142	0.353069	0.000000	-0.279669
80.000	58.326452	0.334244	0.000000	-0.264750
84.000	54.841249	0.316422	0.000000	-0.250626
88.000	51.564299	0.299550	0.000000	-0.237257
92.000	48.483157	0.283578	0.000000	-0.224600
96.000	45.586124	0.268458	0.000000	-0.212619
100.000	42.862199	0.254144	0.000000	-0.201276
104.000	40.301038	0.240593	0.000000	-0.190539
108.000	37.892914	0.227764	0.000000	-0.180374
112.000	35.628684	0.215620	0.000000	-0.170752
116.000	33.499750	0.204123	0.000000	-0.161643
120.000	31.498026	0.193239	0.000000	-0.153020
124.000	29.615913	0.182936	0.000000	-0.144857
128.000	27.846261	0.173181	0.000000	-0.137129
132.000	26.182353	0.163947	0.000000	-0.129814

136.000	24.617869	0.155206	0.000000	-0.122888
140.000	23.146868	0.146930	0.000000	-0.116333
144.000	21.763764	0.139096	0.000000	-0.110126
148.000	20.463305	0.131679	0.000000	-0.104251
152.000	19.240554	0.124658	0.000000	-0.098690
156.000	18.090865	0.118011	0.000000	-0.093425
160.000	17.009875	0.111719	0.000000	-0.088441
164.000	15.993477	0.105762	0.000000	-0.083722
168.000	15.037813	0.100123	0.000000	-0.079256
172.000	14.139253	0.094784	0.000000	-0.075028
176.000	13.294384	0.089730	0.000000	-0.071025
180.000	12.500000	0.084946	0.000000	-0.067236
184.000	11.753083	0.080417	0.000000	-0.063649
188.000	11.050796	0.076129	0.000000	-0.060253
192.000	10.390474	0.072070	0.000000	-0.057039
196.000	9.769608	0.068227	0.000000	-0.053996
200.000	9.185841	0.064589	0.000000	-0.051115
204.000	8.636955	0.061145	0.000000	-0.048388
208.000	8.120868	0.057885	0.000000	-0.045806
212.000	7.635619	0.054798	0.000000	-0.043362
216.000	7.179365	0.051877	0.000000	-0.041049
220.000	6.750373	0.049110	0.000000	-0.038859
224.000	6.347016	0.046492	0.000000	-0.036786
228.000	5.967760	0.044013	0.000000	-0.034823
232.000	5.611166	0.041666	0.000000	-0.032965
236.000	5.275880	0.039445	0.000000	-0.031206
240.000	4.960628	0.037341	0.000000	-0.029542
244.000	4.664214	0.035350	0.000000	-0.027965
248.000	4.385511	0.033465	0.000000	-0.026473
252.000	4.123462	0.031681	0.000000	-0.025061
256.000	3.877071	0.029992	0.000000	-0.023724
260.000	3.645403	0.028393	0.000000	-0.022458
264.000	3.427578	0.026879	0.000000	-0.021260
268.000	3.222769	0.025446	0.000000	-0.020126
272.000	3.030197	0.024089	0.000000	-0.019052
276.000	2.849133	0.022804	0.000000	-0.018035
280.000	2.678887	0.021588	0.000000	-0.017073
284.000	2.518815	0.020437	0.000000	-0.016162
288.000	2.368307	0.019348	0.000000	-0.015300

292.000	2.226793	0.018316	0.000000	-0.014483
296.000	2.093734	0.017339	0.000000	-0.013711
300.000	1.968627	0.016415	0.000000	-0.012979
304.000	1.850995	0.015540	0.000000	-0.012287
308.000	1.740391	0.014711	0.000000	-0.011631
312.000	1.636397	0.013927	0.000000	-0.011010
316.000	1.538617	0.013184	0.000000	-0.010423
320.000	1.446679	0.012481	0.000000	-0.009867

Table 3.2.1 – The test results are shown in the table above.

RESULT ANALYSIS

From the results in Table 3.2.1, we can see that the disproportionality of $\boldsymbol{V_1(t)}$ is equal to zero which means there is no contamination of strontium in the sample of iron while the disproportionality of $\boldsymbol{V_2(t)}$ is not equal to zero which means there is presence of contamination in the form of an isotope.

CONCLUSION

A computer system for operational recognition of the purity of radioactive isotopes was researched upon and created. This system is fully functional and very precise in detecting the radioactive isotopes or presence of impurity in any given sample. This was achieved using functions of disproportionality and has brought back positive results.

This computer system can be used in medicine, agriculture and industry to help improve efficiency, treating and diagnosing diseases and improving our lives on a day to day basis.

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ADDITION

This is the program code for Bachelors.cs:

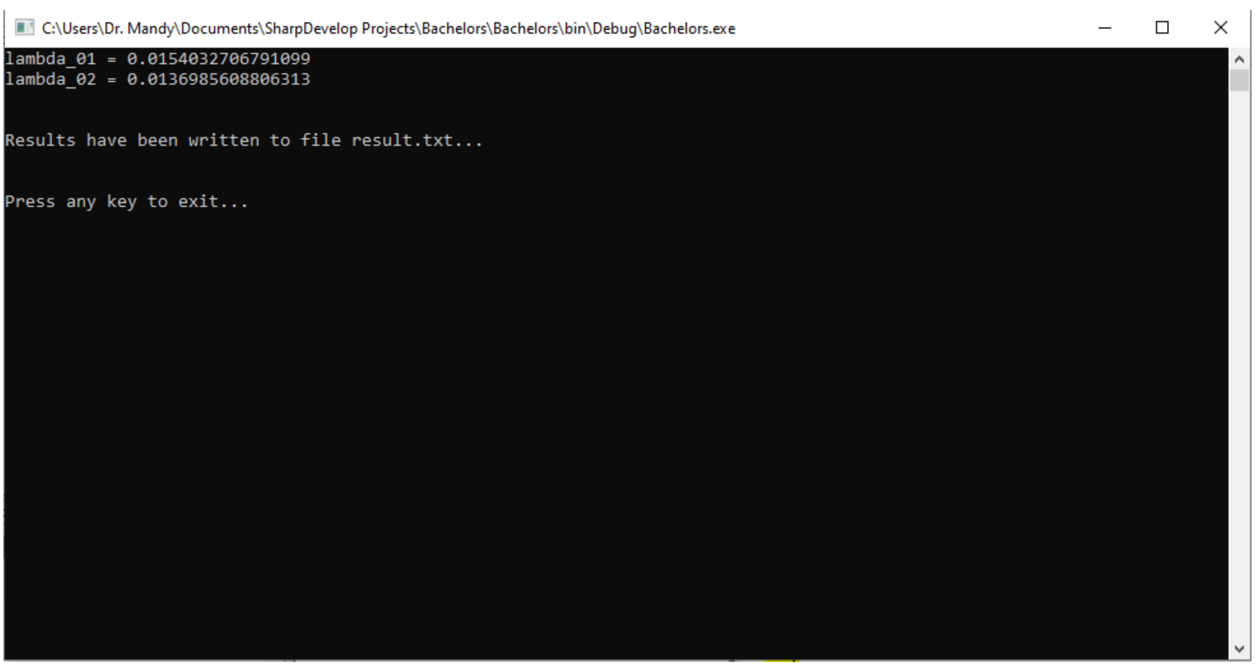
```
using System;
using System.Collections.Generic;
using System.IO;
using System.Text;

namespace Bachelors
{
   public class Isotope
   {
      public double m_01 = 200;
      public double m_02 = 1;
      public double T1 = 45;
      public double T2 = 50.6;
      public double lambda_01;
      public double lambda_02;

      public double a1(double x) {
         double v;
         v = m_01 * Math.Exp(-lambda_01 * x);
         return v;
      }

      public double a11(double x) {
```

```
    double v;
    v = -lambda_01 * m_01 * Math.Exp(-lambda_01 * x);
    return v;
}

public double a12(double x) {
    double v;
    v = lambda_01 * lambda_01 * m_01 * Math.Exp(-lambda_01 * x);
    return v;
}

public double a2(double x) {
    double v;
    v = m_02 * Math.Exp(-lambda_02 * x);
    return v;
}

public double a21(double x) {
    double v;
    v = -lambda_02 * m_02 * Math.Exp(-lambda_02 * x);
    return v;
}

public double a22(double x) {
    double v;
    v = lambda_02 * lambda_02 * m_02 * Math.Exp(-lambda_02 * x);
    return v;
```

```
        }

        static void Main(string[] args)
        {
            double t;
            double t0 = 0;
            double dt = 4;
            double tk = 320;
            double disp1;
            double disp2;
            double v1;
            double v2;
            Isotope ob1 = new Isotope();

            StreamWriter data = new StreamWriter("C:\\Users\\Dr.
Mandy\\Desktop\\Project pictures\\result.txt");
            ob1.lambda_01 = Math.Log(2)/ob1.T1;
            ob1.lambda_02 = Math.Log(2)/ob1.T2;

            Console.WriteLine("lambda_01 = {0}\nlambda_02 =
{1}", ob1.lambda_01, ob1.lambda_02);
            for (t = t0; t <= tk; t += dt)
            {
                disp1 = ob1.a1(t) / ob1.a11(t) - ob1.a11(t) / ob1.a12(t);
                disp2
= (ob1.a1(t) + ob1.a2(t)) / (ob1.a11(t) + ob1.a21(t)) - (ob1.a11(t) + ob1.a21(t)) / (o
b1.a12(t) + ob1.a22(t));
```

```
            v1 = ob1.a1(t) * disp1;
            v2 = (ob1.a1(t) + ob1.a2(t)) * disp2;
            data.Write("t = {0:G3} a1 = {1:G6} a2 = {2:G6} v1 = {3:G6} v2 = {4:G6}\n", t, ob1.a1(t), ob1.a2(t), v1, v2);
        }

        Console.WriteLine("\n\nResults have been written to file result.txt...");
        Console.WriteLine("\n\nPress any key to exit...");
        Console.ReadKey();
    }
  }
}
```

SCREENSHOT OF OUTPUT ON SCREEN



Picture – Screenshot of program showing lambda 1 & 2 values.