Journal of Nano- and Electronic Physics
Vol. **13** No 3, 03022(5pp) (2021)

Журнал нано- та електронної фізики
Том **13** № 3, 03022(5cc) (2021)

Bianchi Type VI Inflationary Cosmological Model with Massive String Source in General Relativity

Laxmi Poonia, Sanjay Sharma*, Sunil Kumawat

Department of Mathematics and Statistics, Manipal University Jaipur, Rajasthan 303007, India

(Received 21 March 2021; revised manuscript received 16 June 2021; published online 25 June 2021)

A massive string source inflationary cosmological model has been investigated in Bianchi type VI metric under consideration of a flat potential. The set of field equations is solved assuming that the shear tensor component is proportional to the expansion of the model, which provides $c = (ab)^k$ relation between metric coefficients, where $k \neq 1$ yields a time-dependent deceleration parameter; provides the universe that shows the transition from early phase of deceleration to a current scenario of accelerated expansion. The obtained result indicates that cosmic strings dominate in early cosmos and disappear for sufficiently late time. We have investigated energy density and tension density in the developed model. The proper volume growths with time, and the Hubble parameter is large at initial condition and tends to a finite value at large T. The kinematical and structural aspects of the model and their physical importance are also discussed.

Keywords: Bianchi type VI, Inflationary cosmological model, Massive string source, General relativity.

DOI: 10.21272/jnep.13(3).03022 PACS numbers: 98.80.Cq, 98.80.Jk, 04.20. – q

1. INTRODUCTION

In recent years, the study of cosmological model under framework of Bianchi becomes most relevant mathematical tool to understand a drastic change occurs after evolution of universe. FRW model represents the basic structure of universe, which is purely isotropic and homogeneous, but this model is not stable near singularity. Einstein's theory provides astronomical observation to analysis the foundation and growth of physical cosmos with explanation of accelerated expanding of the physical universe. The analysis of the cosmic microwave background also favored to the inflationary phenomenon in general relativity. Staronbinsky [1] discussed the early model of inflationary universe, but Guth [2] proposed the structural features of inflationary cosmology by fact it is due to the influence of false vacuum energy. The roles of Higg's field with a flat potential have importance in this discussion. Many authors [3-8] have focused on different features of inflation theory and scalar field in various aspects.

The concept of cosmic string plays a vital role to the study of formation of galactic structures in the early physical universe. It is believed that cosmic string cloud arises due to phase transition state after big bang as temperature cooled down below certain critical temperature observed in the GUT [9, 10]. It is observed symmetry of the universe is breakdown spontaneously during phase transition. This phase transition may pro-duce some vacuum domain structures like domain walls, strings cloud, and monopole. The presences of a huge network of clouds of strings in the initial universe are matches with present day astronomical observations. Cosmic string gives rises to density perturbations that cause the formation of galactic structures. String theory is a hypothetical approach in which particles are considered as a one-dimensional object. The gravitational influence on a cosmic string is important because they have possessed stress-energy and coupled to the field of gravitation. Many cosmologists [11, 12] have investigated cosmic strings, cosmological models, with different contexts in general relativity. Yadav [13] has investigated late-time acceleration in Bianchi type V string space-time. Cosmic strings under gravitation effects have been studied by Letelier [14], who suggested massive string sources are formed due to geometric string clouds with an extension of particles. Reddy [15] studied Bianchi type II model with sources of massive string along with a flat potential. Sharma [16] has constructed inhomogeneous cylindrically Bianchi type I space with a flat potential to describe the inflationary scenario of the present universe. Pradhan et al. [17] have derived locally rotation symmetric Bianchi type II under massive string source. Katore and Chopade [18] have studied Bianchi type VI inflationary space-time using a massless scalar field with a flat potential. Bali et al. [19] have derived spatially homogeneous string cosmological space-time in various manners.

In the present paper, generalized Bianchi type VI string cosmological model has been constructed with a scalar field where potential V is taken constantly. To get an inflationary solution, we assume the shear component is proportional to the expansion factor. Some results for the physical parameter are discussed. This paper is classified into the following sections. Section 2 deals with a system of nonlinear differential equations. Section 3 provides an exact solution of the set of field equations. Section 4 presents the kinematical and structural behavior of the constructed model. Section 5 includes the conclusions and discussion.

2. FIELD EQUATIONS AND THE MODEL

The homogeneous and spatially Bianchi type VI in synchronous coordinates can be described by the line element

$$ds^2 = dt^2 - a^2 e^{-2mz} dx^2 - b^2 e^{2nz} dy^2 - c^2 dz^2, \quad (2.1)$$

* sanjaysharma5416@gmail.com

The results were presented at the International Conference on Innovative Research in Renewable Energy Technologies (IRRET-2021)

where a, b, c are the metric coefficients and the function of time t only, and m is a constant.

The Lagrangian connected to the scalar field potential (φ) is given by

$$L = \int \left(R - \frac{1}{2}\varphi_{,i}\varphi_{,j}g^{ij} - V(\varphi)\right)\sqrt{-g}\ d^4x. \quad (2.2)$$

The Einstein field equation is written as

$$R_i^j - \frac{1}{2}Rg_i^j = -T_i^j. \quad (2.3)$$

(In geometrical unit, $8\pi G = c = 1$).

The energy-momentum tensor T_i^j is given by

$$T_i^j = T_{i\ (\varphi)}^j + T_{i\ (m)}^j, \quad (2.4)$$

where $T_{i\ (\varphi)}^j$ and $T_{i\ (m)}^j$ are the components of the energy momentum tensor corresponding to a massless scalar field with potential V and massive string, respectively, which can be written as

$$T_{i\ (\varphi)}^j = \varphi_{,i}\varphi_{,j} - \left(\frac{1}{2}\varphi_{,l}\varphi^{,l} + V(\varphi)\right)g_{ij},$$

$$T_{i\ (m)}^j = \rho v_i v^j - \lambda x_i x^j,$$

where ρ and λ denote the proper energy density and density of the cosmic strings cloud. In this case, we consider the direction of the string along the z-axis.

Here $v^i = (0,0,0,1)$ and $x^i = \left(0,0,\frac{1}{c},0\right)$, so that $v_i v^j = -x_i x^j = 1$ and $v_i x^j = 0$; ρ_0 denotes the energy density of particles attached to the strings and is given by

$$\rho = \rho_0 + \lambda, \quad (2.5)$$

other symbols have their usual meaning.

The equation of energy conservation is as follows:

$$\varphi_{,i}^i = -\frac{dV}{d\varphi}, \quad (2.6)$$

$$\rho_4 + \left(\frac{a_4}{a} + \frac{b_4}{b} + \frac{c_4}{c}\right)\rho + \lambda(n-m) - \lambda\left(\frac{c_4}{c}\right) = 0. \quad (2.7)$$

Einstein field equation (2.3) for metric (2.1) is written as

$$-\frac{b_{44}}{b} - \frac{c_{44}}{c} + \frac{n^2}{c^2} - \frac{b_4c_4}{bc} = \frac{1}{2}\varphi_4^2 + V(\varphi), \quad (2.8)$$

$$-\frac{a_{44}}{a} - \frac{c_{44}}{c} + \frac{m^2}{c^2} - \frac{a_4c_4}{ac} = \frac{1}{2}\varphi_4^2 + V(\varphi), \quad (2.9)$$

$$-\frac{b_{44}}{b} - \frac{a_{44}}{a} - \frac{mn}{c^2} - \frac{a_4b_4}{ab} = -\lambda + \frac{1}{2}\varphi_4^2 + V(\varphi), \quad (2.10)$$

$$\frac{a_4b_4}{ab} + \frac{b_4c_4}{bc} + \frac{a_4c_4}{ac} + \frac{mn-(m^2+n^2)}{c^2} - \frac{b_4c_4}{bc} = -\rho - \frac{1}{2}\varphi_4^2 + V(\varphi), \quad (2.11)$$

$$m\left(\frac{c_4}{c} - \frac{a_4}{a}\right) + \left(\frac{b_4}{b} - \frac{c_4}{c}\right) = 0. \quad (2.12)$$

From equation (2.6) we have

$$\varphi_{44} + \left(\frac{a_4}{a} + \frac{b_4}{b} + \frac{c_4}{c}\right)\varphi_4 = -\frac{dV}{d\varphi}. \quad (2.13)$$

The physical parameters of the developed model are mostly used to find the solutions of the nonlinear differential equations and discuss features.

The proper volume of the model is given by

$$V = \sqrt{-g} = abc\ exp\big((n-m)z\big),$$

the expansion scalar of the model is written as

$$\theta = v_{,i}^i = \frac{a_4}{a} + \frac{b_4}{b} + \frac{c_4}{c},$$

the shear scalar for the model is the following:

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij},$$

where

$$\sigma_{ij} = \frac{1}{2}\left(v_{i,k}D_j^k + v_{j,k}D_i^k\right) - \frac{1}{3}\theta\left(g_{ij} + v_i v_j\right),$$

$$D_i^j = \delta_i^j - v^i v_j.$$

The components of the shear tensor are given by

$$\sigma_1^1 = \frac{1}{3}\left(2\frac{a_4}{a} - \frac{b_4}{b} - \frac{c_4}{c}\right),$$

$$\sigma_2^2 = \frac{1}{3}\left(2\frac{b_4}{b} - \frac{a_4}{a} - \frac{c_4}{c}\right),$$

$$\sigma_3^3 = \frac{1}{3}\left(2\frac{c_4}{c} - \frac{a_4}{a} - \frac{b_4}{b}\right),$$

$$\sigma_4^4 = 0$$

that provides

$$\sigma = \frac{1}{\sqrt{3}}\left[\left(\frac{a_4}{a}\right)^2 + \left(\frac{b_4}{b}\right)^2 + \left(\frac{c_4}{c}\right)^2 - \frac{a_4b_4}{ab} - \frac{b_4c_4}{bc} - \frac{a_4c_4}{ac}\right]^{\frac{1}{2}}.$$

We obtain the deceleration parameter by the given relation

$$q = -\frac{R_{44}/R}{{R_4}^2/R^2}.$$

The Hubble parameter is given by

$$H = \frac{1}{3}\left(\frac{a_4}{a} + \frac{b_4}{b} + \frac{c_4}{c}\right).$$

3. SOLUTION OF THE FIELD EQUATIONS

In order to obtain inflationary solution of equations (2.8)-(2.11), we consider a flat region, for which the potential V is constant i.e.

$$V(\varphi) = \beta(\text{constant}). \quad (3.1)$$

The system of the field equations is given as

$$-\frac{b_{44}}{b} - \frac{c_{44}}{c} + \frac{n^2}{c^2} - \frac{b_4c_4}{bc} = \frac{1}{2}\varphi_4^2 + \beta, \quad (3.2)$$

$$-\frac{a_{44}}{a} - \frac{c_{44}}{c} + \frac{m^2}{c^2} - \frac{a_4c_4}{ac} = \frac{1}{2}\varphi_4^2 + \beta, \quad (3.3)$$

$$-\frac{b_{44}}{b} - \frac{a_{44}}{a} - \frac{mn}{c^2} - \frac{a_4b_4}{ab} = -\lambda + \frac{1}{2}\varphi_4^2 + \beta, \quad (3.4)$$

$$\frac{a_4b_4}{ab} + \frac{b_4c_4}{bc} + \frac{a_4c_4}{ac} + \frac{mn-(m^2+n^2)}{c^2} - \frac{b_4c_4}{bc} = -\rho - \frac{1}{2}\varphi_4^2 + \beta, \quad (3.5)$$

$$m\left(\frac{c_4}{c} - \frac{a_4}{a}\right) + \left(\frac{b_4}{b} - \frac{c_4}{c}\right) = 0. \quad (3.6)$$

Now equations (3.2)-(3.6) are independent with six unknowns. For this purpose, we need one extra physical condition: since the component of shear σ_3^3 is proportional to the scalar expansion (θ) of the universe, this leads to a relation between matric coefficients

$$\frac{1}{3}\left(2\frac{c_4}{c}-\frac{a_4}{a}-\frac{b_4}{b}\right)=\gamma\left(\frac{a_4}{a}+\frac{b_4}{b}+\frac{c_4}{c}\right)$$

with

$$c=(ab)^k, \tag{3.7}$$

where $k=\frac{3\gamma+1}{2-3\gamma}$.

From equations (3.6) and (3.7)

$$b=k_2\,a^{k_1}, \tag{3.8}$$

where k_1 and k_2 are the constants.

From equations (2.13) and (3.1)

$$\varphi_{44}+\left(\frac{a_4}{a}+\frac{b_4}{b}+\frac{c_4}{c}\right)\varphi_4=0. \tag{3.9}$$

Equations (3.2) and (3.3) lead to

$$\frac{a_{44}}{a}-\frac{b_{44}}{b}+\frac{a_4c_4}{ac}+\frac{n^2-m^2}{c^2}-\frac{b_4c_4}{bc}=0. \tag{3.10}$$

Using equations (3.7) and (3.8), we get

$$a_{44}+\left(k_1+k(1+k_1)\right)\frac{{a_4}^2}{a}=\alpha a^{-2k(1+k_1)+1}, \tag{3.11}$$

where $\alpha=\frac{m^2-n^2}{{k_2}^{2k(1-k_1)}}$.

We take $a_4=f(a)$. Equation (3.11) provides

$$f=\sqrt{\frac{\alpha}{1+k_1}}\,a^{-k(1+k_1)+1}, \tag{3.12}$$

which leads to

$$a=\left[k\sqrt{\alpha(1+k_1)}\,t+k_3\right]^{\frac{1}{k(1+k_1)}}, \tag{3.13}$$

where k_3 is the constant of integration.

Equations (3.7) and (3.8) give

$$b=k_2\left[k\sqrt{\alpha(1+k_1)}\,t+k_3\right]^{\frac{k_1}{k(1+k_1)}},$$

$$c={k_2}^k\left[k\sqrt{\alpha(1+k_1)}\,t+k_3\right].$$

The line element (2.1) can be obtained as

$$ds^2=dt^2-T^{\frac{2}{k(1+k_1)}}e^{-2mz}dx^2-{k_2}^2T^{\frac{2k_1}{k(1+k_1)}}e^{2nz}dy^2-{k_2}^{2k}T^2dz^2,$$

where $T=k\sqrt{\alpha(1+k_1)}\,t+k_3$.

4. PHYSICAL AND GEOMETRICAL FEATURES

The expansion (θ) for the model is given by

$$\theta=(1+k)\sqrt{\alpha(1+k_1)}\frac{1}{T}. \tag{4.1}$$

The shear scalar (σ^2) is written as

$$\sigma^2=\frac{1}{2}[(\sigma_1^1)^2+(\sigma_2^2)^2+(\sigma_3^3)^2+(\sigma_4^4)^2],$$

$$\sigma^2=\frac{1}{6}\left[3\left(1+{k_1}^2\right)+(2k^2-2k-1)(1+k_1)^2\right]\times\left(\frac{\alpha}{1+k_1}\right)\frac{1}{T^2}, \tag{4.2}$$

where the components of the stress are presented as

$$\sigma_1^1=\frac{1}{3}[3-(1+k)(1+k_1)]\sqrt{\frac{\alpha}{1+k_1}}\frac{1}{T},$$

$$\sigma_2^2=\frac{1}{3}[3k_1-(1+k)(1+k_1)]\sqrt{\frac{\alpha}{1+k_1}}\frac{1}{T},$$

$$\sigma_3^3=\frac{1}{3}[(2k-1)(1+k_1)]\sqrt{\frac{\alpha}{1+k_1}}\frac{1}{T}.$$

The deceleration factor for the model is given by

$$q=-1+k\sqrt{\alpha(1+k_1)}\frac{1}{T}. \tag{4.3}$$

The Hubble parameter for the model is written as

$$H=\frac{1}{3}(1+k)\sqrt{\alpha(1+k_1)}\frac{1}{T}. \tag{4.4}$$

From equation (3.9), we obtain

$$\varphi=-\frac{\mu k}{{k_2}^{1+k}}\frac{1}{T^{\frac{1}{k}}}+C_1. \tag{4.5}$$

where μ and C_1 are the constants of integration.

From equations (4.1) and (4.2), we observe that the expansion and the shear scalar in the model are decreasing functions of time. This shows that the universe starts with infinite expansion and infinitely large shear for $k>0$ and approaches to null value with growth of the cosmic time as shown in Fig. 1 and Fig. 2.

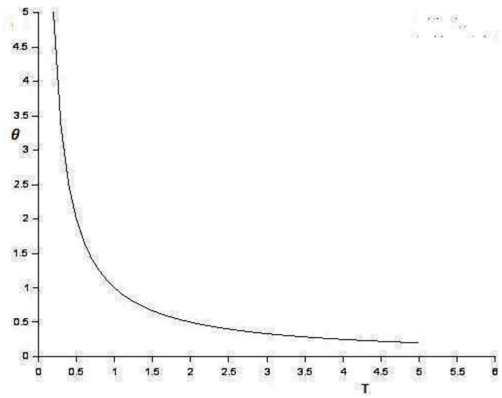

Fig. 1 – Expansion (θ) versus time (T)

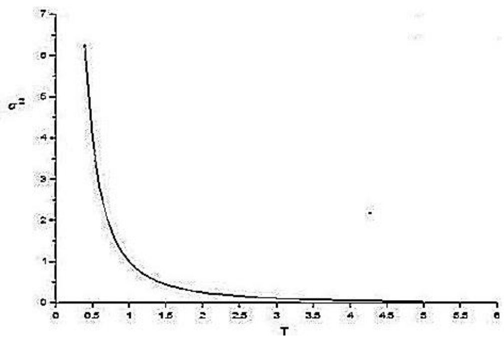

Fig. 2 – Shear scalar (σ^2) versus time (T)

The deceleration parameter becomes negative for infinite large time leads to the de Sitter universe. The model represents the accelerating phase in the current scenario.

The Hubble parameter is also approaching zero for sufficiently large T. The rate of Higg's field decreases, but the universe expands due to the vacuum field energy.

The string tension density of the model is given by

$$\lambda = \left[k(1-k) + \frac{mn+n^2}{{k_2}^{2k}} - k_1\alpha k\right]\frac{1}{T^2}, \qquad (4.6)$$

where m, n are non-negative constants.

The energy density (ρ) for the model is written as

$$\rho = \left[\alpha k_1 \frac{(k_1-1)}{(1+k_1)} - k\alpha(1+k_1) + \frac{m^2-mn}{{k_2}^{2k}}\right]\frac{1}{T^2} + 2\beta. \quad (4.7)$$

The particle density for the model is given by

$$\rho_0 = \left[\alpha k_1 \frac{(k_1-1)}{(1+k_1)} - k(\alpha - k + 1) + \frac{m^2-2mn-n^2}{{k_2}^{2k}}\right]\frac{1}{T^2} + 2\beta. (4.8)$$

The proper volume for the model is represented as

$$V = {k_2}^{k+1} T^{\frac{k+1}{k}} e^{(n-m)z}. \qquad (4.9)$$

From equation (4.6), we observe that the string tension is a decreasing function of time, i.e., becomes divergent at initially epoch and disappears for large T as shown in Fig. 3.

Energy density and particle density of cosmic strings diverge for $T = 0$ and approach toward constant value for infinitely large T as shown in Fig. 4 and Fig. 5. Since $\rho_0/\lambda > 1$ for large T, this means particles dominate the string at late time which agreed with the fact of disappearances of cosmic strings in the present-day observations.

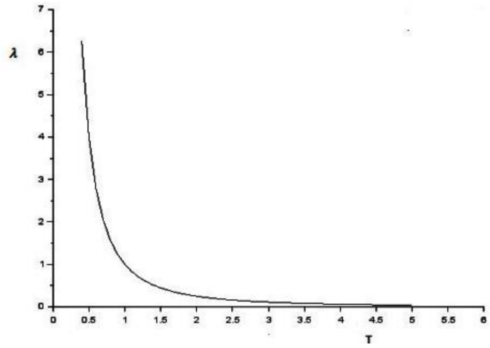

Fig. 3 – The string tension density (λ) versus time (T)

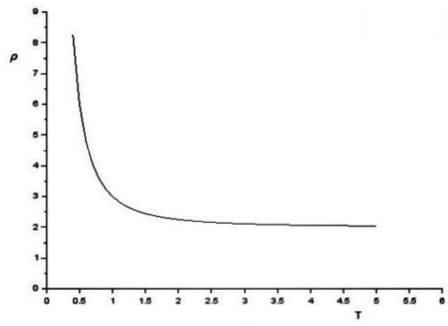

Fig. 4 – The proper energy density versus time ($\beta = 1$)

The proper volume increases with time which indicates accelerated expansion of the universe as shown in Fig. 6.

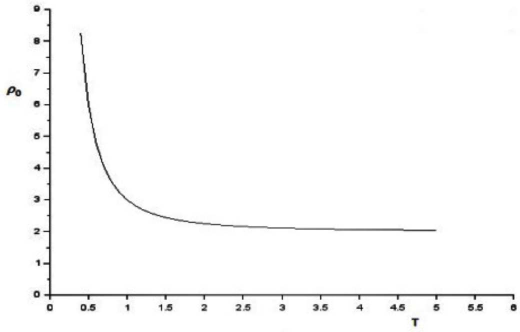

Fig. 5 – The particle density versus time ($\beta = 1$)

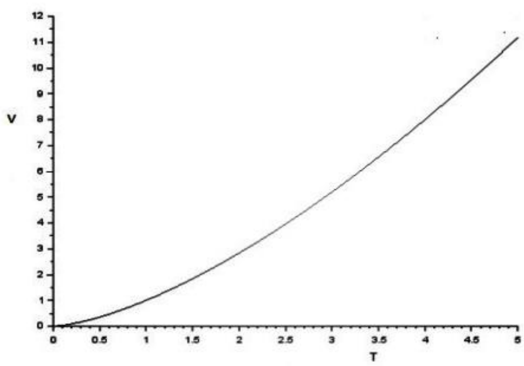

Fig. 6 – The volume versus time

5. CONCLUSIONS AND DISCUSSION

This model represents an inflationary universe in string cosmology where scalar field φ is minimally coupled to gravitation field. The expansion (θ) in the model decreases as T increases and stops as $T \to \infty$. The model starts with infinite shear and tends to zero as T tends to infinity. Deceleration parameter becomes negative as $T \to \infty$ represents an accelerating phase of the model and represents de Sitter universe. DP is time-dependent, so the universe is decelerating in past and an accelerating in the current scenario. The model starts with big bang initially. Proper volume increases with time represent an inflationary scenario in mode. Hubble parameter and string tension become diverge at the initial epoch. Scalar field decreases slowly and becomes constant at late time. The model has no initial singularity. Since σ/θ is constant, the model does not approach isotropy at late time. The decelerating expansions support the formation of the cosmic structure. The particle density and energy density of the cosmic strings diverge at $T = 0$ and become constant for an infinitely large time at late. Cosmic strings dominate over the early universe and become disappeared for sufficiently late time.

REFERENCES

1. A.A. Starobinsky, *Phys. Lett. B* **91** No 1, 99 (1980).
2. A.H. Guth, *Phys. Lett. B* **91**, 99 (1980).
3. A.B. Albrecht, P.J. Steinhardt, *Phys. Rev. Lett.* **4**, 1220 (1982).
4. D. La, P.J. Steinhardt, *Phys. Rev. Lett.* **62**, 376 (1981).
5. R. Wald, *Phys. Rev. D* **28**, 2118 (1983).
6. G.F.R. Ellis, M.S. Madsen, *Class. Quantum Gravitity* **8**, 667 (1991).
7. L. Poonia, S. Sharma, *Annal. Roman. Soc. Cell Biol.* **25** No 2, 1223 (2021).
8. S. Sharma, L. Poonia, *Adv. Math. Sci. J.* **10** No 1, 527 (2021).
9. Ya.B. Zeldovich, *Mon. Not. Roy. Astron. Soc.* **192** No 4,663 (1980).
10. T.W.B. Kibble, *J. Phys. A* **9**, 1387 (1976).
11. R.K. Dubey et al., *Phys. Astron. Int. J.* **2** No 2, 143 (2018).
12. S. Sharma, L. Poonia, *Iop. Conf. Ser. Mater. Sci. Eng.* **1045**, 012025 (2021).
13. A.K. Yadav, *Res. Astron. Astrophys.* **12**, 1467 (2012).
14. P. Letelier, *Phys. Rev. D* **20**, 1294 (1979).
15. D.R.K. Reddy, *Prespacetime J.* **9** No 5, 432 (2018).
16. L. Poonia, S. Sharma, *J. Phys. Conf. Ser.* **1706**, 1742 (2020).
17. A. Pradhan et al., *Fizika B* **18**, 35 (2009).
18. S.D. Katore B.B. Chopade, *Prespacetime J.* **3**, 474 (2012).
19. R. Bali, R.D. Upadhyay, *Astro. Phys. Space Sci.* **288**, 97 (2003).

Інфляційна космологічна модель Б'янкі типу VI з масивним джерелом струн у загальній теорії відносності

Laxmi Poonia, Sanjay Sharma, Sunil Kumawat

Department of Mathematics and Statistics, Manipal University Jaipur, Rajasthan 303007, India

Інфляційна космологічна модель з масивним джерелом струн досліджується в метриці Б'янкі типу VI з урахуванням плоского потенціалу. Система рівнянь поля вирішується в припущенні, що компонента тензора зсуву пропорційна розширенню моделі, забезпечуючи співвідношення $c = (ab)^k$ між метричними коефіцієнтами, де $k \neq 1$ дає залежний від часу параметр уповільнення. Розроблена модель Всесвіту, яка показує перехід від ранньої фази уповільнення до сучасного сценарію прискореного розширення. Отриманий результат вказує на те, що космічні струни домінують у ранньому космосі і зникають на досить тривалий час. Досліджено щільності енергії та натягу в розробленій моделі. Належний об'єм зростає з часом, а параметр Хаббла є великим у початкових умовах і має тенденцію до кінцевого значення при великій T. Також обговорюються кінематичні та структурні аспекти моделі та їх фізичне значення.

Ключові слова: Модель Б'янкі типу VI, Інфляційна космологічна модель, Масивне джерело струн, Загальна теорія відносності.