Parameter Identification of Nonlinear Bearing Stiffness for Turbopump Units of Liquid Rocket Engines Considering Initial Gaps and Axial Preloading

Verbovyi A.[1*][0000-0002-7805-4733], Khomenko V.[1], Neamtu C.[2][0000-0003-0899-0451], Pavlenko V.[1], Cherednyk M.[1], Vashyst B.[1], Pavlenko I.[1][0000-0002-6136-1040]

[1] Sumy State University, 2 Rymskogo-Korsakova St., 40007 Sumy, Ukraine;
[2] Technical University of Cluj-Napoca, 28 Memorandumului St., 400114 Cluj-Napoca, Romania

Abstract. This article is devoted to developing a mathematical model of nonlinear bearing supports for turbopump units of liquid rocket engines considering initial gaps and axial preloading. In addition to the radial stiffness of the bearing support, this model also considers the stiffness of the bearing cage, the rotational speed of the rotor, axial preloading of the rotor (due to which the inner cage shifts relative to the outer, changing the radial stiffness of the support), as well as radial gaps between contact elements of the bearings. This model makes it possible to calculate the stiffness of the bearing supports more accurately. The proposed model is realized using both the linear regression procedure and artificial neural networks. The model's reliability is substantiated by the relatively small discrepancy of the obtained evaluation results with the experimental data. As a result, this model will allow determining the critical frequencies of the rotor with greater accuracy. The results have been implemented within the experience of designing turbopump units for State Company "Yuzhnoye Design Office".

Keywords: bearing support, axial force, radial gap, regression analysis, artificial neural networks.

1 Introduction

One of the reasons for the deterioration of the vibration reliability of pumping units is increased vibrations due to the entry of the machine into resonance, which in turn arises mainly from the coincidence of the operating speed of the rotor with its eigenfrequency.

The radial stiffness of the bearing supports directly affects the natural frequencies of the rotor. At the same time, the bearing housings in which the clips lie have their own stiffness. Thus, the rotor support is a system of two consecutive stiffnesses.

In addition, the stiffness of the bearing is affected by the degree of axial compression of the rotor and the frequency of its rotation, as well as the gap between contact parts.

Therefore, the purpose of this work is to develop an approach for determining the equivalent stiffness of bearing supports considering initial gaps and axial preloading.

For achieving this aim, the following objectives have been formulated. Firstly, a mathematical model of

nonlinear bearing stiffness should be proposed considering axial preloading and radial gaps in contact parts.

Secondly, the regression dependencies for the identification of unknown coefficients should be achieved.

Finally, the proposed approach should be substantiated by comparison with the experimental data.

2 Literature Review

To ensure the vibration reliability of the turbopump rotor, it is necessary to turn to the already existing scientific publications devoted to studying individual influences on the dynamics of the rotor of any of the previously mentioned factors.

Particularly, Sharma et al. [1] presented a review of different models in the analysis of the nonlinear behavior of rotor systems. Xu et al. [2] made an overview of bearings for the next generation of reusable liquid rocket turbopumps.

Bai et al. [3] studied the impact of flexible support stiffness on rotor systems' dynamic characteristics and stability. Xu et al. [4] investigated the impact of bearing

stiffness on the nonlinear dynamics of shafts in drive systems.

Du and Liang [5] evaluated the dynamic performance of hydrostatic bearings. As a result, a better understanding of the performance of water-lubricated hydrostatic bearings was provided. Cao [6] carried out a transient analysis of flexible rotors with nonlinear bearings.

Study [7] describes rotor dynamics of the rotation of moving parts and the compliance of bearing support elements. This article also describes an approach to studying nonlinear reactions in rotor supports. The design model of the rotating rotor considers the rotation of the shaft, which manifests itself in the form of centrifugal inertia forces acting on the inner cage of the rolling bearing. After numerical simulation in ANSYS, diagrams of the dependence of the movements of the inner cage on the force applied to it in the radial direction were constructed from the calculated points. Thus, the authors of this article propose a more advanced method for determining the nonlinear stiffness characteristics of bearing supports.

In the article [8], using the example of the oxidizer rotor of a liquid rocket engine, it is shown how to consider the gaps in the rolling bearing, as well as the axial compression of the shaft when modeling the dynamics of the rotor, which directly affects the rigidity of rolling bearings due to the displacement of the inner cage relative to the outer one. The simulation was carried out using the multi-purpose ANSYS software package. In work, contact spots were obtained between the rolling elements, the contact angle was determined. The authors propose nonlinear stiffness as the tangent of the angle of inclination of the curve diagram of the relationship between radial load and radial displacement. Thus, the "radial load – radial displacement" graph was constructed according to the calculated points because of numerical simulation. Also, because of approximation, analytical expressions describing this dependence were determined.

The article [9] is devoted to a general approach to the use of neural networks to determine the parameters that affect the dynamics of the rotor. The neural networks described by the authors allow us to consider the nonlinear regression dependences of the stiffness of radial bearings on the speed of rotation of the rotor. Thus, it was possible to develop algorithms for identifying the stiffness of bearings based on previously known critical frequencies of the system.

The work described in the article [10] is devoted to considering such factors as the nonlinear weakening of the support during operation and the nonlinearity of the stiffness of the support.

Finally, numerical methods for calculating the stiffness of elements of arbitrary shape are described in [11, 12].

Overall, the described approaches to the calculation process in this article allow for modal analysis of the system, harmonic, and allow for virtual balancing without the need to have a significant time to prepare the source data and allow you to significantly reduce the machine time of the account, having an insignificant loss an accuracy. Nevertheless, a comprehensive approach to determining the equivalent stiffness of bearing supports should be developed to eliminate recent publications' advantages.

3 Research Methodology

3.1 A regression model of the bearing stiffness

Considering an experience of designing turbopump units ordered by the State Company "Yuzhnoye Design Office" (Dnipro, Ukraine), the equivalent bearing stiffness can be described by the following dependence:

$$c(\omega, r, F_a) = c_0 + \alpha\omega^2 + \beta r + \gamma F_a, \qquad (1)$$

where c_0 – initial stiffness, N/m; ω – rotational speed, rad/s; r – radial displacement of a rotor, m; F_a – axial preloading of a rotor, N; α, β, γ – unknown coefficients, N·s^2/m, N/m^2, and m^{-1}, respectively.

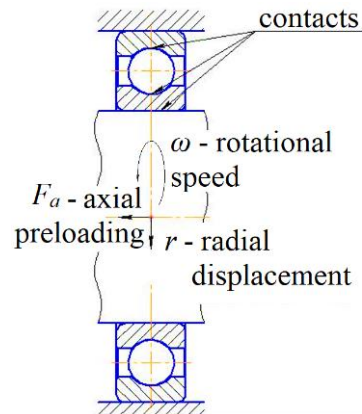The corresponding design scheme is presented in Figure 1.

Figure 1 – The design scheme of the bearing support

This nonlinear model significantly improves the quasilinear model proposed previously in [8]. Its identification is now based on evaluating unknown parameters α, β, and γ.

For finding these coefficients, they can be presented as independent variables of the following error function $R(\alpha, \beta, \gamma)$, compiled according to the least-squares method:

$$R(\alpha, \beta, \gamma) = \sum_{i=1}^{n}(c_0 + \alpha\omega_i^2 + \beta r_i + \gamma F_{ai} - c_i)^2, \quad (2)$$

where i – a current number of experimental points; n – the total number of experimental points.

The value of this error function should be minimal ($R \rightarrow \min$). In this case, the following conditions can be written:

$$\begin{cases} \frac{\partial R}{\partial \alpha} = 2\sum_{i=1}^{n}(c_0 + \alpha\omega_i^2 + \beta r_i + \gamma F_{ai} - c_i)\omega_i^2 = 0; \\ \frac{\partial R}{\partial \beta} = 2\sum_{i=1}^{n}(c_0 + \alpha\omega_i^2 + \beta r_i + \gamma F_{ai} - c_i)x_i = 0; \\ \frac{\partial R}{\partial \gamma} = 2\sum_{i=1}^{n}(c_0 + \alpha\omega_i^2 + \beta r_i + \gamma F_{ai} - c_i)y_i = 0, \end{cases} \quad (3)$$

that can be reduced to the following linear equation:

$$[K]\{A\} = \{C\}, \qquad (4)$$

where $[K]$ – symmetrical quadratic 3×3 matrix; $\{C\}$ – 3×1 column-vector; $\{A\} = \{\alpha, \beta, \gamma\}^T$ – 3×1 column-vector of evaluated parameters.

Elements of quadratic matrix $[K]$ and column-vector $[C]$ take the following forms:

$$K_{11} = \sum_{i=1}^{n} \omega_i^4; \; K_{12} = K_{21} = \sum_{i=1}^{n} \omega_i^2 r_i;$$
$$K_{13} = K_{31} = \sum_{i=1}^{n} \omega_i^2 F_a; \; K_{22} = \sum_{i=1}^{n} r_i^2; \quad (5)$$
$$K_{23} = K_{32} = \sum_{i=1}^{n} r_i F_a; \; K_{33} = \sum_{i=1}^{n} F_{ai}^2;$$

$$C_1 = \sum_{i=1}^{n} (c_i - c_0)\omega_i^2; \; C_2 = \sum_{i=1}^{n} (c_i - c_0)r_i; \quad (6)$$
$$C_3 = \sum_{i=1}^{n} (c_i - c_0)F_{ai}.$$

Based on the transverse matrix approach, the column-vector of the evaluated parameters is determined as follows:

$$\{A\} = [K]^{-1}\{C\}. \quad (7)$$

Then, after determining the obtained coefficients α, β, and γ and substituting them into expression (1), and analytical dependence of the equivalent stiffness can be obtained considering the rotational speed of the rotor, radial gaps in the bearing support, and axial preloading.

3.2 The use of artificial neural networks

The above-described algorithm for determining equivalent stiffness assumes the linear bearing stiffness model. However, artificial neural networks can be applied if a linear mathematical model is not suitable (especially if there are more factors affecting the equivalent stiffness). The corresponding comprehensive approach for such modeling is presented in [9].

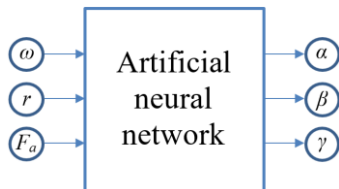Particularly, for the considered case study, an artificial neural network architecture is presented in Figure 2.

Figure 2 – An architecture of the artificial neural network

4 Results and Discussion

The design scheme of the rotor of the oxidizer turbopump of the liquid rotor engine is presented in Figure 3 [7].
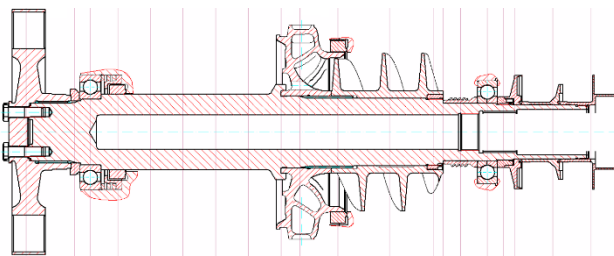

Figure 3 – An architecture of the artificial neural network

For this case study, the experimental results data is presented in Table 1 for bearings of types 45-216 and 45-276214.

Table 1 – Experimental data for different types of bearings

i	ω_i, rad/s	r_i, 10^{-6} m	F_a, 10^3 N	Bearing stiffness, 10^8 N/m	
				45-216	45-276214
1	0			1.88	2.10
2	1100	0	0	1.90	2.10
3	1963			1.93	2.12
4	2215			1.94	2.13
5	0			2.02	2.25
6	1100	95	4	2.04	2.26
7	1963			2.07	2.27
8	2215			2.08	2.27

As a result of numerical calculation, the following values for elements of quadratic matrix (5) have been obtained:

$$K_{11} = 8.077 \cdot 10^{13}; \; K_{12} = K_{21} = 9.471 \cdot 10^2;\cdot$$

$$K_{13} = K_{31} = 3.988 \cdot 10^{10};\cdot K_{22} = 3.610 \cdot 10^{-8};$$

$$K_{23} = K_{32} = 1.520;\cdot K_{33} = 6.400 \cdot 10^7.$$

Additionally, for the bearing 45-216, the column-vector (6) is as follows:

$$C_1 = 2.418 \cdot 10^{14}; \; C_2 = 6.555 \cdot 10^3; \; C_3 = 2.760 \cdot 10^3,$$

and for the bearing 45-276214:

$$C_1 = 1.931 \cdot 10^{14}; \; C_2 = 6.080 \cdot 10^3; \; C_3 = 2.560 \cdot 10^3.$$

After considering the regression dependence (7), the unknown parameters α, β, and γ have been evaluated and summarized in Table 2.

Table 2 – The evaluated parameters

Bearing	α, N·s^2/m	β, N/m^2	γ, m^{-1}	Maximum error, %
45-216	0.72	$1.37 \cdot 10^{11}$	4100	7.0
45-276214	0.05	$1.37 \cdot 10^{11}$	0	1.8

Comparison of the data presented in Table 1 with the proposed mathematical model (1) for the data presented in Table 2 substantiates the reliability of the proposed approach. Particularly, the relative error of the parameter identification does not exceed 7 % for the bearing 56-216 and 2 % for the bearing 45-276214.

5 Conclusions

In this article, the analysis of existing studies on the influence of various factors on the equivalent stiffness of rolling bearings and various methods of considering these factors when developing a reliable mathematical model was carried out.

Based on these studies, an algorithm for evaluating the equivalent stiffness has been developed. The proposed model considers the impact of the rotational speed of the shaft, radial gaps, and axial preloading of the shaft.

The reliability of the proposed method is proved by the fact that the relative error of the parameter identification

does not exceed 7 % for the bearing 56-216 and 2 % for the bearing 45-276214.

In the case of large nonlinearities (e.g., for the bearing 56-216), a different approach based on artificial neural networks has allowed developing a reliable mathematical model considering other factors affecting the equivalent stiffness of the bearing support instead of determining the coefficients of influence.

6 Acknowledgments

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