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# Influence of Mechanical and Geometric Characteristics on Thermal Buckling of Functionally Graded Sandwich Plates

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Functionally graded materials (FGM) are a new range of composite materials having a gradual and continuous variation of the volume fractions of each of the constituents (in general, metal and ceramic) in thickness, which accordingly causes changes in the overall thermomechanical properties of the structural elements they constitute. The interest of this work is the use of a high-order plate theory for the study of thermal buckling of FGM plates resting on Winkler-Pasternak type elastic foundation. The present method leads to a system of differential equations, where the number of unknowns is five. The material properties of FGM plate such as Young's modulus and coefficient of thermal expansion are assumed to be variable through the thickness according to the Mori-Tanaka distribution model. The thermal loading is assumed to be uniform, linear and nonlinear through the thickness of the plate. A parametric study is thus developed to see the influence of the geometric and mechanical characteristics, in particular, the geometric ratio (*a*/*b*), thickness ratio (*a*/*h*) and the material index (*k*), as well as the impact of the Winkler and Pasternak parameters on the critical buckling load.

**Keywords:** Functionally graded materials (FGM), High order theory, Mori-Tanaka model, Elastic foundation, Thermal buckling.

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## 1. INTRODUCTION

Functional graded materials (FGM) are a class of composites in which the material properties change gradually in one or more Cartesian directions [1]. This class of composite materials has attracted considerable attention in the engineering community, especially in high temperature applications such as nuclear reactors, aerospace and energy industries [2]. Over the past two decades, many research reports have been published on thermal stress, failure, thermomechanical response, buckling, free vibration, etc. of structural elements in FGM [3].

Due to the importance and wide technical applications of FGM structures, they have been addressed by many researchers [4]. Reddy [5] studied the third-order plate shear deformation theory (TSDT). Touratier [6] elaborated the sinusoidal shear plate strain theory (SSDT), and Karama [7] developed the exponential shear plate strain theory (ESDT). Sankar and Tzeng [8] obtained an elasticity solution for FGM beams with exponential variation of properties subjected to transverse loads. Kapania and Raciti [9] provided a detailed review of shear deformation theories used for static, vibration and buckling analysis of beams and plates.

## 2. GEOMETRIES AND MATERIALS

Consider an FGM plate of thickness $h$, length $a$, and width $b$, mentioned with respect to the rectangular Cartesian coordinates ($x$, $y$, $z$). The $x$-$y$ plane is taken to be the mid-plane of the undeformed plate plane and the $z$-axis is perpendicular to the $x$-$y$ plane (Fig. 1).

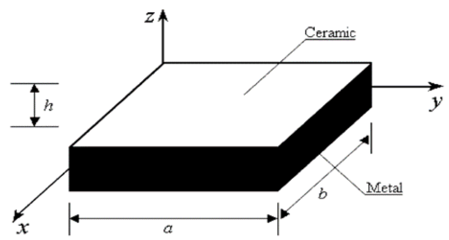


**Fig. 1** – FGM plate geometry

According to the Mori-Tanaka homogenization scheme, the material properties of the FGM plate are given by:

$$E(z)=\frac{E_m+(E_c-E_m)V_c}{1+(1-V_c)(E_c/E_m-1)(1+\nu)/(3-3\nu)},$$
$$\alpha(z)=\frac{\alpha_m+(\alpha_c-\alpha_m)V_c}{1+(1-V_c)(\alpha_c/\alpha_m-1)(1+\nu)/(3-3\nu)}, \tag{2.1}$$

where $E_m$ and $E_c$ are the corresponding properties of metal and ceramic, respectively, and $k$ is the material parameter. The volume fractions of the ceramic constituent $V_c$ and the metallic constituent $V_m$ can be written in the form [10]:

$$V_c=(z/h+1/2)^k,\ V_m=1-V_c. \tag{2.2}$$

### 2.1 Displacement Field

The displacements of a material point located at ($x$, $y$, $z$) in the plate can be written [11]:

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$$u(x,y,z) = u_0 - z\left(\partial w_0 / \partial x\right) + \Psi(z)\phi_1$$
$$v(x,y,z) = v_0 - z\left(\partial w_0 / \partial y\right) + \Psi(z)\phi_2 \qquad (2.3)$$
$$w(x,y,z) = w_0$$

where *u, v, w,* $\varphi_1$, and $\varphi_2$ are five unknowns of the displacements, representing a warping function that defines the variation of transverse shear strains and stresses through the thickness.

In this work, the warping function is defined by the first-order shear strain theory (FSDPT) [12]:

$$\Psi(z) = z\,. \qquad (2.4)$$

The third-order deformation plate theory (HPT) from Reddy JN (2000) is obtained as follows [13]:

$$\Psi(z) = z\left(1 - \left(4z^2/3h^2\right)\right). \qquad (2.5)$$

Zenkour's sinusoidal shear strain plate theory (SSDPT) is obtained by [12]:

$$\Psi(z) = \left(h/\pi\right)\sin\left(\pi z/h\right). \qquad (2.6)$$

The present plate shear strain theory (HPT) given as:

$$f(z) = \left[\ln\left(\pi\exp(1/20)\right) - (0.1407)^{5/4}\cosh\left(\pi z\right)\right]z\,. \qquad (2.7)$$

### 2.2 Strain Field

The strain field associated with the displacement field of equation (3.1) is given by:

$$\begin{Bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \varepsilon_{xy}\end{Bmatrix} = \begin{Bmatrix}\varepsilon^0_{xx}\\ \varepsilon^0_{yy}\\ \varepsilon^0_{xy}\end{Bmatrix} + z\begin{Bmatrix}k_{xx}\\ k_{yy}\\ k_{xy}\end{Bmatrix} + \psi(z)\begin{Bmatrix}\eta_{xx}\\ \eta_{yy}\\ \eta_{xy}\end{Bmatrix} \qquad (2.8)$$
$$\varepsilon_{xx} = 0, \qquad \begin{Bmatrix}\gamma_{yz}\\ \gamma_{xz}\end{Bmatrix} = \frac{\partial\psi(z)}{\partial z}\begin{Bmatrix}\gamma^0_{yz}\\ \gamma^0_{xz}\end{Bmatrix}$$

### 2.3 Constitutive Equations

The constitutive relations of an FGM plate, taking into account thermal effects, can be expressed by [14]:

$$\begin{Bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \tau_{xy}\\ \tau_{xz}\\ \tau_{yz}\end{Bmatrix} = \begin{bmatrix}c_{11} & c_{12} & 0 & 0 & 0\\ c_{12} & c_{22} & 0 & 0 & 0\\ 0 & 0 & c_{44} & 0 & 0\\ 0 & 0 & 0 & c_{55} & 0\\ 0 & 0 & 0 & 0 & c_{66}\end{bmatrix}\begin{Bmatrix}\varepsilon_{xx}-\alpha T\\ \varepsilon_{yy}-\alpha T\\ \gamma_{xy}\\ \gamma_{xz}\\ \gamma_{yz}\end{Bmatrix} \qquad (2.9)$$

where $\sigma_{xx}$, $\sigma_{yy}$, $\tau_{xy}$, $\tau_{xz}$, $\tau_{yz}$ and $\varepsilon_{xx}$, $\varepsilon_{yy}$, $\gamma_{xy}$, $\gamma_{xz}$, $\gamma_{yz}$ are the stress and strain components, respectively. The stiffness coefficients $C_{ij}$ of the FGM plate are expressed by:

$$c_{11} = c_{22} = E(z)\big/\left(1-\nu^2\right)$$
$$c_{12} = \nu c_{11} \qquad (2.10)$$
$$c_{44} = c_{55} = c_{66} = E(z)\big/2\left(1+\nu^2\right)$$

### 2.4 Stability Equations

The strain potential energy of the system is composed of the strain energy of the plate and the potential energy of the two-parameter elastic foundation, which can be written as [10]:

$$V = U + U_f\,. \qquad (2.11)$$

Here $U$ is the total strain energy of the plate, given by:

$$U = \frac{1}{2}\int_V \begin{bmatrix}\sigma^{(n)}_{xx}(\varepsilon_{xx}-\alpha^{(n)}T)+\sigma^{(n)}_{yy}(\varepsilon_{yy}-\alpha^{(n)}T)\\ +\sigma^{(n)}_{xy}\gamma_{xy}+\sigma^{(n)}_{yz}\gamma_{yz}+\sigma^{(n)}_{xz}\gamma_{xz}\end{bmatrix} dV\,. \qquad (2.12)$$

And $U_F$ is the strain energy due to the Pasternak elastic foundation, determined by the expression [15]:

$$U_F = \frac{1}{2}\int_\Omega \left[K_w w_0^2 + K_g\left(w_{0,x}^2 + w_{0,y}^2\right)\right] d\Omega\,. \qquad (2.13)$$

Using the Euler equations, the stability equations are thus obtained:

$$\begin{aligned}
&N^1_{x,x} + N^1_{xy,y} = 0\\
&N^1_{xy,x} + N^1_{y,y} = 0\\
&M^1_{x,xx} + 2M^1_{xy,xy} + M^1_{y,yy} + N^0_{x,}w^1_{,xx} + N^0_{y,}w^1_{,yy} - Kw + Kg\nabla^2 w = 0\\
&S^1_{x,x} + S^1_{xy,y} + Q^1_{xz} = 0\\
&S^1_{xy,x} + S^1_{y,y} + Q^1_{yz} = 0
\end{aligned} \qquad (2.14)$$

where 1 and 0 refer to the state of stability and state of equilibrium, respectively. The terms $N^0_x$, $N^0_y$ and $N^0_{xy}$ are the pre-buckling forces obtained as:

$$N^0_x = N^0_y = A_T\big/(1-\nu);\, N^0_{xy} = 0\,. \qquad (2.15)$$

## 3. ANALYTICAL SOLUTION

FGM plates are generally classified according to the type of support used. For the analytical solution of equations (14), the Navier's method is used under the specified boundary conditions:

$$\begin{Bmatrix}u^1\\ v^1\\ w^1\\ \varphi^1_1\\ \varphi^1_2\end{Bmatrix} = \sum_{m,n=1}^{\infty}\begin{Bmatrix}U^1_{mn}\cos(\lambda x)\sin(\mu y)\\ V^1_{mn}\sin(\lambda x)\cos(\mu y)\\ W^1_{mn}\sin(\lambda x)\sin(\mu y)\\ X^1_{mn}\cos(\lambda x)\sin(\mu y)\\ Y^1_{mn}\sin(\lambda x)\cos(\mu y)\end{Bmatrix} \qquad (3.1)$$

where $U^1_{mn}, V^1_{mn}, W^1_{mn}, X^1_{mn}$ and $Y^1_{mn}$ are arbitrary parameters to be determined, $\lambda$ and $\mu$ are defined as follows:

$$\lambda = m\pi / a, \qquad \mu = n\pi / bv\,. \qquad (3.2)$$

## 4. VARIATION OF TEMPERATURE

### 4.1 Uniform Variation

The temperature variation is given by:

$$T(z) = T_f - T_i = \Delta T\,. \qquad (4.1)$$

### 4.2 Linear Variation

The linear temperature distribution along the thickness of the FGM plate is considered:

$$T(z) = \Delta T\left(\left(z/h\right)+\left(1/2\right)\right)+T_m; \Delta T = T_c - T_m . \quad (4.2)$$

### 4.3 Nonlinear Variation

The nonlinear temperature distribution along the thickness of the FGM plate is considered:

$$T(z) = \Delta T\left(z/h+1/2\right)^p + T_m;\ \Delta T = T_c - T_m , \quad (4.3)$$

where $p$ is the temperature exponent.

The dimensionless parameters retained are:

$$K_g = k_g D / a^2,\ K_w = k_w D / a^4,$$
$$T_{cr} = 10^{-3}\Delta t_{cr},\ D = E_c h^3 / 12\left(1-\nu^2\right).$$

## 5. NUMERICAL RESULTS AND DISCUSSION

### 5.1 Comparative Analysis

The FGM plate is considered aluminum and alumina with the following material properties. Metal (aluminum): Young's modulus and coefficient of thermal expansion are $E_m$ = 70 GPa and $\alpha_m$ =23.106/°c. Ceramic (alumina): Young's modulus and coefficient of thermal expansion are $E_c$ = 380 GPa and $\alpha_c$ = 7.4.10 $^{-6}$/°c. For linear and nonlinear temperature distributions through the thickness $T_t$ = 5°c.

**Table 1** – The critical buckling load $T_{cr}$ of a simply supported FGM plate under uniform temperature variation for different values of the power index $k$ and different values of the ratio $a/b$ with $a$ = 100 h

| **k** | **Theory** | ***a/b* = 1** | ***a/b* = 2** | ***a/b* = 3** | ***a/b* = 4** | ***a/b* = 5** |
|---|---|---|---|---|---|---|
| **0** | A. BOUHADRA et al. | 17.0894 | 42.6876 | 85.2553 | 144.6496 | 220.6721 |
| | ZENKOUR et al. SPT | 17.0894 | 42.6876 | 85.2554 | 144.6500 | 220.6729 |
| | ZENKOUR et al. HPT | 17.0894 | 42.6875 | 85.2551 | 144.6490 | 220.6706 |
| | ZENKOUR et al. FPT | 17.0894 | 42.6875 | 85.2551 | 144.6489 | 220.6704 |
| | Present FSPT | 17.0910 | 42.6975 | 85.2950 | 144.7640 | 220.9385 |
| | Present HPT | 17.0894 | 42.6876 | 85.2554 | 144.6500 | 220.6729 |

**Table 2** – The critical buckling load $T_{cr}$ of a simply supported FGM plate under linear temperature rise for different values of the power index $k$ and different values of the ratio $a/b$ with $a$ = 100 h)

| **k** | **Theory** | ***a/b* = 1** | ***a/b* = 2** | ***a/b* = 3** | ***a/b* = 4** | ***a/* = 5** |
|---|---|---|---|---|---|---|
| **0** | A. BOUHADRA et al. | 24.1789 | 75.3752 | 160.5107 | 279.2993 | 431.3442 |
| | ZENKOUR et al. SPT | 24.1789 | 75.3753 | 160.5109 | 279.3000 | 431.3459 |
| | ZENKOUR et al. HPT | 24.1789 | 75.3751 | 160.5102 | 279.2980 | 431.3412 |
| | ZENKOUR et al. FPT | 24.1789 | 75.3751 | 160.5102 | 279.2979 | 431.3409 |
| | Present FSPT | 24.1821 | 75.3951 | 160.5901 | 279.5281 | 431.8770 |
| | Present SPT | 24.1789 | 75.3753 | 160.5109 | 279.3000 | 431.3459 |

**Table 3** – The critical buckling load $T_{cr}$ of a simply supported FGM plate under nonlinear temperature rise for different values of the power index $k$ and different values of the $a/b$ ratio, $a$ = 10 h, $p$ = 2.5

| **k** | **Theory** | ***a/b* = 1** | | ***a/b* = 2** | | ***a/b* = 3** | |
|---|---|---|---|---|---|---|---|
| | | $\beta$ = 2 | $\beta$ = 5 | $\beta$ = 2 | $\beta$ = 5 | $\beta$ = 2 | $\beta$ = 5 |
| **0** | ZENKOUR SPT | 4.8414 | 9.6829 | 11.2294 | 22.4589 | 20.0164 | 40.0328 |
| | ZENKOUR HPT | 4.8410 | 9.6821 | 11.2269 | 22.4538 | 20.0066 | 40.0133 |
| | ZENKOUR FPT | 4.8408 | 9.6817 | 11.2246 | 22.4492 | 19.9919 | 39.9838 |
| | Present FSPT | 4.8844 | 9.7689 | 11.4609 | 22.9219 | 20.7533 | 41.5066 |
| | Present HPT | 4.8414 | 9.6829 | 11.2294 | 22.4589 | 20.0164 | 40.0328 |

### 5.2 Parametric Study

**Table 4** – Variation of the critical buckling temperature $T_{cr}$ as a function of the power index $k$ and the ratio $a/h$ = 100 for different theories

| **Theory** | ***a/b*** | **Material index** | | | | |
|---|---|---|---|---|---|---|
| | | ***k* = 0** | ***k* = 0.5** | ***k* = 1** | ***k* = 2** | ***k* = 3** |
| **HPT** | 1 | 17.0894 | 11.1626 | 9.8089 | 8.9653 | 8.6333 |
| | 2 | 42.6875 | 27.8849 | 24.5019 | 22.3919 | 21.5608 |
| | 3 | 85.2551 | 55.6980 | 48.9359 | 44.7124 | 43.0469 |
| | 4 | 144.6490 | 94.5160 | 83.0299 | 75.8416 | 73.0025 |
| | 5 | 220.6729 | 144.2201 | 126.6716 | 115.6616 | 111.3045 |
| **FPT** | 1 | 17.0910 | 11.1637 | 9.8101 | 8.9669 | 8.6350 |
| | 2 | 42.6975 | 27.8917 | 24.5095 | 22.4014 | 21.5712 |
| | 3 | 85.2950 | 55.7250 | 48.9662 | 44.7501 | 43.0885 |
| | 4 | 144.7640 | 94.5938 | 83.1174 | 75.9503 | 73.1221 |
| | 5 | 220.9385 | 144.4012 | 126.8753 | 115.9144 | 111.5827 |

**Table 5** – Effect of the Pasternak elastic foundation on the critical buckling temperature of FGM square plates under uniform temperature variation for different values of geometric ratio "*a*/*h*"

| **(*kw*, *kg*)** | ***k*** | ***a*/*h* = 5** | ***a*/*h* = 10** | ***a*/*h* = 15** | ***a*/*h* = 20** | ***a*/*h* = 25** | ***a*/*h* = 30** |
|---|---|---|---|---|---|---|---|
| **(0, 0)** | 0 | 5.5834 | 1.6186 | 0.7413 | 0.4215 | 0.2711 | 0.1888 |
| | 1 | 3.2130 | 0.9297 | 0.4256 | 0.2419 | 0.1556 | 0.1083 |
| | 2 | 2.8571 | 0.8429 | 0.3876 | 0.2207 | 0.1420 | 0.0989 |
| **(100, 0)** | 0 | 6.5479 | 1.8598 | 0.8485 | 0.4818 | 0.3097 | 0.2155 |
| | 1 | 4.6405 | 1.2866 | 0.5842 | 0.3311 | 0.2127 | 0.1480 |
| | 2 | 4.3815 | 1.2240 | 0.5570 | 0.3160 | 0.2030 | 0.1413 |
| **(100, 10)** | 0 | 8.4517 | 2.3357 | 1.0600 | 0.6008 | 0.3858 | 0.2684 |
| | 1 | 7.4582 | 1.9909 | 0.8972 | 0.5072 | 0.3254 | 0.2263 |
| | 2 | 7.3898 | 1.9763 | 0.8913 | 0.5040 | 0.3233 | 0.2248 |

**Table 6** – Effect of the Pasternak elastic foundation on the critical buckling temperature of FGM square plates under uniform temperature variation for different values of geometric ratio

| **(*kw*, *kg*)** | ***k*** | ***a*/*b* = 0.5** | ***a*/*b* = 1** | ***a*/*b* = 1.5** | ***a*/*b* = 2** | ***a*/*b* = 2.5** | ***a*/*b* = 3** |
|---|---|---|---|---|---|---|---|
| **(0, 0)** | 0 | 10.6832 | 17.0894 | 27.7605 | 42.6877 | 61.8577 | 61.8577 |
| | 1 | 6.13187 | 9.80896 | 15.9339 | 24.5019 | 35.5055 | 48.9358 |
| | 2 | 5.60468 | 8.96538 | 14.5629 | 22.3920 | 32.4450 | 44.7118 |
| **(100, 0)** | 0 | 14.5412 | 19.5007 | 29.2443 | 43.6522 | 62.5228 | 85.7375 |
| | 1 | 11.8411 | 13.3772 | 18.1298 | 25.9290 | 36.4900 | 49.6492 |
| | 2 | 11.7011 | 12.7757 | 16.9076 | 23.9159 | 33.4958 | 45.4737 |
| **(100, 10)** | 0 | 19.3007 | 24.2603 | 34.0042 | 48.4118 | 67.2823 | 90.4965 |
| | 1 | 18.8847 | 20.4208 | 25.1734 | 32.9727 | 43.5336 | 56.6928 |
| | 2 | 19.2222 | 20.2966 | 24.4287 | 31.4371 | 41.0170 | 52.9947 |

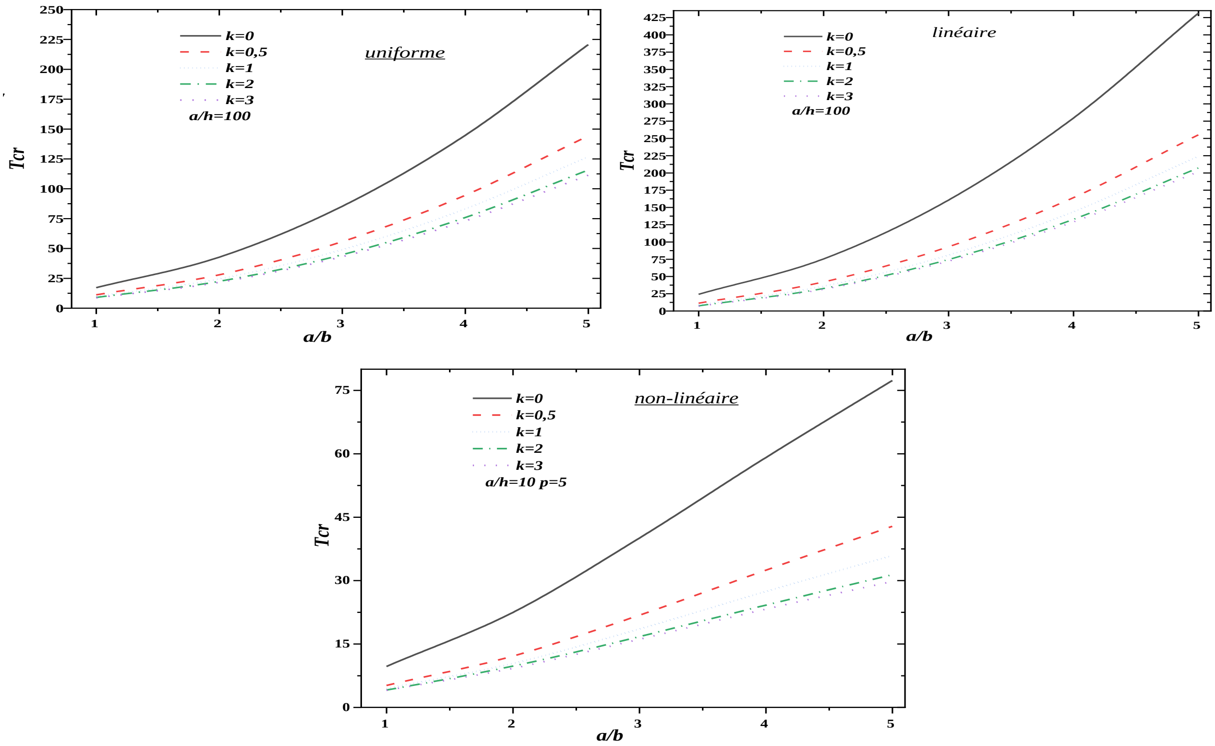


**Fig. 2** – Variation of the critical buckling temperature $T_{cr}$ as a function of the *a*/*b* ratio for different values of the power index *k* with *a*/*h* = 10

Fig. 2 represents the variation of the critical buckling temperature as a function of the geometric ratio (*a*/*b*) according to three temperature variations through the thickness (uniform, linear and nonlinear), and as shown in the figure, the temperature $T_{cr}$ increases as the geometric parameter increases.

Fig. 3 represents the variation of the critical buckling temperature as a function of the thickness ratio (*a*/*h*) according to three temperature variations through the thickness (uniform, linear and nonlinear), and as shown in the figure, the temperature $T_{cr}$ decreases as the thickness parameter increases.

Fig. 4 represents the variation of the critical buckling temperature as a function of the material index *k* under the effect of the elastic foundation according to two temperature variations through the thickness (uniform and linear), and as shown in the figure, the temperature $T_{cr}$ decreases as the power index increases, and the values obtained increase with an increase in the two coefficients of the elastic foundation.

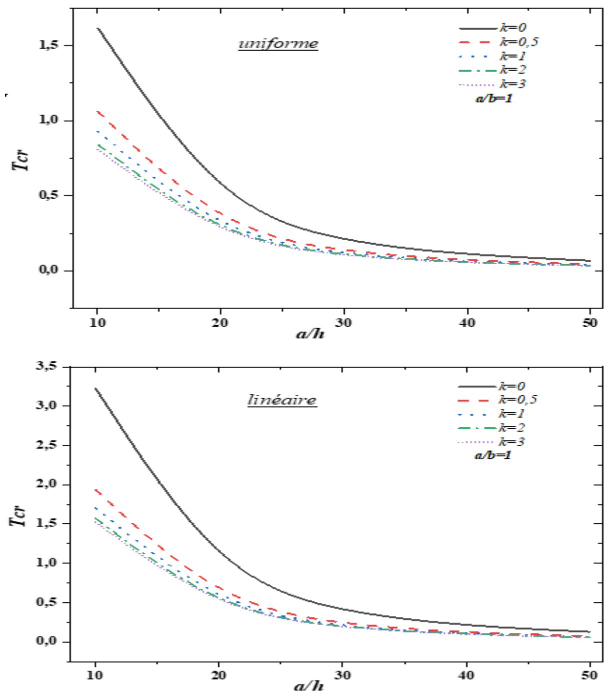


**Fig. 3** – Variation of the critical buckling temperature $T_{cr}$ as a function of the $a/h$ ratio for different values of the power index $k$

## 6. CONCLUSIONS

In the present study, the thermal buckling behavior of simply supported functionally graded material (FGM) plates resting on the Winkler-Pasternak type elastic foundation was studied. The accuracy of the Mori-Tanaka model of the material property distribution through the thickness of the FGM plate and the theory used is determined by comparison with other high-order shear strain theories, where excellent agreement was observed in all cases.

In addition, the influence of the plate parameters such as power index $k$, aspect ratio $a/b$, variation of the parameter of the elastic foundation $kw$, ratio $a/h$ and

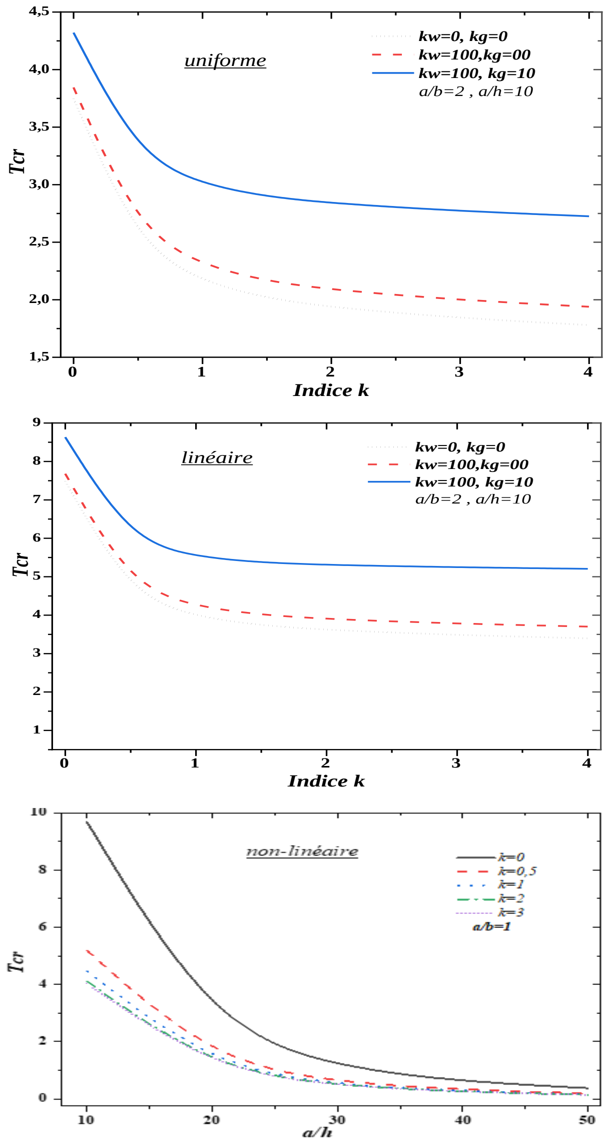


**Fig. 4** – Effect of the gradient index on the critical buckling temperature $T_{cr}$ of an FGM plate resting on elastic foundations

the type of thermal loading on the critical buckling load of the FGM plate have been fully investigated.

## REFERENCES


1. F. Bouazza, K. Abdelhakim, A.O. Tounsi, A.B. El Abbas, *J. Mech. Sci. Technol.* **26** No 12, 4073 (2012).
2. M.B. Bouiadjra, M. Houari, A. Sid, A.O. Tounsi, *J. Therm. Stress.* **35** No 8, 677 (2012).
3. K. Ravikiran, A. Kashif, N. Ganesan, *Appl. Math. Model.* **32**, 2509 (2008).
4. A. Bouhadra, S. Benyoucef, A.O. Tounsi, F. Bernard, R. Bouiadjra, M. Houari, A. Sid, *J. Therm. Stress.* **38** No 6, 630 (2015).
5. J.N. Reddy, *Mechanics of Laminated Composites Plates: Theory and Analysis* (CRC Press: Boca Raton: 1997).
6. M. Touratier, *Int. J. Eng. Sci.* **29** No 8, 901 (1991).
7. M. Karama., K.S. Afaq., S. Mistou, *Int. J. Solids Struct.* **40** No 6, 1525 (2003).
8. B.V. Sankar, J.T. Tzeng, *AIAA J.* **40** No 6, 1228 (2002).
9. R.K. Kapania, S. Raciti, *AIAA J.* **27** No 7, 923 (1989).
10. Y. Kiani, E. Bagherizadeh, M.R. Eslami, *Zamm Math. Mech.* **91** No 7, 581 (2011).
11. A.M. Zenkour, *J. Eng. Math.* **50**, 75 (2004).
12. A.M. Zenkoura, M. Sobhy, *Compos. Struct.* **93**, 93 (2010).
13. K.H. Khayat, M. Sonebi, A. Yahia, C.B. Skaggs, *Proceed. Int. RILEM Conf. Production Methods and Workability of Concrete*, 463 (Eds. P.J.M. Bartos, D.L. Marrs, D.J. Cleland) (CRC Press: 1996).
14. J.N. Reddy, *Int. J. Non-Linear Mech.* **25** No 6, 677 (1990).

## Вплив механічних та геометричних характеристик на термічне жолоблення функціонально градуйованих сендвіч пластин

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Функціонально градуйовані матеріали (FGM) – це новий клас композиційних матеріалів, які мають поступову та безперервну зміну об'ємних часток кожного з компонентів (загалом металу та кераміки) з товщиною, що відповідно спричиняє зміну загальних термомеханічних властивостей структурних елементів. Інтерес даної роботи полягає у використанні теорії високого порядку для дослідження термічного жолоблення FGM пластин, що спираються на пружну основу типу Вінклера-Пастернака. Цей метод веде до системи диференціальних рівнянь, де кількість невідомих дорівнює п'яти. Властивості матеріалу FGM пластин, такі як модуль Юнга та коефіцієнт теплового розширення, вважаються змінними з товщиною відповідно до моделі розподілу Морі-Танака. Теплове навантаження вважається рівномірним, лінійним і нелінійним по товщині пластини. Таким чином, проведено параметричне дослідження, щоб побачити вплив геометричних і механічних характеристик, зокрема, геометричного співвідношення ($a/b$), співвідношення товщин ($a/h$) та індексу матеріалу ($k$), а також впливу параметрів Вінклера і Пастернака на критичне навантаження на вигин.

**Ключові слова:** Функціонально градуйовані матеріали (FGM), Теорія високого порядку, Модель Морі-Танаки, Пружна основа, Термічне жолоблення.