Diffraction of *E*-polarized Photons on Periodic Grating of Metal Strips

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The problem of diffraction of *E*-polarized light at normal falling on a grating of infinitely thin metallic strips is solved. Light is represented as a flux of particles – photons. The problem of determining the psi-function of a photon scattered by the grating is led down to the Riemann-Hilbert boundary problem. A strict solution is obtained in the form of a convergent infinite system of linear algebraic equations. The system equations are valid for any relation between wavelength and period of the structure and any relation between slit width and strip width. As follows from a comparison of the de Broglie representation of the psi-function and its decomposition into Fourier series, the possible values of the photon momentum component perpendicular to its initial direction of motion are determined by even values of the "quantum" of momentum, whose magnitude is determined by the grating period. Photons passed through or reflected by the grating get discrete values of momentum when interacting with the grating and deviate at discrete angles. Numerical calculations show that the diffraction maxima are located in front of the slit and have some internal structure that depends on the ratio between the grating period and the photon wavelength. As the ratio of the grating period to the photon wavelength increases, the diffraction peak splits. When the ratio becomes less than unity, the diffraction pattern disappears, we have a uniform illumination. Therefore, the value of the specified ratio, equal to one, is the threshold.

Keywords: Diffraction, Grating, Quantum, Psi-function, Probability amplitude, Diffraction pattern, Photon.

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1. INTRODUCTION

There has been continuous attention to the problem of electromagnetic wave scattering by a strip grating. This is explained by both the interest of theorists and a wide range of applications. In the given paper, the problem of diffraction of *E*-polarized photons in the case of normal incidence of a flux on a grating formed by an unlimited sequence of infinitely thin metal strips is resolved. In the quantum formulation, the problem of determining the Ψ-function of a photon passed and reflected by a grating is considered, which is lead down to the Riemann-Hilbert boundary value problem. The solution of the problem is presented in the form of an infinite system of linear algebraic equations for determining the Fourier coefficients of the Ψ-function.

2. PROBLEM STATMENT

A homogeneous flux of photons falls normally from above on a grating located in the *XOY* plane. The slit width is *d*, the grating period is *l,* so the strip width is $l-d$ (Fig. 1). It is necessary to define the flux intensity (probability density $|\Psi|^2$) of photons above and below the grating. Scattering of *E*-polarized photons having E_x-component of the electric field is considered. In this case, the photon function according to de Broglie [1] can be represented as

$$\Psi = E_x = E_o e^{-ikz}. \quad (1)$$

A stationary process is considered, so the time multiplier $e^{-i\varpi t}$ is absent in this expression.

In the region above $(z \le 0)$ and below $(z \ge 0)$ the grating, the Ψ-function of the scattered photon must satisfy the two-dimensional Schrödinger equation, which for a photon has the view:

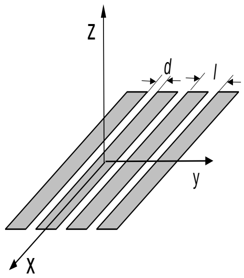

Fig. 1 – Diffraction grating

$$\frac{\partial^2\Psi}{\partial y^2} + \frac{\partial^2\Psi}{\partial z^2} + k^2\Psi = 0 \quad (2)$$

and coincides with the wave equation for an electromagnetic wave $k = 2\pi/\lambda$, λ is a wavelength. Due to the periodicity of the structure, the Ψ-function must be a periodic function with a period l in the direction of the *Y*-axis. Thus, it can be expanded in a Fourier series:

$$\Psi(x,y) = \sum_{n=-\infty}^{\infty} E_n(z) e^{i\frac{2\pi n}{l}y}. \quad (3)$$

We will assume that photons cannot penetrate the metallic strips. In electromagnetic theory, in this case one speaks of the ideal conductivity of the metal. Thus, in metal slits $\Psi \equiv 0$.

According to the requirements of finiteness in the upper half-space, the Ψ-function will have the form

$$\Psi^{(I)}(y,z) =$$
$$= e^{-ikz} + \sum_{n=-\infty}^{\infty} a_n exp\left(i\sqrt{k^2 - \left(\frac{2\pi n}{l}\right)^2} z\right) \cdot exp\left(i\frac{2\pi n}{l} y\right), \quad (4)$$

where the first constituent corresponds to the amplitude of the probability of a photon falling on the grating, the second – to the remote one.

In the lower half-space, the amplitude of the probability of a photon passed through a grating

$$\Psi^{(II)}(y,z) = \sum_{n=-\infty}^{\infty} b_n exp\left(-i\sqrt{k^2-\left(\frac{2\pi n}{l}\right)^2} z\right)\cdot exp\left(i\frac{2\pi n}{l}y\right). \quad (5)$$

Here, $\gamma_n = \sqrt{k^2-\left(\frac{2\pi n}{l}\right)^2}$ will count the value of the root, which has a positive imaginary part, and if it is equal to zero – a positive real part.

3. SOLUTION OF THE PROBLEM

On metal strips, the Ψ-function equals to zero, and on slits – the Ψ-function and its derivative are continuous. And therefore, the right parts of expressions (4), (5) must be identically equal at $z = 0$ on the full period, from which equality (6) follows, which is valid on the entire period

$$b_o = 1 + a_o, \quad b_n = a_n, \quad (6)$$

and equalizations (on metal)

$$\sum b_n exp\left(i\frac{2\pi n}{l}y\right) = 0. \quad (7)$$

As the derivative $\frac{\partial\Psi}{\partial z}$ is continuous on the slit, we also get (on the slits)

$$-k + \sum b_n\left(\sqrt{k^2-\left(\frac{2\pi n}{l}\right)^2}\right)\cdot exp\left(i\frac{2\pi n}{l}y\right) = 0. \quad (8)$$

Equalizations (7), (8), are reduced to the Riemann-Hilbert problem, the exact solution of which is represented as an infinite system of linear algebraic equations relative to the coefficients b_n [2]

$$\begin{aligned} -b_o &= i\delta b_o V_\sigma^o - i\delta V_\sigma^o + \sum_{n=1}^{\infty} x_n \chi_n\left(V_\sigma^n + V_\sigma^{-n}\right) + 2cR_\sigma, \\ 0 &= i\delta b_o V_o^o - i\delta V_o^o + \sum_{n=1}^{\infty} x_n \chi_n\left(V_o^n + V_o^{-n} + 2cR_o\right), \\ x_m &= i\delta b_o V_m^o - i\delta V_m^o + \sum_{n=1}^{\infty} x_n \chi_n\left(V_m^n + V_m^{-n}\right) + 2cR_m, \\ &(m = 1, 2, 3...), \quad x_n = b_n n, \end{aligned} \quad (9)$$

where $\chi_n = 1 + i\sqrt{\frac{\delta^2}{n^2} - 1}$, $x_n = nb_n$, $\delta = \frac{l}{\lambda}$.

Expressions for the $V_o^o, V_\sigma^o, V_\sigma^n, V_m^o, V_m^n, R_o, R_m$ coefficients in the Legendre polynomials are represented in the work [3]. Due to the parameter χ_n tending to 0 as $\chi_n = 0(1/n^2)$, at $n \to \infty$ system (9) converges and allows to apply the reduction method.

4. DISCUSSION OF THE OBTAINED RESULTS

Here we must make a reservation comparing the representations of the Ψ-function for the *E*- and *H*-polarizations [2]. They have the same form, and the same conclusions follow from their analysis. Let us represent the Ψ-functions (4), (5) in the de Broglie form

$$\begin{aligned} \Psi_r &= \sum a_n exp\frac{i}{\hbar}\left(-p_z z + p_y y\right), \\ \Psi_t &= \sum b_n exp\frac{i}{\hbar}\left(p_z z + p_y y\right), \end{aligned} \quad (10)$$

where p_z, p_y are the z- and y-components of the photon momentum, respectively,

$$p_y = 2\frac{\pi n\hbar}{l}, \; p_z = \hbar\sqrt{k^2-\left(\frac{2\pi n}{l}\right)^2}.$$

The following conclusions follow from analysis (10). In classical electrodynamics [3], the field of plain wave scattering on a grating is interpreted as a sum of spatial harmonics (4), (5) with amplitudes a_n, b_n. Each of them propagates at an angle, the tangent of which is determined by the relation

$$\mathrm{tg}\alpha_n = \frac{2\pi n/l}{\sqrt{k^2-\left(\frac{2\pi n}{l}\right)^2}}. \quad (11)$$

After simple trigonometric transformations we get from expressions (11) the known diffraction grating equation [3]:

$$l\sin\alpha_n = n\lambda. \quad (12)$$

According to (10), relation (11) can be represented as

$$\mathrm{tg}\alpha_n = \frac{p_y}{p_z}. \quad (13)$$

Expression (13) can be explained as follows. A photon passed through the slit receives a momentum component in the direction perpendicular to its primary direction of motion as a result of elastic collision with an electron of a strip. An expression that coincides with equation (12) was also obtained in [4] for the diffraction of photons by two slits when slit width $d \to 0$. In this case, the approach was based on application of laws of conservation of energy and momentum at the elastic collision of a photon with an electron moving in the strip (what can be considered as practically free movement in an infinitely deep potential well [5]). As noted in [2], the result follows from the analysis of the y-component of the momentum p_y, which can be examined as some rule of selection of possible values of the y-component of the photon momentum acquired as a result of collision with an electron. The final one is determined by the even values of the "quantum" of the momentum $\pi\hbar/l$ of an electron moving in the metal strip. It should be noted that in [4] it was assumed that the width of the potential well is equal to the width of the metal strip $l - d$. But the results obtained in this work and in [2] clarify this assumption. The width of the potential well is determined by the grating period.

Concerning the diffraction pattern, we emphasize once again that photons passing through the slit acquire discrete values of the momentum upon collision with electrons deviating by discrete angles determined by (12) or (13). So, we have intensity maxima at some spots where photons came, and intensity minima in the spots where photons did not come.

It should be noted that the results obtained clearly confirm the dual nature of particles. So, when determining the possible values of the electron momentum in a metal strip (which we treated as a potential well), we use the wave properties of the electron. The phenomenon of interaction of a photon with an electron is based on the corpuscular properties of particles. In the end, the dual nature of particles is contained in the de Broglie representation (see (10)), according to which it is both a plane wave and a particle with momentum.

Now let us compare expressions (10) with the representations of the components of electromagnetic waves in classical optics. Based on the quantum concepts formulated above, the coherence conditions introduced in wave optics acquire a different meaning. Let us consider the classical form of receiving coherent beams with the help of the Young experiment, a description of which can be founded in [3]. As known, two beams are coherent if their phase difference $\Delta\varphi$ is constant in time, and the interference maximum is observed when $\Delta\varphi = 2\pi m$. In terms of quantum mechanics, the phase difference $\Delta\varphi = 2\pi m$ corresponds to the difference between the y-components of the momenta of any two photons determined by the expression $\Delta p = \frac{2\pi\hbar}{l}m$ (see representations (10)).

To obtain a distinguishable interference pattern in the Young installation, the distance L between the screen and the sources must be much greater than the distance b between the sources (two slits in the non-transparent screen). In this case, the position of the intensity maxima is observed at the value of the x coordinate, according to [2], equal to

$$x_{max} = \pm m\,\frac{b}{L}\lambda,\ m = 0,\ 1,\ 2,\ \ldots \qquad (14)$$

Here $\lambda = \lambda_0/n$, λ_0 is the light wavelength in vacuum, n is the refractive index of the medium. Let us transform (10), so

$$\frac{x_{max}}{L} = \pm m\,\frac{\lambda}{b} = \pm m\,\frac{\lambda}{b}\frac{2\pi\hbar}{2\pi\hbar} = \frac{p_y}{p} = \mathrm{tg}\alpha, \qquad (15)$$

or finally $\mathrm{tg}\alpha = \pm m\lambda$.

At small diffraction angles α, the obtained expression coincides with (12). So, we see that in this case we are dealing with a classical example of the diffraction phenomenon. It can be concluded that the phenomenon of interference for particles does not exist. This conclusion agrees with the experimental results on diffraction of electrons represented in [6]. According to the results of this experiment, there can be no question of interference of each electron reached the screen individually.

We now turn to a discussion of the numeral results. First at all, we are interested in the dependence of the coefficients b_n, and then the diffraction pattern on the wavelength and slit width, or more precisely on $\delta = l/\lambda$ and the fill factor d/l.

Numerical calculations were performed using a computer program that allows to obtain the basic characteristics of the grating within a wide range of change of the parameters. Substituting $\chi_n = 0$ into the system of equations (9) for all $n > \delta$, it is possible to get from (9) the limiting system. The computer program allows calculation within d/l from 0 to 1, and within δ from 0 to 4.1. The psi-function of a photon passed throw the grating is represented by expression (5). At a large distance from the grating plane $z >> l$ in the sum (5), there will be only those coefficients b_n for which $n < \delta$

$$\Psi^{(II)}(y,z) = \sum_{n=0}^{N} b_n exp\left(-i\frac{2\pi}{l}\sqrt{\delta^2 - n^2}\,z\right)\cdot cos\left(\frac{2\pi n}{l}y\right), \qquad (16)$$

where $N = \delta$. For all other coefficients with $n \geq \delta$, all γ_n become imaginary and disappear at a great distance from the grating. It should be noted that relation (16) holds at a large distance between the grating and the screen.

Fig. 2 and Fig. 3 show the dependences of $|\Psi|^2$ on the y-coordinate (y/l) for one period of the structure, since the diffraction pattern is repeated with period l. Fig. 2 represents $|\Psi|^2$ at $\delta = 1.9$ and different values of the fill factor d/l. Fig. 3 shows $|\Psi|^2$ at $d/l = 0.5$ and different values of δ.

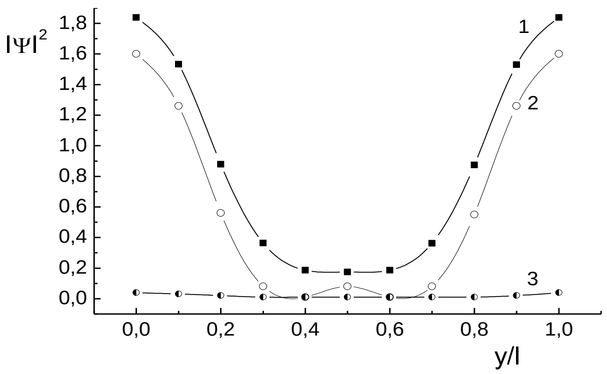

Fig. 2 – Diffraction patterns of distribution $|\psi|^2$ at different values of the fill factor d/l: 1 – 0.9; 2 – 0.5; 3 – 0.1; at $l/\lambda = 1.9$

As evident from the charts in Fig. 2, the diffraction pattern has one maximum over the period length located opposite the slit. The value of the maximum depends on the ratio between the slit width and the grating period. With an increase in the slit width, the height of the maximum grows.

From a comparison of the dependences for different d/l ratios, the probability of photon passage through the grating decreases with a decrease in the slit width, and when $d \to 0$, $|\Psi|^2 \to 0$.

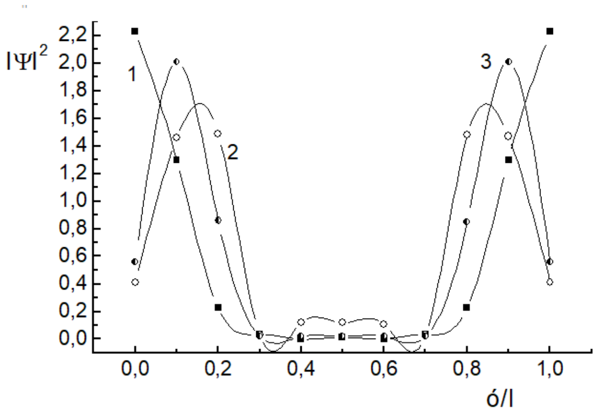

Fig. 3 – Diffraction patterns of distribution $|\psi|^2$ at different values of l/λ: 1 – 2.1; 2 – 3.1; 3 – 4.1; at $d/l = 0.5$

From a comparison of the dependences for different d/l ratios, the probability of photon passage through the grating decreases with a decrease in the slit width, and when $d \to 0$, $|\Psi|^2 \to 0$.

Glance at Fig. 3 give the possibility to analyze the dependence diffraction pattern on the parameter l/λ. In contrast to H-polarization [2], the bifurcation of the maximum begins with the value $\delta = 2$ and not with $\delta = 1$.

5. CONCLUSIONS

In the given paper, the problem of diffraction of E-polarized photons by an infinite grating of infinitely thin metallic strips at the normal falling is solved for an arbitrary ratio between the slit width and the structure period, and an arbitrary ratio between the wavelength and the grating period is obtained. The solution is based on a strict method for solving boundary-value problems – the Riemann-Hilbert method. The exact solution for the probability amplitude Ψ of the transmission of E-polarized photons during diffraction by a grating is represented as an infinite system of linear algebraic equations with respect to the Fourier expansion coefficients. At $n = \to \infty$, the system converges, its coefficients tend to zero as $1/n^2$, and the reduction method can be applied.

It is demonstrated that the condition of coherence in optics is reduced in quantum mechanics to the condition of possible discrete values of momentum acquired by a photon as a result of its interaction with a grating. So, in terms of quantum mechanics, the phenomenon of interference for particles does not exist. This conclusion is consistent with the electron diffraction experiment [6], in which the time of flight of an electron from the screen with a hole to the observation screen was significantly less than the time interval between the appearance of two electrons sequentially following one after the other. So, the diffraction phenomenon is a consequence of the quantum nature of light.

The numerical results allow to assert that the probability of a photon arriving to any point of the screen located behind the grating has some maxima, the number of which depends on the relation between the wavelength and the structure period.

It should also be noted that this theory, in addition to optical phenomena, can to some extent qualitatively help explain the behavior of the resonant absorption of high-frequency waves in various crystals [7].

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Дифракція *E*-поляризованих фотонів на нескінченній ґратці металевих стрічок

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Розв’язана задача про дифракцію *E*-поляризованих фотонів при нормальному падінні на ґратку, утворену необмеженою послідовністю нескінченно тонких металевих стрічок. Світло представляється як потік частинок – фотонів. Задача знаходження псі-функції фотона, розсіяного ґраткою, зводиться до граничної задачі Рімана-Гільберта. Розв’язок задачі отримано у вигляді нескінченної системи лінійних алгебраїчних рівнянь, що сходиться. Система придатна для будь-яких співвідношень між довжиною хвилі та періодом структури та будь-яких співвідношень між шириною щілини та періодом ґратки. Як витікає із порівняння де Бройлівського представлення псі-функції з її розкладом у ряд Фур’є, можливі значення складової імпульсу фотона, перпендикулярної до первинного напрямку руху, визначаються парними значеннями "кванта" імпульсу, величина яких залежить від періода ґратки. Фотони, пропущені або відбиті ґраткою, отримують дискретні значення імпульсу внаслідок взаємодії з ґраткою і відхиляються на дискретні кути від первинного напрямку. Як випливає з чисельних розрахунків, дифракційні максимуми розташовуються перед щілиною і мають деяку внутрішню структуру залежно від співвідношення між довжиною хвилі та періодом ґратки. При збільшенні відношення періоду ґратки до довжини хвилі дифракційний пік розділяється. Дифракційна картина спостерігається, коли це відношення більше одиниці. Коли воно стає менше одиниці, дифракційна картина зникає, маємо однорідну освітленість. Отже, значення вказаного відношення рівне одиниці є пороговим.

Ключові слова: Дифракція, Ґратка, Квант, Псі-функція, Амплітуда ймовірності, Дифракційна картина, Фотон.