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An Evolutionary Anisotropic Behavior for DC04 Sheet Using Hill48 Function under Non-Associated Flow Rule Hypothesis

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In this paper, the evolution of the anisotropic behavior of a DC04 sheet during work hardening is analyzed based on a quadratic Hill48 function and under the application of two plasticity approaches: associated and non-associated flow rule approaches (AFR and NAFR, respectively). The mechanical properties have been modeled, such as the uniaxial flow stresses $\sigma(\theta)$ and the coefficient of anisotropy $r(\theta)$ (or the Lankford parameter). In the identification of anisotropic parameters, the decoupling of the two tensors (strain and stress) under the assumption of non-associated plasticity gives better predictions compared with the experimental. With the purpose of introducing the evolutionary anisotropic path of plastically equivalent characteristics for the uniaxial flow stresses $\sigma(\theta)$, the isotropic hardening of mechanical strain hardening function is presented using the following empirical law based on Voce model. Therefore, the evolution of plastic potential was described by polynomial function based on NAFR approach.

Keywords: Hill48 function, Anisotropic metal, Non-associated flow rule, Rolling metal sheet, Evolution yield hardening.

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1. INTRODUCTION

The rigid transformation of sheet metal is widely used todays in industry (such as automotive, aeronautics technology areas). The rolling and deep drawing processes are strongly influenced by the structural and mechanical behavior, in particular the anisotropic properties of these products. The rolled sheet modeling is very important for simulation of manufacturing and forming processes. However, for many sheet metals obtained by cold-rolling, a strong gradient of crystallographic texture exhibits in plane and thickness of metal [1]. This is accurate concerning ferritic steels (FSS), for two causes: (i) during hot working, continuous recrystallization happens, which precludes randomization of textural components and (ii) during cooling process, the phase transformation is totally negligible. In addition, cold rolling and/or subsequent heat treatments do not completely eradicate or even modify such textures [2]. Orthotropic symmetry is characterized by three mutually orthogonal planes in the rolling sheets. However, it is important to note that the rolling process favors the existence of an induced anisotropy, which exhibited a remarkable improvement in the drawability features, sheet metal forming and thereby enhancement of desired dimensions and shape [3]. Therefore, the anisotropic mechanical behavior of materials is well identified via several proposed (quadratic and non-quadratic) functions. Earlier, von Mises [4] developed a criterion for quadratic yield plasticity based on the second invariant of deviatoric stress J_2 according to the isotropic material assumption. Hill [5] introduced the first anisotropic function, which is a basic generalization of von Mises' isotropic plasticity, to give a quadratic formulation mainly associated with the flow rule based on Drucker's postulate using thus the r-value coefficients of Lankford as model calibration parameters, and further Hosford [6] generalized the isotropic approach for a non-quadratic Yield function. Numerous other successful efforts have been made in recent decades to improve phenomenological anisotropy models and extended to a planar anisotropic one more recently. Barlat et al. [7-12] used two linear transformations, while Banabic et al. [13-16] proposed a non-quadratic Yield function in order to probe the anisotropic features of metals, provided that more data are required for identification, including in-plane anisotropy.

Spitzig et al. [17] confirmed that the influence of mean stress on metal flow stress is not impacted by the expected plastic expansion required by the associated flow rule (AFR) in common steel and aluminum alloys. Consequently, the authors have specified and confirmed that AFR is not properly exact. However, it was mentioned by other researchers [18-20] that the material anisotropy through a new mathematical model resulted in decoupled predictions of yield stress and strain ratios (r-values). These models based on non-associated flow rule (non-AFR) lead to an improvement in hardening consistency. The phenomenological data incompatible among the requirement of associated plasticity (AFR) with respect to the dependence on spherical pressure (average stress of hydrostatic tensor) of yield stress during the initial perfect plasticity (associated plasticity) has been indicated. Under non-AFR, the final volume calculated by the components of the strain tensor based on the associated plasticity (AFR) is no significant from that measured, where plastic potential is decoupled from the Yield stress. Stoughton et al. [18] introduced a mean stress-sensitive formula under non-AFR for rolling process. They considered the Drucker's stability postulate conditions in metal forming of non-AFR [19]. Safaei et al. [20] under a modern non-AFR approach proposed a model based on Barlat's yield criterion (Barlat et al., 2003). However, Lian et al. [21] proposed a non-associated formalism based on Hill-48 plasticity criterion representing anisotropic hardening as well as r-values evolution and its application to form a limit diagram for ferritic drawing steel.

2. YIELD FUNCTION FOR SHEET METALS

2.1 Hill48 Yield Criteria

One of the most commonly used models of metal forming or yield plastic potential is the Hill 1948 yield criterion, which can be stated in a general form as the in-plane stress state:

$$F\sigma_{22}^2 + G\sigma_{11}^2 + H(\sigma_{11} - \sigma_{22})^2 + 2N\sigma_{12}^2 = \sigma_0^2 . \qquad (1)$$

In the sheet plane, the plastic anisotropy can be identified by four parameters: F, G, H, and N. The coefficients of anisotropy can be calculated from: i) microtextural components, ii) measured through mechanical testing (i.e., standard uniaxial tensile and simple shearing), as shown $N = 3G = 3F = 3H = 3$. The Hill48 yield function is reduced to the Mises yield function in this case. Based on the Hill48 quadratic function, the $\sigma(\theta)$ yield mechanical parameter and the ratio of strains across width and thickness (r-value or the Lankford parameter) were computed as:

$$\sigma(\theta) = \frac{\sigma_0}{\left(F\sin^4\theta + G\cos^4\theta + H\cos^2 2\theta + 2N\sin^2\theta\cos^2\theta\right)^{1/2}},$$
$$r(\theta) = \frac{H\cos^2 2\theta - (F + G - 2N)\cos^2\theta\sin^2\theta}{F\sin^2\theta + G\cos^2\theta} . \qquad (2)$$

The material constants F, G, H and N of the Hill48 model can be calculated by two methods:

$$\mathrm{F} = \frac{1}{2}\left(\frac{2\sigma_0^2}{\sigma_{90}^2(1 + r_{90})}\right),$$
$$\mathrm{G} = 1 - \frac{1}{2}\left(\frac{2\sigma_0^2 r_{90}}{\sigma_{90}^2(1 + r_{90})}\right), \qquad (3)$$
$$\mathrm{H} = \frac{1}{2}\left(\frac{2\sigma_0^2 r_{90}}{\sigma_{90}^2(1 + r_{90})}\right), \quad \mathrm{N} = \frac{1}{2}\left(\frac{4\sigma_0^2}{\sigma_{45}^2} - 1 + \frac{\sigma_0^2(r_{90} - 1)}{\sigma_{90}^2(1 + r_{90})}\right).$$

a) **Associative Flow Rule (AFR)** as a classic approach in material plasticity where coupling between yield stresses and the plastic potential operates.

b) **Non-Associative Flow Rule**

1- **Stress-Based Hill48 Model:** yield stress requires 04 experimental monoaxial flow stresses according to the rolling (RD), transverse (TD), diagonal (DD) directions and the balanced biaxial stress. Material anisotropic coefficients can be calibrated as following:

$$F^y = \frac{1}{2}\left[\frac{\sigma_0^2}{\sigma_{90}^2} + \frac{\sigma_0^2}{\sigma_b^2} - 1\right],$$
$$G^y = \frac{1}{2}\left[1 - \frac{\sigma_0^2}{\sigma_{90}^2} + \frac{\sigma_0^2}{\sigma_b^2}\right], \qquad (4)$$
$$H^y = \frac{1}{2}\left[1 + \frac{\sigma_0^2}{\sigma_{90}^2} - \frac{\sigma_0^2}{\sigma_b^2}\right],$$
$$N^y = \frac{1}{2}\left[\frac{4\sigma_0^2}{\sigma_{45}^2} - \frac{\sigma_0^2}{\sigma_b^2}\right],$$

where σ_0, σ_{45}, σ_{90} are monodirectional flow loadings of 0°, 45° and 90° according to the rolling direction (RD) (therefore, $G^y + H^y = 1$), σ_b is the balanced biaxial flow stress measured experimentally by a biaxial tensile test. Noting that, all these variants imply $\sigma_{ref} = \sigma_0$.

2- **Lankford-Based Hill48 Criterion (Plastic Potential):** The plastic potential function involves three uniaxially measured Lankford r-values along the rolling (RD), transverse (TD) and diagonal (DD) directions. The experimental plastic potential function r-values are given by:

$$F^p = \frac{r_0}{r_{90}(1 + r_0)},$$
$$G^p = \frac{1}{(1 + r_0)}, \qquad (5)$$
$$H^p = \frac{r_0}{(1 + r_0)},$$
$$N^p = \frac{(1 + 2r_{45})(r_0 + r_{90})}{2r_{90}(1 + r_0)} .$$

The Lankford coefficients r_0, r_{45}, r_{90} are the anisotropic ratios of the plastic strain rate in the three directions 0°, 45° and 90° in the sheet plane (therefore, $G^p + H^p = 1$).

3. EXPERIMENTAL PROCEDURE

In this study, the low carbon steel was used, known as DC04 (ASTM A620, NE10130-2006). The studied sheet is of a fine thickness of 1.5 mm, provided by the Algerian company of tractors (ETRAG). The chemical composition is shown in Table 1.

Table 1 – Chemical composition of DC04 steel (in wt. %), Fe balance

C	Si	Mn	P	S	Mo	Al	Cu	Ti
0.06	0.03	0.19	0.012	0.011	0.01	0.039	0.051	0.004

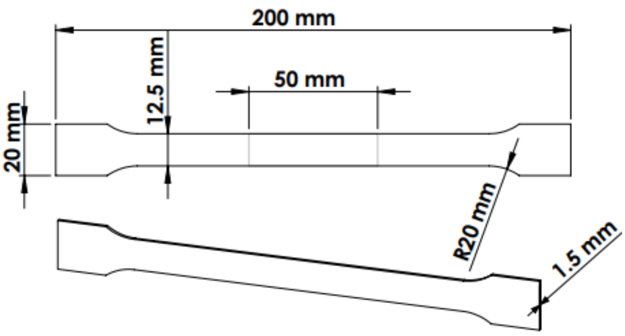

Fig. 1 – The shape and dimensions of rectangular samples

The evolution of experimental parameters in the sheet plane, principally the $\sigma(\theta)$ yield stress and the $r(\theta)$ anisotropic ratio, was obtained by uniaxial loading tests. The tensile specimen was maintained in the following directions: 0°, 45° and 90° relative to the direction of rolling. The specimens were carefully prepared and cut by laser (Fig. 1).

4. COEFFICIENTS FOR DIFFERENT YIELD FUNCTIONS

Table 2 shows the results of Hill48 (F, G, H and N) anisotropic coefficients under two hypotheses.

Table 2 – Identification of coefficients under two approaches

AFR	**F**	**G**	**H**	**N**
	0.472	0.556	0.443	1.286
NAFR-*s*	F	G	H	N
	0.520	0.436	0.479	1.321
NAFR-*r*	**F**	**G**	**H**	**N**
	0.491	0.537	0.462	1.132

Based on Hill48, the predicted mechanical parameters ($\sigma(\theta)$ and $r(\theta)$) at various metal orientations are picked in Fig. 2.

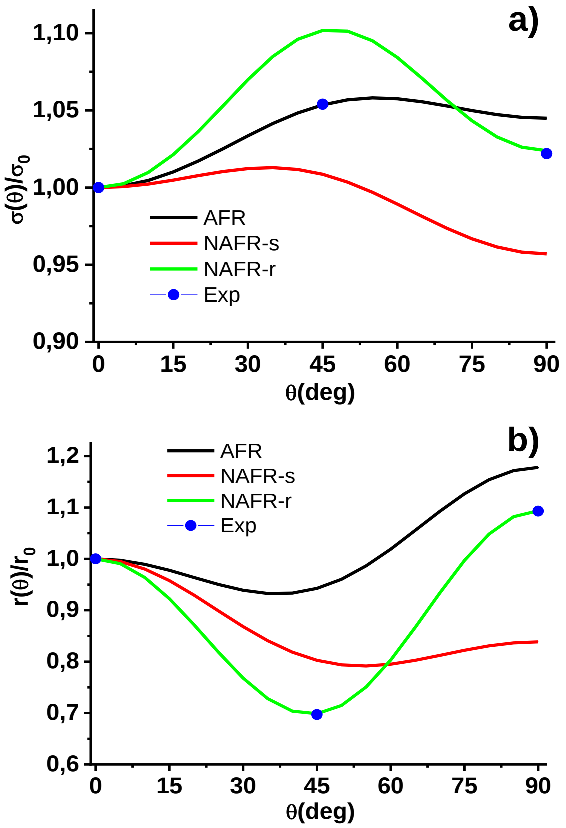

Fig. 2 – a) Normalized yield stress, b) normalized Lankford coefficient predicted with Hill48 under AFR and non-AFR approaches and experiment results

5. THE EVOLVING OF ANISOTROPIC MODEL IN TERMS OF LONGITUDINAL EQUIVALENT PLASTIC STRAIN AMOUNT UNDER NON-ASSOCIATED FLOW RULES (NON-AFR)

5.1 Evolution of $\sigma(\theta)$ Yield Stress

Based on the optimization of the Voce's law of the experimental uniaxial tensile curve in the rolling direction, a dataset interval between the values of the plastic longitudinal strain [0.001 to 0.3] is proposed, with a step size of 0.1.

5.2 Evolution of $r(\theta)$-value

The anisotropic parameter of Lankford r-value can be defined as follows:

$$r(\theta)_{\text{instantanous}} = \frac{\varepsilon_w(\theta)}{\varepsilon_t(\theta)} \Rightarrow \frac{\varepsilon_2(\theta)}{\varepsilon_3(\theta)} = \frac{\varepsilon_2(\theta)}{-(\varepsilon_1(\theta)+\varepsilon_2(\theta))} = = -\frac{m(\theta)}{1+m(\theta)}$$

with

$$\varepsilon_1(\theta)+\varepsilon_2(\theta)+\varepsilon_3(\theta)=0\,, \qquad (6)$$

where ε_1, ε_2 and ε_3 are the true plastic strains according to the longitudinal, transverse and thickness orientations.

For all three specimens with selected orientations, transverse and longitudinal plastic strains are related by the ratio (slope $m(\theta)$) determined by the fitted linear regression curve (Fig. 3).

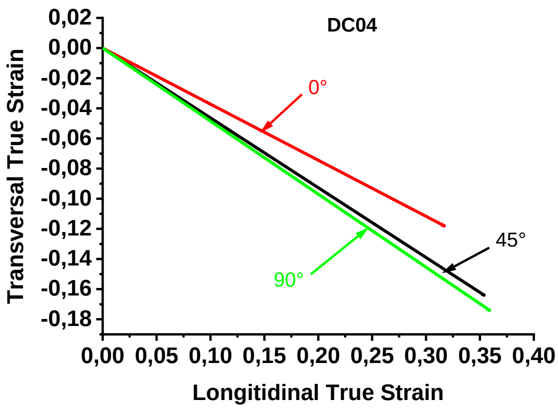

Fig. 3 – Transverse and longitudinal true plastic strain ratios measured experimentally at three orientations of DC04

To obtain the instantaneous r-value, the third-order polynomial of the relationship between transverse and longitudinal plastic deformations was proposed as follows:

$$\text{Poly3}(\bar{\varepsilon}^p) = a_1(\bar{\varepsilon}^p)^3 + a_2(\bar{\varepsilon}^p)^2 + a_3(\bar{\varepsilon}^p) + a_4\,. \qquad (7)$$

The constants of the 3rd order polynomial (Poly3) fitted in three directions are depicted in Table 3.

The optimal yield stress and r-value parameters are given in Table 4.

Under the non-associated approach according to the three amounts of equivalent plastic deformations, the predictions of the anisotropy mechanical parameters (stress $\sigma(\theta)$ and $r(\theta)$) in terms of the normalized values are given in Fig. 4.

Table 3 – Constants of the polynomial function

Angle	a_1	a_2	a_3	a_4
0°	– 1.2981	1.3805	– 0.8569	– 6E-06
45°	– 0.7283	0.6896	– 0.5177	– 2E-06
90°	– 1.4548	– 1.4548	– 0.8569	– 6E-06

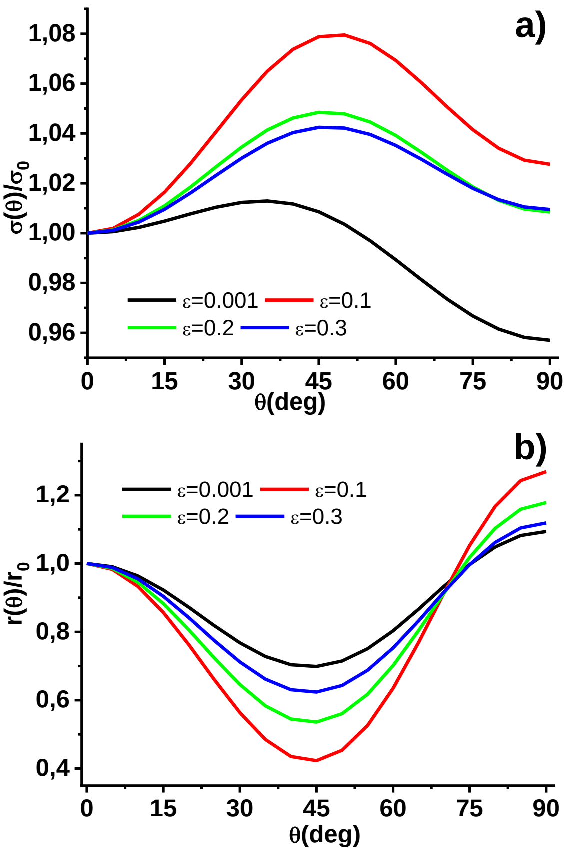

Fig. 4 – Comparison of the angular response of: a) normalized values of $\sigma(\theta)$, b) normalized values of $r(\theta)$

6. CONCLUSIONS

In the current contribution, mathematical functions based on the quadratic yield function Hill48 were presented and analyzed to capture the anisotropic mechanical behavior of the DC04 sheet steel. Under associated and non-associated flow rule related to Hill48 criterion, the yield tensor and plastic potential have been computed. Based on the mathematical formulation and experimental tests, the principal conclusions of this paper can be provided as follows:

– The anisotropy coefficient is perfectly optimized and predicted by Hill's criterion in the framework of non-associated plasticity (non-AFR-*r*) in comparison with the experiment.
– For the evolutionary stress tensor anisotropy and on the basis of the extrapolation of the isotropic work hardening, the model of Voce is adopted in order to provide a better optimization with the results of experimental tensile tests.
– For the Lankford coefficient, the evolutionary strain tensor anisotropy is driven by the polynomial regression of degree 3 (Poly3) of the ratio of longitudinal strain to transverse strain. The evolution of the geometry of the tensile specimen is controlled by this polynomial function along the three directions of characterization.

Table 4 – Constants of Hill48 under non-AFR approach

	$\bar{\varepsilon}^p$	*F*	*G*	*H*	*N*
NAFR-*s*	0.1%	0.520	0.436	0.479	1.321
	10%	0.486	0.542	0.513	1.299
	20%	0.495	0.512	0.504	1.345
	30%	0.495	0.514	0.504	1.369
	$\bar{\varepsilon}^p$	*F*	*G*	*H*	*N*
NAFR-*r*	0.1%	0.491	0.537	0.462	1.132
	10%	0.264	0.335	0.664	0.802
	20%	0.370	0.436	0.536	0.961
	30%	0.454	0.508	0.491	1.061

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Еволюційна анізотропна поведінка листа DC04 з використанням функції Hill48 за гіпотезою неасоційованого правила потоку

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У роботі аналізується еволюція анізотропної поведінки листа DC04 під час зміцнення на основі квадратичної функції Hill48 та застосування двох підходів пластичності: підходи асоційованого та неасоційованого правила потоку (AFR та NAFR відповідно). Було змодельовано механічні властивості, такі як одноосьові напруги потоку $\sigma(\theta)$ і коефіцієнт анізотропії $r(\theta)$ (або параметр Ланкфорда). При ідентифікації анізотропних параметрів, розчеплення двох тензорів (деформації та напруги) за припущенням неасоційованої пластичності дає кращі прогнози порівняно з експериментальними. З метою введення еволюційного анізотропного шляху пластично еквівалентних характеристик для одноосьових напружень течії $\sigma(\theta)$, ізотропне зміцнення функції механічного зміцнення представлене з використанням емпіричного закону на основі моделі Voce. Тому еволюцію пластичного потенціалу було описано поліноміальною функцією на основі підходу NAFR.

Ключові слова: Функція Hill48, Анізотропний метал, Неасоційоване правило потоку, Листовий прокат.