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Inflationary Cosmological Model of Bianchi Type II in General Relativity

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A string cosmic model with a perfect fluid distribution model has been researched in general relativity. It is based on the notion of Bianchi II homogeneous bifurcation. During the early universe's inflation, the cosmos expanded at an accelerated rate, stretching out space-time and smoothing out any kinks that may have occurred. The behavior of gravity in terms of the curvature of space-time serves as the foundation for this model. This brief epoch lasted just a fraction of a second, yet it had a tremendous impact on the universe's eventual history. The universe is uniform and isotropic on large sizes, but inflation happened in the early universe, and the world is homogenous and isotropic on small scales. The universe's energy was liberated in the form of particles and radiation. The equation of state parameter, which connects cosmic fluid pressure and energy density, is supposed to remain constant throughout the universe's development. This assumption simplifies the mathematical explanation of the universe's development compared to models with a time-varying equation of state. The initial event that led to the formation of galaxies and stars was the phase change. The assumption is that the homogeneous generalisation of Bianchi type II with stress-energy-momentum, density, and pressure may be used to solve the Einstein metric field equations. The resulting model will depict the cosmos expanding, shearing, and spinning. Discuss the geometrical and physical properties of the model as well to help you understand how it works.

Keywords: Bianchi Type II, Space-time, Cosmic-sting, General relativity.

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1. INTRODUCTION

The cosmological model Bianchi type II is a solution to the Einstein field equations of general relativity that describes an anisotropic, homogeneous universe. This means that the universe is the same in all directions at every point, but the properties of the universe can be different in different directions. In the Bianchi type II model, the universe is composed of three orthogonal planes, each of which expands at a different rate. This results in a shearing motion of the cosmic fluid that is present in the universe, which leads to the anisotropy of the universe. The expansion rate can be different in each plane and is governed by a scale factor.

The cosmic string has a significant influence on the formation of structures in cosmology [9]. They appear when the phase transition in the early universe (10-36) [13], as suggested by grand unified theories (GUT) [1, 2, 14], takes place when the temperature falls below a crucial temperature (TGUT = 1028k). It is thought that the density shifts caused by the vacuum strings are essential for the formation of galaxies. Letelier had started the lengthy string assertion (lines 8–9). These threads are a representation of the gravitational field and convey stress energy.

In various studies, different researchers have looked into cosmological models for different types of Bianchi spacetimes, including type II, VIII, and IX. One such study by Chakraborty, Nandi [10], and Kriori et al.[4] and highlight the importance of the cosmological model Bianchi type-II in general relativity. Another research Asseo with Sol [3] in the field of general relativity analyzed the Bianchi type-I cosmological model of magnetization. The study of the integration of cosmic strings in various spacetime models such as Bianchi type-IX, II, and VII has been conducted by Patel, Maharaj, and Leach [5]. Additionally, Rao et al.[15] studied the Saez-Ballester theory of gravity about accurate models of Bianchi type-IX, VIII, and II.

The relationship between the scalar field and the cosmological constant was used by Agarwal and Singh [12] to explore Bianchi type-IX, VIII, & II models inside the scalar-tensor theory. Wang examined the behavior of Bianchi type-III LRS cosmological model of a cloud string with a viscosity [16-18]. Additionally, Bianchi type-II LRS cosmological models describe clouds of both geometrical and heavy strings, which have been studied by Banerjee and Roy[11].

In this study, the authors focus on Bianchi type II spacetime and analyze its physical and geometric properties. They examine the effect of energy-momentum viscosity on the model and present a graphical representation of its variations over time.

2. METRIC WITH FIELD EQUATION

Let the space-time metric equation of Bianchi Type – II is

$$ds^2 = -dt^2 + R^2(t)\left[dx^2 + dz^2\right] + S^2(t)\left[dy - xdz\right]^2 \quad (1)$$

Where $R(t)$ & $S(t)$ are a function of t.

Energy-momentum tensor and bulk viscosity are related.

$$T_i^j = \rho \upsilon_i \upsilon^j - \lambda x_i x^j - \xi \vartheta \upsilon_i^j (g_i^j + \upsilon_i \upsilon^j) \quad (2)$$

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The Ricci tensor R_i^j and Weyl conformal curvature tensor for this metric has been given by

$$R_i^j - \frac{1}{2} R g_i^j = T_i^j \tag{3}$$

The T_i^j describes the distribution of energy and momentum in the universe.

$$T_i^j = -\rho u_i u^j + \lambda x_i x^j \tag{4}$$

Relation between u_i & x_i is

$$u_i u^j = -x_i x^j = -1 \qquad \& \quad x_i u^i = 0 \tag{5}$$

The energy of the cloud string at rest is ρ

$$\rho = \rho_p + \lambda \tag{6}$$

The condition is determined with the space-like unit vector and the matter's four-velocity vector in the direction of anisotropy.

$$g_{ij} u^i u^j = -1 \tag{7}$$

Einstein tensor is obtained by

$$R_i^j - \frac{1}{2} R g_i^j = -T_i^j \tag{8}$$

Equation (8) & (1) gives the following system of equation

$$2\frac{R_{44}(t)}{R(t)} + \left(\frac{R_4(t)}{R(t)}\right)^2 - \frac{3}{4}\left(\frac{S(t)}{R^2(t)}\right)^2 = \xi\vartheta \tag{9}$$

$$\frac{R_{44}(t)}{R(t)} + \frac{S_{44}(t)}{S(t)} + \frac{R_4(t)S_4(t)}{R(t)S(t)} + \frac{1}{4}\left(\frac{R(t)}{S^2(t)}\right)^2 = \xi\vartheta \tag{10}$$

$$\left(\frac{R_4(t)}{R(t)}\right)^2 + 2\frac{R_4(t)S_4(t)}{R(t)S(t)} - \frac{1}{4}\frac{S^2(t)}{R^4(t)} = 0 \tag{11}$$

$$2\frac{R_4(t)}{R(t)} + \frac{S_4(t)}{S(t)} = 0 \tag{12}$$

The physical parameters of the developed model, which are used extensively for solving field equations and discussing geometric features, are given by Spatial volume(V) for the developed model (1) is

$$V = \sqrt{-g} = R^2(t)S(t) \tag{13}$$

The scalar of expansion (θ) is

$$\theta = 2\frac{R_4(t)}{R(t)} + \frac{S_4(t)}{S(t)} \tag{14}$$

Shear scalar is

$$\sigma^2 = \frac{1}{3}\left(\frac{R_4(t)}{R(t)} - \frac{S_4(t)}{S(t)}\right)^2 \tag{15}$$

Deceleration parameter is

$$q = \frac{R_{44}(t) \big/ R(t)}{R_4^2(t) \big/ R^2(t)} \tag{16}$$

Hubble parameter is

$$H = \frac{1}{3}\left(2\frac{R_4(t)}{R(t)} + \frac{S_4(t)}{S(t)}\right) \tag{17}$$

Where the sub-indices 4 in $R(t)$, and $S(t)$ denote partial derivative w.r.t variables t.

3. FIELD EQUATION WITH SOLUTION

Each solution describes a different aspect of the universe, Equations (9-12) are the independent equation with some unknown variables, so we require some extra conditions such as coefficient of bulk viscosity

$$(\xi)\ \alpha \frac{1}{\exp asion\ scalar\ (\theta)}$$

$$\xi\vartheta = \phi\ (\text{constant}) \tag{18}$$

And

$$R(t) = S^M(t)\ ;\ \mathrm{M} > 1 \tag{19}$$

From the equations (9) and (10)

$$\frac{R_{44}(t)}{R(t)} - \frac{S_{44}(t)}{S(t)} + \left(\frac{R_4(t)}{R(t)}\right)^2 - \frac{R_4(t)S_4(t)}{R(t)S(t)} - \left(\frac{S(t)}{R^2(t)}\right)^2 = 0 \tag{20}$$

Equations (19) and (20) give

$$2S_{44}(t) + 4M\frac{S_4^2(t)}{S(t)} = \frac{2}{M-1}S^{3-4M}(t) \tag{21}$$

We consider

$$S_4(t) = f(S) \tag{22}$$

$$f = S_4(t) = \left(\frac{S^{4-4M}(t)}{2\times(M-1)} + A\times S^{-4M}(t)\right)^{\frac{1}{2}} \tag{23}$$

Equation (2.10) leads to

$$\int\left(\frac{S^{4-4M}(t)}{2\times(M-1)} + A\times S^{-4M}(t)\right)^{-\frac{1}{2}} dS(t) = \pm(t - t_0) \tag{24}$$

Where t_0 is an arbitrary constant

The metric equation (1) can be reduced to the form

$$ds^2 = -\frac{dT^2}{\left[\frac{T^{4(1-M)}}{2(M-1)} + A\times T^{-4M}\right]} + T^{2M}\left[dX^2 + dZ^2\right] + T^2\left[dY - X.dZ\right]^2 \tag{25}$$

Using transformation

$x = X$, $S(t) = T$, $y = Y$, and $z = Z$

4. PHYSICAL AND GENERAL FEATURES

Physical features include properties such as the density and pressure of matter, the flux of energy and momentum, and the presence of fields such as the electromagnetic field. These properties determine how matter and energy are distributed in the universe, and how they interact with one another. Geometrical features include properties such as the distance between points in spacetime, the angles between vectors, and the curvature of spacetime. These properties determine the geometry of the universe, and how objects move and interact within it.

The shear σ^2 is

$$\sigma^2=\frac{1}{3}\times(M-1)^2\times\left[\frac{T^{2(1-2M)}}{2\times(M-1)}+A\times T^{-2(2M+1)}\right] \quad (26)$$

The scalar of the expansion is

$$\vartheta=(2M+1)\sqrt{\left[\frac{T^{2(1-2M)}}{2\times(M-1)}+A\times T^{-2(2M+1)}\right]} \quad (27)$$

Hubble parameter

$$H=\frac{(2M+1)}{3}\sqrt{\left[\frac{T^{2(1-2M)}}{2\times(M-1)}+\frac{A}{T^{2(2M+1)}}\right]} \quad (28)$$

The deceleration parameter

$$q=-1+\frac{3}{2M+1} \quad (29)$$

In terms of geometrical features, Bianchi Type II models have a hyperbolic spatial geometry, meaning that the spatial section has a negative curvature. These hyperbolic spatial slices are locally isometric to the 3-dimensional hyperbolic space. Additionally, the spacetime is not maximally symmetric, which means that the curvature tensor does not have the maximum number of independent components.

Spatial volume is

$$V=T^{2M+1} \quad (30)$$

The bulk viscosity coefficient is

$$\xi=\frac{\frac{\beta}{2M+1}}{\sqrt{\left[\frac{1}{2(M-1)T^{2(2M-1)}}+AT^{-2(2M+1)}\right]}} \quad (31)$$

4.1 Graphical Representation of Physical and Geometrical Parameters

Table 1 – Value of scalar shear with different powers

S. No.	T	Scalar Shear (A = 1.5) $\sigma^2=\frac{1}{3}\times(M-1)^2\times\left[\frac{T^{2(1-2M)}}{2\times(M-1)}+A\times T^{-2(2M+1)}\right]$		
		$M=n=1.5$	$M=n=2.5$	$M=n=3.5$
1	1	0.6250	4.1250	10.6250
2	1.1	0.3457	1.4253	2.4386
3	1.2	0.2078	0.5530	0.6473
4	1.3	0.1335	0.2368	0.1945
5	1.4	0.0905	0.1104	0.0651
6	1.5	0.0640	0.0553	0.0239
7	1.6	0.0469	0.0295	0.0095
8	1.7	0.0353	0.0165	0.0041
9	1.8	0.0272	0.0097	0.0019
10	1.9	0.0214	0.0059	0.0009
11	2.0	0.0171	0.0038	0.0004
12	2.1	0.0138	0.0024	0.0002
13	2.2	0.0114	0.0016	0.0001
14	2.3	0.0094	0.0011	0.0001
15	2.4	0.0079	0.0008	0.0000

Graph is drawn by Table 1 data

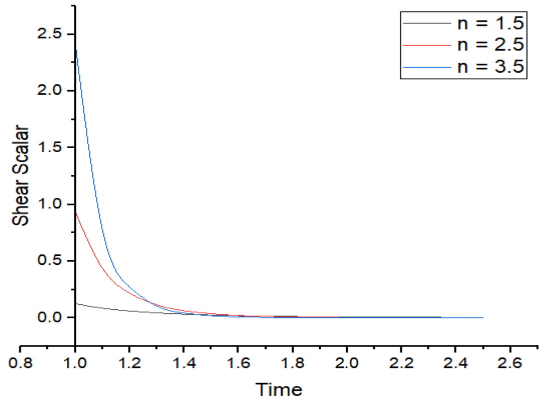

Fig. 1 – Graph between shear scalar vector & time

Table 2 – Value of scalar shear with different powers

S. No.	T	The scalar of expansion (A = 1.5) $\vartheta=(2M+1)\sqrt{\left[\frac{T^{2(1-2M)}}{2\times(M-1)}+A\times T^{-2(2M+1)}\right]}$		
		$M=n=1.5$	$M=n=2.5$	$M=n=3.5$
1	1	6.3246	8.1240	10.4307
2	1.1	4.7037	4.7754	4.9971
3	1.2	3.6466	2.9744	2.5745
4	1.3	2.9230	1.9465	1.4114
5	1.4	2.4065	1.3288	0.8164
6	1.5	2.0241	0.9404	0.4948
7	1.6	1.7321	0.6865	0.3123
8	1.7	1.5032	0.5145	0.2042
9	1.8	1.3198	0.3944	0.1377
10	1.9	1.1701	0.3083	0.0955
11	2	1.0458	0.2451	0.0677
12	2.1	0.9414	0.1977	0.0491
13	2.2	0.8525	0.1615	0.0363
14	2.3	0.7761	0.1334	0.0272
15	2.4	0.7100	0.1113	0.0207

Graph is drawn by Table 2 data

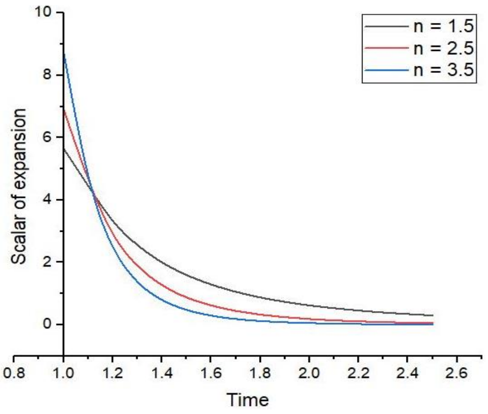

Fig. 2 – Graph between scalar of expansion & time

Table 3 – Value of Hubble parameter with different powers

S. No.	T	Hubble parameter (A = 1.5) $H=\frac{(2M+1)}{3}\sqrt{\left[\frac{T^{2(1-2M)}}{2\times(M-1)}+\frac{A}{T^{2(2M+1)}}\right]}$		
		$M=n=1.5$	$M=n=2.5$	$M=n=3.5$
1	1	2.1082	2.7080	3.4769
2	1.1	1.5679	1.5918	1.6657
3	1.2	1.2155	0.9915	0.8582
4	1.3	0.9743	0.6488	0.4705
5	1.4	0.8022	0.4429	0.2721
6	1.5	0.6747	0.3135	0.1649
7	1.6	0.5774	0.2288	0.1041
8	1.7	0.5011	0.1715	0.0681
9	1.8	0.4399	0.1315	0.0459
10	1.9	0.3900	0.1028	0.0318
11	2	0.3486	0.0817	0.0226
12	2.1	0.3138	0.0659	0.0164
13	2.2	0.2842	0.0538	0.0121
14	2.3	0.2587	0.0445	0.0091
15	2.4	0.2367	0.0371	0.0069

Graph is drawn by Table 3 data

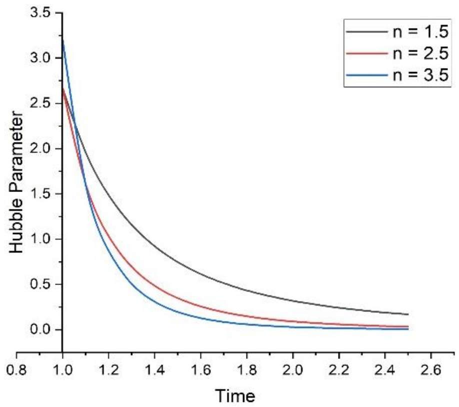

Fig. 3 – Graph between Hubble parameter & time

5. OBSERVATIONS

The initial spital volume (V) is zero and grows along with the time (T) as well as becomes infinite at a late time, which indicates that cosmic inflation is possible in a developed model, which shows that when T approaches 0, then the spatial volume (V) also approaches to 0, and as T approaches infinity then the spatial volume is also approaching infinity ($V\to\infty$). It shows that when $T\to 0$ then (spatial volume V) $\to$ 0, and when $T\to\infty$ then spatial volume becomes $V\to\infty$.

Table 4 – Value of Spital Volume with different powers

S. No.	T	Spital Volume $V=T^{2M+1}$		
		$M=n=1.5$	$M=n=2.5$	$M=n=3.5$
1	1	1.0000	1.0000	1.0000
2	1.1	1.4641	1.7716	2.1436
3	1.2	2.0736	2.9860	4.2998
4	1.3	2.8561	4.8268	8.1573
5	1.4	3.8416	7.5295	14.7579
6	1.5	5.0625	11.3906	25.6289
7	1.6	6.5536	16.7772	42.9497
8	1.7	8.3521	24.1376	69.7576
9	1.8	10.4976	34.0122	110.1996
10	1.9	13.0321	47.0459	169.8356
11	2	16.0000	64.0000	256.0000
12	2.1	19.4481	85.7661	378.2286
13	2.2	23.4256	113.3799	548.7587
14	2.3	27.9841	148.0359	783.1099
15	2.4	33.1776	191.1030	1100.7531

Graph is drawn by Table 4 data

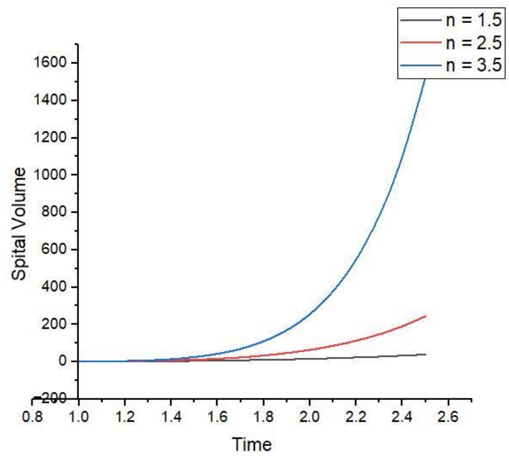

Fig. 4 – Graph between Volume & time

Based on these findings, it may be concluded that the cosmos expands with a volume of zero and explodes at limitless distances in the past and future. In the early phases of cosmic history, in particular, the significance of bulk viscosity appears to be important because the model-maintained anisotropy late in time, but at M = 1 bulk viscosity is absent for ξ = 0, the model became shear-free and isotropic. The expansion scalar, represented by θ, tends to continuously increase and become infinitely large as the time approaches infinity ($T\to\infty$). At $T\to 0$ (Initial Stage), the σ (shear scalar) approaches infinity, but eventually reaches zero at $T\to\infty$. This model depicts an expanding and non-rotating universe with a big-bang origin, as initial energy density is infinite ($\rho\to\infty$) and gradually decreases to zero as time passes ($\rho\to 0$ when $T\to\infty$). The negative dark pressure in this model indicates an accelerating phase in the cosmos. The Hubble parameter and scalar of expansion become divergent at the initial stage but eventually approach to 0 as the time approaches infinity. The shear scalar decreases over time and reaches zero for late times. The high field also decreases slowly and becomes finite after a long period. The presence of a bulk viscosity coefficient in this scenario results in cosmic inflation.

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Інфляційна космологічна модель Б'янкі типу II у загальній теорії відносності

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Космічна модель струни з ідеальною моделлю розподілу рідини, яка заснована на понятті однорідної біфуркації Б'янкі типу II, була досліджена в загальній теорії відносності. Під час ранньої інфляції Всесвіту космос розширювався з прискореною швидкістю, розтягуючи простір-час і згладжуючи будь-які перегини, які могли виникнути. Поведінка гравітації в термінах кривизни простору-часу служить основою для цієї моделі. Ця коротка епоха тривала лише частку секунди, але вона мала величезний вплив на подальшу історію Всесвіту. Всесвіт є однорідним та ізотропним у великих розмірах, але інфляція відбулася в ранньому Всесвіті, і світ однорідний та ізотропний у малих масштабах. Енергія Всесвіту була звільнена у формі частинок і випромінювання. Рівняння параметра стану, яке зв'язує тиск космічної рідини та щільність енергії, має залишатися постійним протягом усього розвитку Всесвіту. Це припущення спрощує математичне пояснення розвитку Всесвіту порівняно з моделями зі змінним у часі рівнянням стану. Початковою подією, яка призвела до утворення галактик і зірок, була зміна фази. Припущення полягає в тому, що однорідне узагальнення Б'янкі типу II з напругою-енергією-імпульсом, щільністю та тиском може бути використано для вирішення рівнянь метричного поля Ейнштейна. Отримана модель буде зображати космос, що розширюється, зсувається та обертається.

Ключові слова: Біанкі типу II, Простір-час, Космічне жало, Загальна теорія відносності.