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Bianchi Type V Inflationary Cosmological Model with Bulk Viscosity in General Relativity

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Our goal in the current paper is to develop Bianchi Type V space time undertaken in framework of massless scalar field with flat potential. Model produces an inflationary universe scenario provided bulk viscosity is present. The Bianchi Type V cosmological model is a homogeneous and anisotropic solution of the Einstein field equations, which describes the evolution of the universe. In this model, the geometry of the universe is characterized by three mutually orthogonal spatial directions, and the time evolution is determined by the matter content of the universe an appropriate transformation is used to solve the Einstein field equations. In order to develop a physical universe model we consider $\xi\theta = \alpha$ (constant) and we have assumed a supplementary condition $BC = \mu$ and $B/C = v$ between metric potential. We investigate the geometrical and physical aspects of the model with in presence of bulk viscosity. The model isotropies under specific conditions and the increase in spatial volume over time describe the inflationary phase of the universe. The model in general represents anisotropic space-time but isotropies at late time. The Hubble parameter decreases with time. At time $T = 0$, the model possesses a Point Type singularity.

Keywords: Bulk viscosity, Bianchi Type V, Inflation.

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1. INTRODUCTION

In recent literature, Einstein theory becomes an intriguing topic due to its efficacy in explaining the physical universe's rapid cosmic expansion. The early cosmos's extremely rapid exponential expansion, which increased its volume by at least 10^{78}, which is produced by a negative pressure vacuum energy density, is described as inflation. The cosmos continued to expand after the inflationary epoch, although at a significantly slower rate. Guth [1] was the first to put forth the inflationary hypothesis. Inflationary theory is capable of providing a comprehensive solution to a multitude of cosmological issues, including isotropy, homogeneity, monopoles, and flatness there are numerous inflationary situations that can be found in the physical cosmos as studied by Linde [2]. The roles of Higg's field with a flat potential have importance in this discussion. Many authors [3-9] have focused on different features of inflation theory and scalar field in various aspects. Weinberg [10] looked at the standard equation for bulk and shear viscosity to explain cosmological rate production.

Poonia and Sharma [11-13] discussed the cosmic phenomenon while taking bulk viscosity into account and looked at how it can affect evolution to understand inflation and the early universe's structure, Reddy [14] have developed spatially, anisotropic, and homogeneous Bianchi Type V space time. When examining the FRW model, Khalatnikov and Belinskii [15] used bulk viscosity as a function of energy density.In resent paper, to address the inflationary scenario of the universe and cosmic acceleration, we have presented a Bianchi Type V inflationary space time with bulk viscosity in the presence of flat potential. The growth of the spatial volume over time indicates the universe's accelerating phase. The model's geometrical and physical behaviour likewise discussed.

2. METRIC AND FIELD EQUATIONS

$$ds^2 = -dt^2 + A^2dx^2 + e^{2x}(B^2dy^2 + C^2dz^2) \quad (2.1)$$

Where A, B and C are only a cosmic time t function given by, the gravitational field action is given as a minimally coupled scalar field with potential $V(\varphi)$:

$$L = \int -\sqrt{g}\left[R - \tfrac{1}{2}g^{ij}\partial_i\varphi\partial_j\varphi - V(\varphi)\right]d^4x \quad (2.2)$$

The Einstein field equation can be referenced as

$$R_{ij} - \tfrac{1}{2}Rg_{ij} = -T_{ij} \quad (2.3)$$

With

$$T_{ij} = \partial_i\varphi\partial_j\varphi - \left[\tfrac{1}{2}\partial_l\varphi\partial^l\varphi + V(\varphi)\right]g_{ij} - \theta\xi(g_{ij} + v_iv_{j)} \quad (2.4)$$

And

$$\frac{1}{\sqrt{-g}}\partial_u\left(\sqrt{-g}\partial^u\varphi\right) = -\frac{\partial V}{\partial\varphi} \quad (2.5)$$

Field equation is

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4C_4}{BC} - \frac{1}{A^2} = -\frac{1}{2}\varphi_4^2 + V(\varphi) + \xi\theta \quad (2.6)$$

$$\frac{A_{44}}{A} + \frac{C_{44}}{C} + \frac{A_4C_4}{AC} - \frac{1}{A^2} = -\frac{1}{2}\varphi_4^2 + V(\varphi) + \xi\theta \quad (2.7)$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4B_4}{AB} - \frac{1}{A^2} = -\frac{1}{2}\varphi_4^2 + V(\varphi) + \xi\theta \quad (2.8)$$

$$\frac{A_4B_4}{AB} + \frac{A_4C_4}{AC} + \frac{B_4C_4}{BC} - \frac{3}{A^2} = \frac{1}{2}\varphi_4^2 + V(\varphi) + 2\xi\theta \quad (2.9)$$

$$2\frac{A_4}{A} - \frac{B_4}{B} - \frac{C_4}{C} = 0 \quad (2.10)$$

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In order to develop a physical universe model we consider $\xi\theta = \alpha$ (constant) from Brevik et al. [16].
From equation (2.10)

$$A^2 = BC \tag{2.11}$$

For scalar field equation (2.5) leads to

$$\varphi_{44} + \left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right)\varphi_4 = \frac{dV}{d\varphi} \tag{2.12}$$

Consider flat potential

$V(\varphi) = constant = k$

$$\varphi_{44} + \left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right)\varphi_4 = 0 \tag{2.13}$$

$$\varphi_4 = \frac{k_1}{ABC} \tag{2.14}$$

The scalar factor line element (2.1) leads to

$$R^3 = ABC \tag{2.15}$$

From equation (2.11)

$$R^3 = A^3 \tag{2.16}$$

From equation (2.7) and (2.8)

$$\frac{B_{44}}{B} - \frac{C_{44}}{C} + \frac{A_4}{A}\left(\frac{B_4}{B} - \frac{C_4}{C}\right) = 0 \tag{2.17}$$

Which provide

$$\frac{(CB_4 - BC_4)_4}{(CB_4 - BC_4)} = -\frac{1}{2}\left(\frac{B_4}{B} + \frac{C_4}{C}\right) \tag{2.18}$$

Which provide to

$$C^2\left(\frac{B}{C}\right)_4 = \frac{k_2}{(BC)^{1/2}} \tag{2.19}$$

k_2 is a constant of integration.
For solution of equation (2.18), we consider

$BC = \mu$ And $\frac{B}{C} = v$

Equation (2.19) provides to

$$\frac{v_4}{v} = \frac{k_2}{\mu^{3/2}} \tag{2.20}$$

From equation (2.6) and (2.9)

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{2B_4C_4}{BC} + \frac{A_4B_4}{AB} + \frac{A_4C_4}{AC} - \frac{4}{A^2} = 2k + 3\alpha \tag{2.21}$$

Taken $2k + 3\alpha = 2L$

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{1}{2}\left(\frac{B_4}{B} + \frac{C_4}{C}\right)^2 + \frac{2B_4C_4}{BC} - \frac{4}{A^2} = 2L \tag{2.22}$$

Equation (22) provides to

$$2\mu_{44} + \frac{1}{\mu}\mu_4^2 = 4L\mu + 8 \tag{2.23}$$

Consider $\mu_4 = f(\mu)$, $\mu_{44} = f'f$

Where $f' = \frac{\partial f}{\partial \mu}$
This provides to

$$f^2 = \frac{4L}{3}\mu^2 + 4\mu + \frac{n_1}{\mu} \tag{2.24}$$

After suitable Transformation metric (2.1) leads to

$$ds^2 = \frac{T}{\frac{4L}{3}T^3 + 4T^2 + n_1}dT^2 + TdX^2 + e^{2X}T\left\{e^{\int \frac{k_1}{T\sqrt{\frac{4L}{3}T^3 + 4T^2 + n_1}}dT + N} dY^2 + e^{-\int \frac{k}{T\sqrt{\frac{4L}{3}T^3 + 4T^2 + n_1}}dT + N} + dZ^2\right\} \tag{2.25}$$

Where $\mu = T$

$$v = e^{\int \frac{k_1}{T\sqrt{\frac{4L}{3}T^3 + 4T^2 + n_1}}dT + N} \tag{2.26}$$

Where N is a constant of integration

$$A^2 = BC = \mu = T \tag{2.27}$$

$$B^2 = \mu v = Te^{\int \frac{k_1}{T\sqrt{\frac{4L}{3}T^3 + 4T^2 + n_1}}dT + N} \tag{2.28}$$

$$C^2 = \mu/v = Te^{-\int \frac{k_1}{T\sqrt{\frac{4L}{3}T^3 + 4T^2 + n_1}}dT + N} \tag{2.29}$$

Particular Case

Put $n_1 = 0$ in equation (2.24)

$$f^2 = \frac{4L}{3}\mu^2 + 4\mu \tag{2.30}$$

$$\frac{d\mu}{\left(\mu + \frac{2}{\beta^2}\right)^2 - \left(\frac{2}{\beta^2}\right)^2} = \beta\, dt \tag{2.31}$$

Where

$$\beta^2 = \frac{4L}{3} \tag{2.32}$$

Equation (2.31) provides

$$\mu = \frac{4}{\beta^2}sinh^2\left(\frac{\beta t + \gamma}{2}\right) \tag{2.33}$$

From equation (2.19)

$$\frac{v_4}{v} = \frac{k_1\beta^3}{8\,sinh^3\left(\frac{\beta t + \gamma}{2}\right)} \tag{2.34}$$

Which provides to?

$$v = aexp\, b\left\{-cosec\, h\left(\frac{\beta t + \gamma}{2}\right)coth\left(\frac{\beta t + \gamma}{2}\right) + log\left[cosech\left(\frac{\beta t + \gamma}{2}\right) + coth\left(\frac{\beta t + \gamma}{2}\right)\right]\right\} \tag{2.35}$$

Where $b = \frac{k_1\beta^2}{8}$ and a is the integration's constant

$$As\ a\ result\ \ \frac{\beta t + \gamma}{2} = T$$

And

$$dS^2 = -\frac{4}{\beta^2}dT^2 + sinh^2TdX^2 + e^{\beta X}sinh^2T\left\{e^{b[-cosechTcothT + \log(cosechT + cothT)]}dY^2 + e^{-b[-cosechTcothT + \log(cosecht + cothT))}dZ^2\right\} \tag{2.36}$$

Where

$$\frac{2}{\beta}x = X, \frac{2\sqrt{a}}{\beta}y = Y, \frac{2}{\beta\sqrt{a}}z = Z$$

3. GEOMETRICAL AND PHYSICAL CHARACTERISTICS OF THE MODEL

From equation (2.15)
The appropriate volume is provided by

$$R^3 = A^3 = T^{\frac{3}{2}} \tag{3.1}$$

The rate of the higs's field is given by

$$\phi_4 = \frac{K_1}{ABC} = \frac{K_1}{A^3} = \frac{K_1}{T^{\frac{3}{2}}} \tag{3.2}$$

The expansion

$$\theta = \frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C} \tag{3.3}$$

$$\theta = \frac{3}{2}\frac{\mu_4}{\mu}$$

$$\theta = 3\frac{\sqrt{\frac{4L}{3}T^3 + 4T^2 + n_1}}{2T^{3/2}} \tag{3.4}$$

Shear

$$\sigma = \frac{1}{2}\left(\frac{B_4}{B} - \frac{C_4}{C}\right)$$

$$\sigma = \frac{1}{2}\frac{\gamma_4}{\gamma} = \frac{L}{2T^{3/2}} \tag{3.5}$$

And

$$\frac{\sigma}{\theta} = \frac{K_1}{3\sqrt{\frac{4L}{3}T^3 + 4T^2 + n_1}} \tag{3.6}$$

Hubble parameter is given by

$$H = \frac{R_4}{R}$$

$$H = \frac{\sqrt{\frac{4L}{3}T^3 + 4T^2 + n_1}}{2T^{3/2}} \tag{3.7}$$

4. DISCUSSION AND CONCLUSION

We have looked at the Bianchi Type V Inflationary Universe with Bulk Viscosity in this discussion the universe's stages of acceleration and deceleration are represented in this model. The spatial volume increases with time, illustrating an inflationary scenario. With time, decreasing the Hubble parameter .Higgs field rate decreases over time and A Point Type singularity exists in the model at $T = 0$.

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Інфляційна космологічна модель Біанкі типу V з об'ємною в'язкістю в загальній теорії відносності

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У статті представлена розробка просторово-часової моделі Біанкі типу V в рамках безмасового скалярного поля з плоским потенціалом. Модель створює сценарій інфляційного всесвіту за умови наявності об'ємної в'язкості. Космологічна модель Біанкі типу V є однорідним і анізотропним рішенням рівнянь поля Ейнштейна, яке описує еволюцію Всесвіту. У цій моделі геометрія Всесвіту характеризується трьома взаємно ортогональними просторовими напрямками, а часова еволюція визначається вмістом матерії у Всесвіті, відповідне перетворення використовується для вирішення рівнянь поля Ейнштейна. Щоб розробити фізичну модель Всесвіту, ми розглядаємо $\xi\theta = \alpha$ (константа) і припускаємо додаткову умову $BC = \mu$ і $B/C = \upsilon$ між метричним потенціалом. Були досліджені геометричні та фізичні аспекти моделі за наявності об'ємної в'язкості. Модельні ізотропії за певних умов і збільшення просторового об'єму з часом описують інфляційну фазу Всесвіту. Модель загалом представляє анізотропний простір-час, але ізотропії в пізній час. Параметр Хаббла з часом зменшується. У момент часу $T = 0$ модель має сингулярність типу Point.

Ключові слова: Об'ємна в'язкість, Б'янкі типу V, Інфляція.