The Effect of the Generated Pump Electric Field on the Amplification Properties of a Superheterodyne Parametric Free-Electron Laser

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In this work, within the framework of the quadratic nonlinear approximation, we analysed the influence of the generated pump electric field on the amplification properties of a parametric superheterodyne laser (FEL). The high amplification properties of such a device are ensured by implementing two interconnected three-wave parametric resonances. Due to the first of these resonances between the electromagnetic wave of the signal, the *H*-ubitron pump magnetic field and the slow space charge wave (SCW), the first one is amplified. The second of these resonances between the longitudinal pump electric field, slow and fast SCWs allows us to increase the slow SCW more. The connection between the two parametric resonances is ensured by the slow SCW common to resonances. In the work, the growth increments of the waves of the above-mentioned three-wave resonances are analysed separately and for the entire system. It has been demonstrated that generating an additional pump electric field significantly affects the growth increment of the second parametric resonance of longitudinal waves, increasing it by 33 %. Due to this, the growth increment of the entire FEL increases by $28-10$ %, depending on the system parameters. It was found that the influence of the generated pump electric field is most effective at high frequencies and relatively low energies of the electron beam when the amplification of the electromagnetic signal due to the first parametric resonance is significantly less compared to the amplification of longitudinal waves due to the second parametric resonance.

Keywords: Superheterodyne Free-electron lasers, Space charge waves, Three-wave parametric interactions, Growth increment, Electrostatic Undulator

1. INTRODUCTION

Free electron lasers (FEL) are used as sources of THz radiation for different domains of human activity, such as remote monitoring of gases, medical inspection, imaging, security inspection, etc. [1-6]. Such FELs have crucial features, e.g., their tunability in a wide frequency range and high power of the THz radiation. The disadvantages of such devices include their larger size and difficulty in controlling their operation. These problems have been well-known for a long time, and researchers and engineers have been struggling with them [1-6].

In this article, we explore THz FEL with higher amplification characteristics – superheterodyne parametric FEL [7, 8]. This superheterodyne FEL to achieve higher growth increments, uses an additional pumping device called an electrostatic undulator. The electrostatic undulator using parametric instability additionally amplifies the longitudinal space charge waves (SCWs) in the relativistic electron beam [7, 8]. Therefore, superheterodyne parametric FEL has higher amplification characteristics.

Recently we analysed the dynamics of SCWs in the field of the electrostatic undulator [8]. We discovered an interesting effect of generating an additional electric field by an electron beam under the influence of an external undulator electric field of pumping [8]. We called this field a generated pump electric field. This field has a significant amplitude and is in phase with the undulator field of the external pump. Therefore, the resulting pump field increases significantly (~ 30 % compared to the external pump field) [8].

In article [8], we analysed the SCW behaviour in the electrostatic undulator without the electromagnetic signal and transverse *H*-ubitron pumping. The presented work aims to study the influence of the generated electric pump field on the amplification of the electromagnetic signal in the superheterodyne parametric FEL already in the presence of the electromagnetic signal and transverse *H*-ubitron pumping. This analysis in the presented work was carried out in a quadratic nonlinear approximation regarding the amplitudes of interacting waves.

2. MODEL

We consider the behaviour of waves in the superheterodyne parametric FEL whose scheme is shown in Fig. 1. This FEL contains two devices: *H*-ubitron undulator 2 and electrostatic undulator 3. *H*-ubitron undulator creates a periodic magnetic field B_2, that induction is perpendicular to the relativistic electron beam (REB) (position 1, Fig. 1). Electrostatic undulator creates a longitudinal reversible periodic electrostatic field E_{20}, the strength of which is parallel to passing REB (position 2, Fig. 1).

The input electromagnetic signal is a plane monochromatic electromagnetic wave with electric E_1 and magnetic fields B_1 propagating along the REB.

In the FEL under study, there are two three-wave parametric resonances. The first resonance involves the transverse electromagnetic wave, the magnetic field of *H*-ubitron pumping B_2, and the longitudinal slow SCW E_α. Due to this resonance, a slow space charge wave E_α is excited. The slow E_α, fast E_β space charge waves and the electrostatic pumping field of the undulator E_{20} take part in the second resonance.
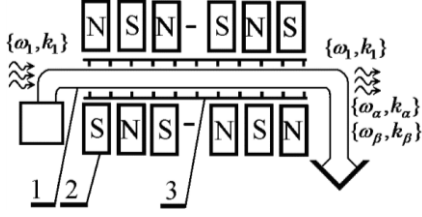
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Fig. 1 – Simplified scheme of a superheterodyne parametric FEL: 1 – relativistic electron beam; 2 – *H*-ubitron undulator; 3 – electrostatic undulator

The electric field strengths *E* and magnetic field inductions *B* that take part in parametric resonances have the following form

$$\mathbf{B}_2 = \left[B_2 \exp(ip_{2,B}) + c.c. \right] \mathbf{e}_y , \tag{1}$$

$$\mathbf{E}_{20} = [E_{20} \exp(ip_{2,E}) + c.c.] \mathbf{e}_z , \tag{2}$$

$$\mathbf{E}_1 = \left[E_1 \exp(ip_1) + c.c. \right] \mathbf{e}_x , \tag{3}$$

$$\mathbf{B}_1 = \left[B_1 \exp(ip_1) + c.c. \right] \mathbf{e}_y , \tag{4}$$

$$\mathbf{E}_\alpha = \sum_{m=1}^{N} \left[E_{\alpha,m} \exp\left(ip_{\alpha,m}\right) + c.c. \right] \mathbf{e}_z , \tag{5}$$

$$\mathbf{E}_\beta = \sum_{m=1}^{N} \left[E_{\beta,m} \exp\left(ip_{\beta,m}\right) + c.c. \right] \mathbf{e}_z . \tag{6}$$

In these equations B_2, E_{20}, E_1, B_1, $E_{\alpha,m}$, $E_{\beta,m}$ are the complex amplitudes of the correspond fields, $p_{2,B}$, $p_{2,E}$, p_1, $p_{\alpha,m}$, $p_{\beta,m}$, are their phases, e_x, e_y, e_z are unit vectors on the corresponding axis, *m* is the number of the *m*th harmonic.

Phases of the input electromagnetic signal p_1, slow $p_{\alpha,m}$ and fast $p_{\beta,m}$ SCWs are defined as follows:

$$p_1 = \omega_1 - k_1 z , \quad p_{2,E} = -k_{2,E} z , \quad p_{2,B} = -k_{2,B} z ,$$
$$p_{\alpha,m} = m\omega_\alpha - k_{\alpha,m} z , \tag{7}$$
$$p_{\beta,m} = m\omega_\beta - k_{\beta,m} z .$$

The dispersion conditions determine wave numbers of the input signal k_1, slow $k_{\alpha,m}$ and fast $k_{\beta,m}$ SCWs.

$$k_1 = \sqrt{\omega_1^2 - \omega_p^2 / \gamma_0^2} / c ,$$
$$k_{\alpha,m} = m\omega_\alpha / \upsilon_{z0} + \omega_p / \left(\gamma_0^{3/2} \upsilon_{z0} \right) , \tag{8}$$
$$k_{\beta,m} = m\omega_\beta / \upsilon_{z0} - \omega_p / \left(\gamma_0^{3/2} \upsilon_{z0} \right) .$$

where ω_p is the Langmuir frequency, υ_{z0} is the averaged electron beam velocity, γ_0 is its Lorentz factor.

As we mentioned above, the superheterodyne FEL has two three-wave parametric resonances. The first one is the interaction between the input electromagnetic signal and the powerful pump magnetic field that perturbs the slow SCW in the electron beam. The condition for the three-wave parametric resonance of this interaction is the following phases relationship:

$$p_{\alpha,1} = p_1 + p_{2,B} \tag{9}$$

or putting $\omega_\alpha = \omega_1$:

$$k_{\alpha,1} = k_1 + k_{2,B} \tag{10}$$

The second interaction is between the slow SCW, fast SCW and the powerful pump electrostatic field that amplifies the slow SCW and creates and amplifies the fast SCW. Similarly, for the second three-wave resonance, we have the following condition for their phases:

$$p_{\alpha,m} - p_{\beta,m} = p_2 \tag{11}$$

or putting $\omega_\alpha = \omega_\beta$:

$$k_{\alpha,n} - k_{\beta,m} = k_{2,E} \tag{12}$$

Using (7), (8) and (12), we get the wave number $k_{2,E}$ of the electrostatic undulator pump field:

$$k_{2,E} = 2\omega_p / \left(\gamma_0^{3/2} \upsilon_{z0} \right)$$

We easily obtain the undulation period Λ of the electrostatic undulator [3] from the last equation:

$$\Lambda = \pi \gamma_0^{3/2} \upsilon_{z0} / \omega_p \tag{13}$$

In article [8], we found an interesting effect. The relativistic electron beam passing through the longitudinal periodic electrostatic field generates its own pump electric field with the same phase as the original. This field is called the generated pump electric field. The strength of this field is defined by the formula:

$$\mathbf{E}_2^g = \mathbf{E}_{20} / \left(\left(k_2^2 \upsilon_{z0}^2 \gamma_0^3 / \omega_p^2 \right) - 1 \right) . \tag{14}$$

Let us consider a qualitative picture of the occurrence of the generated pump electric field E₂. The electrostatic undulator creates a periodically reversing external longitudinal electric pump field. As a result, areas of positive and negative potential appear in the FEL. When moving through the space in the negative potential region, the electron beam is decelerated; in the positive potential area, it is accelerated. Based on considerations of the continuity equation, we can conclude that the concentration of beam electrons becomes greater in the negative potential region. In the area of positive potential, it becomes smaller. Such a modulated electron beam generates its own pump electric field, which has the same phase as the external one. Therefore, the total pump field, that is the sum of the external and generated pump fields, becomes more than the external one.

Please note that the described phenomenon occurs only in the case of transit beam motion through the electrostatic undulator.

The total pump electric field we present as a superposition of the electrostatic undulator pump field and generated one in the form:

$$\mathbf{E}_2 = \mathbf{E}_{20} + \mathbf{E}_2^g + \left[E_2 \exp(ip_{2,E}) + c.c. \right] \mathbf{e}_z . \tag{15}$$

3. BASIC EQUATIONS

As initial equations, we use the hydrodynamic equation to describe electron motion in electric and magnetic fields, the continuity equation that defines the beam's local charge concentration, and Maxwell's equation, which describes electric strength.

We solve it with the hierarchical asymptotical method [9]. Applying this method to our model, we obtain another system of differential equations for the amplitudes of electric field strengths.

$$C_{1,1}\frac{dE_1}{dz} = C_{3,1,1}^I E_{\alpha,1} B_2^* ,$$

$$C_{1,\alpha,m}\frac{dE_{\alpha,m}}{dz} = C_{3,\alpha,m}^I E_1 B_2 \cdot \delta_{m,1} + C_{3,\alpha,m}^{II} E_{\beta,m} E_2 ,$$

$$C_{1,\beta,m}\frac{dE_{\beta,m}}{dz} = C_{3,\beta,m}^{II} E_{\alpha,m} E_2^* ,$$

$$C_{1,2}\frac{dE_2^g}{dz} + D_2 E_2^g = \left(C_0 E_{20} + \sum_{n=1}^{N} C_{3,2,n} E_{\alpha,n}^* E_{\beta,n} \right)\delta_{m,1} . (16)$$

Coefficients C are defined as follows:

$$C_{1,1} = \frac{\partial D_1}{\partial(-ik_1)} ,$$

$$C_{3,1,m}^I = \frac{\omega_p^2 m \cdot \omega_1 \cdot e}{2\Omega_{\alpha,m} m_e \gamma_0^4 k_1 k_{2,m} c^3 \upsilon_{z0}^2} \left(\frac{\upsilon_{z0}^2 \gamma_0^2}{c^2} - \frac{k_{\alpha,m}}{\Omega_{\alpha,m}} \right) ,$$

$$C_{1,\chi,m} = \frac{\partial D_{\chi,m}}{\partial(-ik_{\chi,m})} ,$$

$$C_{3,\alpha,m}^I = \frac{\omega_p^2 e k_{\alpha,m}}{\upsilon_{z0}^2 \Omega_{\alpha,m}^2 m_e \gamma_0^2 c^2 k_{2,m}} \left(\frac{\upsilon_{z0}}{c} - \frac{k_{\alpha,m} c}{m \cdot \omega_\alpha} \right) ,$$

$$C_{3,\alpha,m}^{II} = \frac{-k_{\alpha,m}\omega_p^2 e / m_e}{\Omega_{\alpha,m}\Omega_{\beta,m}\Omega_{2,m}\gamma_0^6 \upsilon_{z0}^3} \times \left(\frac{k_{\alpha,m}}{\Omega_{\alpha,m}} + \frac{k_{\beta,m}}{\Omega_{\beta,m}} + \frac{k_{2,m}}{\Omega_{2,m}} - \frac{3\upsilon_{z0}^2\gamma_0^2}{c^2} \right) ,$$

$$C_{3,\beta,m}^{II} = -k_{\beta,m} C_{3,\alpha,m}^{II} / k_{\alpha,m} ;$$

$$C_{3,2,m}^{II} = -k_{2,m} C_{3,\alpha,m}^{II} / k_{\alpha,m} ;$$

$$C_0 = -ik_2\omega_p^2 / (\Omega_{2,1}^2\gamma_0^2)$$

Dispersion functions D_1 and D_χ are defined as follows:

$$D_1 \equiv \left[k_1^2 - \frac{\omega_1^2}{c^2} - \frac{\omega_p^2}{\gamma_0 c^2} \right] = 0 ,$$

$$D_{\chi,m} \equiv -ik_{\chi,m}\left(1 - \frac{\omega_p^2}{\Omega_{\chi,m}^2\upsilon_{z0}^2\gamma_0^3} \right).$$

Other parameters of the plasma frequency ω_p, Lorentz factor γ_0, and $\Omega_{\chi,m}$ are defined as follows:

$$\omega_p^2 = 4\pi n_0 e^2 / m_e , \quad \gamma_0 = 1 / \sqrt{1 - (\upsilon_{z0} / c)^2} ,$$

$$\Omega_{\chi,m} = m\omega_\chi / \upsilon_{z0} - k_{\chi,m} .$$

Here $\delta_{m,1}$ is Kronecker's symbol, χ denotes α, β; D_1, D_χ are dispersion equations for the input signal, slow ($\chi = \alpha$, $D_\alpha = 0$), fast ($\chi = \beta$, $D_\beta = 0$) SCWs and generated

pump electric field ($\chi = 2$, $D_2 \neq 0$) respectively; n_0 is averaged electron concentration in the beam, c is the speed of light in vacuum; $e = -|e|$, m_e are charge and mass of the electron respectively.

4. ANALYSIS

Let's study the case when the input signal is a monochromatic electromagnetic wave. Then we put $m = 1$ and $n = 1$ in (16), and the system of equations will have the following form:

$$C_{1,1}\frac{dE_1}{dz} = C_{3,1,1}^I E_{\alpha,1} B_2^* ,$$

$$C_{1,\alpha,1}\frac{dE_{\alpha,1}}{dz} = C_{3,\alpha,1}^I E_1 B_2 + C_{3,\alpha,1}^{II} E_{\beta,1} E_2 ,$$

$$C_{1,\beta,1}\frac{dE_{\beta,1}}{dz} = C_{3,\beta,1}^{II} E_{\alpha,1} E_2^*$$

$$C_{1,2}\frac{dE_2^g}{dz} + D_2 E_2^g = C_0 E_{20} + C_{3,2,1} E_{\alpha,1}^* E_{\beta,1} . \quad (17)$$

To analyse the amplifying properties of the studied model, we plotted the dependence of the signal electric strength amplitude E_1 on the longitudinal coordinate z (Fig. 2), the growth increment Γ on the Lorentz factor γ_0 (Fig. 3) and on input monochromatic electromagnetic signal frequencies ω_1 (Fig. 4). For the numerical solution of the model, we set the initial values of the signal frequency $\omega_1 = 3.0 \cdot 10^{12}$ s^{-1}, electron beam plasma frequency $\omega_p = 3.0 \cdot 10^{10}$ s^{-1}, Lorentz factor $\gamma_0 = 3.0$, undulator pump electric field strength amplitude $|E_2| = 56\ kV/cm$, undulator pump magnetic field strength amplitude $|B_y| = 0.08$ T. Initially, only the electromagnetic wave of the signal is fed into the system. The slow and fast SCWs are excited during the interaction of the REB with the electric and magnetic fields in the amplification section.

Using (17), we can analyse the dependence of the amplitude of the electric field strength of the first harmonic of the electromagnetic wave signal on the longitudinal coordinate z. Fig. 2. shows these dependencies under two conditions: without the effect of the generated pump electric field (curve 1) and with this effect (curve 2). We see that the generated pump electric field significantly increases the growth increment of the electromagnetic signal (curve 2) compared to the case when there is no such effect (curve 1).

From Fig. 2 we see that the increase in the electromagnetic wave occurs according to an exponential law, that is $E_1 \sim E_{10}\exp(\Gamma z)$, where Γ is the growth increment of the electromagnetic wave in the FEL. Using the data from Fig. 2, it is easy to calculate the growth increments of the electromagnetic wave both in the case of the presence of a generated pump electric field (curve 2) and without it (curve 1). The growth increment characterises the amplification properties of a superheterodyne parametric FEL, which simultaneously uses two interconnected parametric resonances.

Also, using such growth increments, we can evaluate the amplification characteristics of each of the two parametric resonances separately. So, if we assume that the strength of the pump electric field, which is

created by the electrostatic undulator, is equal to zero ($E_2 = 0$), then using the system of equations (17), it is possible to plot dependencies like those shown in Fig. 2. From this figure we can obtain the growth increment of the electromagnetic wave of the signal, which defines the amplification characteristics of a three-wave parametric resonance, in which only the electromagnetic signal, the H-ubitron magnetic pump field and the slow wave of the SCW are involved. We denote such an increment by Γ^{I}.
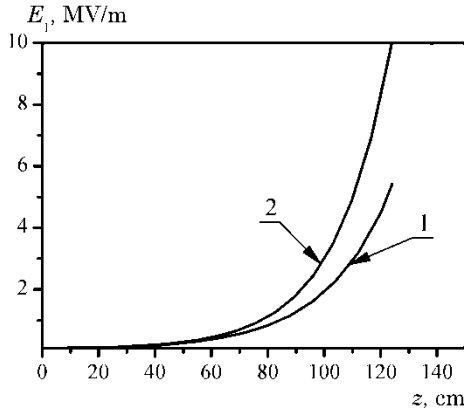

Fig. 2 – The dependence of the electric strength amplitude of the electromagnetic signal on the longitudinal coordinate z under two conditions: without the effect of the generated pump electric field (curve 1); with this effect (curve 2).

If we consider the case when the induction of the H-ubitron pump magnetic field is zero ($B_2 = 0$) and a modulated electron beam is supplied to the input of the studying device, we can plot, using system (17), the dependence of the strength amplitude of the slow SCW wave on the longitudinal coordinate and determine the growth increment of this wave. This growth increment defines the amplification characteristics of a three-wave parametric resonance, in which only the longitudinal slow and fast SCWs and the longitudinal pump electric field are involved. We denote such an increment by Γ^{II}.

Using the entered increments Γ, Γ^{I}, Γ^{II}, we will further analyse the amplification characteristics of a superheterodyne parametric FEL at various parameters, both with and without the effect of the generated pump electric field.

In Fig. 3 we can see the wave growth increments dependencies (Γ, Γ^{I}, Γ^{II}) on the Lorentz factor γ_0. These dependencies were obtained by taking into account the influence of the generated pump electric field (curves 1, 4, 5) and without such taking into account (curves 1, 2, 3).

Analysing this figure, we can draw the following conclusions. The generated pump electric field significantly enhances the growth increment Γ^{II}, while it does not affect the Γ^{I}. As a result, the total increment Γ also increases by approximately $28 - 10\%$ at the beam Lorentz factor range $2.0 - 4.0$. The generated pump electric field most significantly increases the growth increment of a superheterodyne parametric FEL Γ at relatively low Lorentz factor values.

Fig. 4 shows the wave growth increments dependencies (Γ, Γ^{I}, Γ^{II}) on the signal frequency ω_1. The de-

pendences Γ, Γ^{I}, Γ^{II} on the signal frequency ω_1 were obtained both for the case of taking into account the influence of the generated pump electric field (curves 1, 4, 5) and without such taking into account (curves 1, 2, 3). We can see that only the growth increment Γ^{I} depends on the input signal frequency in inverse proportion (curve 1), but the component Γ^{II} (curves 2, 4) remains unchanged. As in the case Fig. 3, the generated pump electric field significantly enhances the Γ^{II} growth increment, which allows us to significantly increase the total growth increment Γ in a wide frequency range of the input signal (see Fig. 4). At the signal frequency $\omega_1 = 1.5\ THz$ the growth increment Γ with the generated pump electric field is by 10 % bigger than in the case when this effect is absent. This difference increases with increasing signal frequency, reaching an upper limit of approximately 30 % for the signal in the upper-frequency region.
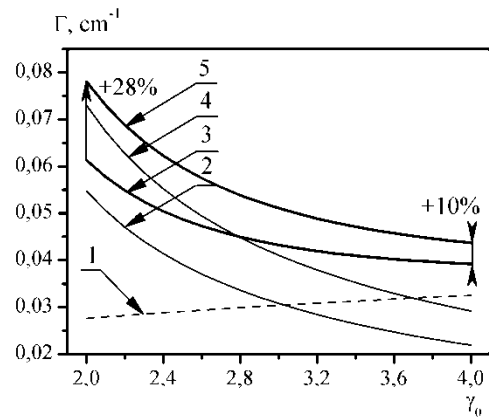

Fig. 3 – The dependencies of the wave growth increments (Γ, Γ^{I}, Γ^{II}) on the Lorentz factor γ_0. Curve 1 corresponds to the dependence $\Gamma^{\mathrm{I}} = \Gamma^{\mathrm{I}}(\gamma_0)$, curves 2 and 4 correspond to the dependencies $\Gamma^{\mathrm{II}} = \Gamma^{\mathrm{II}}(\gamma_0)$, curves 3 and 5 correspond to the dependencies $\Gamma = \Gamma(\gamma_0)$. Curves 1, 2, 3 are obtained for the case of the absence of the pump electric field generation effect, and curves 1, 4, 5 are obtained for the case when the generated pump electric field is considered.
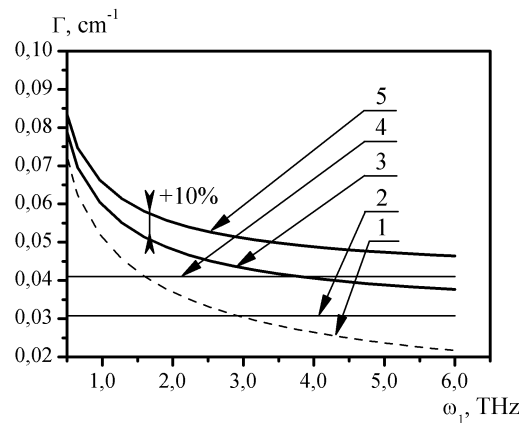

Fig. 4 – The dependencies of the wave growth increments (Γ, Γ^{I}, Γ^{II}) on the signal frequency ω_1. Curve 1 corresponds to the dependence $\Gamma^{\mathrm{I}} = \Gamma^{\mathrm{I}}(\omega_1)$, curves 2 and 4 correspond to the dependencies $\Gamma^{\mathrm{II}} = \Gamma^{\mathrm{II}}(\omega_1)$, curves 3 and 5 correspond to the dependencies $\Gamma = \Gamma(\omega_1)$. Curves 1, 2, 3 are obtained for the case of the absence of the pump electric field generation effect, and curves 1, 4, 5 are plotted for the case when the generated pump electric field is considered.

5. CONCLUSIONS

Thus, in our work, within the framework of the quadratic nonlinear approximation, we obtained a system of equations for the strength amplitudes of interacting waves. Analysing the dynamics of the waves, we determine the growth increments for each of the two three-wave parametric resonances (Γ^{I} and Γ^{II}) and the growth increment of the entire system (Γ).

We demonstrated that the generated pump electric field significantly affects the amplification of the longitudinal space charge waves in the electrostatic undulator, increasing their growth increment Γ^{II} by 33 %. Due to this, the growth increment of the entire FEL increases by $28 - 10$ %, depending on the system parameters. Our analysis shows that the most significant influence of the generated pump electric field occurs at high frequencies of the electromagnetic signal and relatively low energies of the electron beam. This happens because it is precisely under this condition that the growth increment of the electromagnetic signal Γ^{I} significantly decreases compared to the growth increment of the slow SCW Γ^{II}.

REFERENCES

1. Y.U. Jeong, K.-H. Jang, S. Bae, V. Pathania, J. Mun, K. Lee, *J. Korean Phys. Soc.* **80**, 367 (2022).
2. H.P. Freund, M.V. Fazio, P.G. O'Shea, R.B. True, *IEEE Trans. Plasma Sci.* **50**, 888 (2023).
3. V. Pathania, S. Bae, K.-H. Jang, K. Lee, Y.U. Jeong, *Opt. Continuum, OPTCON* **1**, 91 (2022).
4. L. Yan, Z. Liu, *Nat. Photon.* **16**, 404 (2022).
5. A. Fisher, Y. Park, M. Lenz, A. Ody, R. Agustsson, T. Hodgetts, A. Murokh, P. Musumeci, *Nat. Photon.* **16**, 441 (2022).
6. N. Sei, T. Sakai, Y. Hayakawa, Y. Sumitomo, K. Nogami, T. Tanaka, K. Hayakawa, *Sci. Rep.* **11**, 3433 (2021).
7. V.V. Kulish, A.V. Lysenko, I.V. Gubanov, A. Yu. Brusnik, Pat. 87750 , Ukraine, MPK(2009): H05H 9/00, publ. 10.08.2009, bull. No 15/2009.
8. A.V. Lysenko, S.S. Ilin, *J. Nano- Electron. Phys.* **14**, 05006 (2022).
9. V.V. Kulish, *Hierarchic Electrodynamics and Free Electron Lasers: Concepts, Calculations, and Practical Applications* (CRC Press: 2011).

Вплив генерованого електричного поля накачки на підсилювальні властивості супергетеродинного параметричного лазера на вільних електронах

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У роботі в рамках квадратичного нелінійного наближення проведено аналіз впливу генерованого електричного поля накачки на підсилювальні властивості супергетеродинного параметричного лазера на вільних електронах (ЛВЕ). Високі підсилювальні властивості такого пристрою забезпечуються за рахунок реалізації двох пов'язаних між собою трихвильових параметричних резонансів. Завдяки першому з цих резонансів між електромагнітною хвилею сигналу, Н-убітронним полем накачки та повільною хвилею просторового заряду (ХПЗ) відбувається підсилення першої. Другий з цих резонансів між поздовжніми електричним полем накачки, повільною і швидкою хвилями ХПЗ дозволяє додатково підсилювати повільну ХПЗ. Зв'язок між двома параметричними резонансами забезпечується загальною для обох резонансів повільною ХПЗ. У роботі проаналізовано інкременти зростання хвиль кожного з вищезгаданих трихвильових резонансів як окремо, так і всієї системи в цілому. Продемонстровано, що ефект генерації додаткового електричного поля накачки істотно впливає на інкремент зростання другого параметричного резонансу, збільшуючи його на 33 %. Завдяки цьому інкремент зростання всього ЛВЕ збільшується на 28 – 10 % залежно від параметрів системи. З'ясовано, що вплив генерованого електричного поля накачки найбільш ефективний при високих частотах і порівняно низьких енергіях електронного пучка, коли підсилення електромагнітного сигналу за рахунок першого параметричного резонансу істотно менше порівняно з підсиленням поздовжніх хвиль за рахунок другого параметричного резонансу.

Ключові слова: Супергетеродинний лазер на вільних електронах, Хвилі просторового заряду, Трихвильова параметрична взаємодія, Інкремент зростання, Електростатичний ондулятор.