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Entanglement Properties of a Three-Mode Atom-Molecule Bose-Einstein Condensates: System Considering the Interactions due to the *s*-wave Intramodal Elastic Scattering

S.K. Giri*

Department of Physics, Panskura Banamali College (Autonomous), 721152 Panskura, India

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Ultracold atoms in the atomic Bose-Einstein condensate (ABEC) state can form molecular Bose-Einstein condensate (MBEC) through photoassociation. In the atom-molecule Bose-Einstein condensates (BECs), two or more atoms in the ABEC can combine to form a molecule in the MBEC and again a molecule from a MBEC can decompose to atoms in the ABEC. The Bose-stimulated Raman adiabatic passage is an efficient mechanism for conversion of an atomic BEC to a molecular BEC. A three-mode atom-molecule Bose-Einstein condensates system can be prepared through the photoassociative Bose-stimulated Raman adiabatic passage. In our system, three modes are one ABEC, one excited MBEC, and one stable MBEC. The intramodal interactions due to the $\chi^{(3)}$ nonlinearity is present in all three BEC modes along with ABEC-excited MBEC and excited MBEC-stable MBEC intermodal couplings. The quantum mechanical Hamiltonian of the system is constructed considering all three intraspecies interactions and intermodal couplings among the modes. The Hamiltonian of the system is solved analytically using a special intuitive approach which is more general and gives more accurate result than the well-known short time approximation method. The correctness of the solution is verified through the equal time commutation relation. Staring from a three-mode composite coherent state we compute the time evolution of the field annihilation operators of all three modes in presence of all possible interactions and couplings. Using these solutions, we investigate the quantum entanglement properties of the system for all three two-mode combinations. Entanglement is found for two combinations of modes, where as one combination is always separable. Also, we study the dependence of the entanglement properties of the system with the interaction and coupling constants.

Keywords: Bose-Einstein condensates, Bose-stimulated Raman adiabatic passage, Quantum entanglement, Qubit.

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1. INTRODUCTION

The basic resource for realization of quantum computation, quantum communication is the quantum entanglement. There are different physical systems that can generate quantum entanglement which have practical importance. Such as, quantum dots, cavity quantum electrodynamics, nuclear magnetic resonance, ion trap, BEC of dilute gas. The BEC system may be an ABEC system, may be a MBEC system, or may be an atom-molecule BEC system [1]. The entangled states of BECs of weakly interacting gas have tremendous applications in quantum communication and quantum computation [2, 3]. A two mode BEC system can be used to realize qubit which is the basic building block of quantum communication, quantum information processing [4]. Two weakly coupled BECs can produce the Josephson charged qubits [4], quantum algorithm can be implemented using BECs [5], two-component BECs coupled via optical fibre can transfer quantum states [6], and Quantum entanglement is the only way to achieve the Heisenberg-limited sensitivity. BEC can be used as a quantum probe to enhance measurement sensitivity in quantum metrology [7]. To use the atoms in the BEC as a quantum probe, we require two-mode BEC which will act as qubit. The BEC qubits are macroscopic as in a BEC large numbers of bosons occupy the same state. A three-mode BEC system may be considered as a three qubits quantum system [8].

In this study, we consider a three-mode atom-molecule BECs system. One mode is atomic BEC and the other two are molecular BECs. This is an isolated three-mode system, involving no decaying terms, any irreversible loss to the environment. Such system can be formed through the two-colour free-bound-bound photoassociation, where the first laser initially prepares a molecular BEC in the exited state, then the second laser removes the molecule from the excited molecular BEC state to a stable molecular BEC state [1]. The atom-atom and the molecule-molecule elastic interactions give raise the intramodal interactions via $\chi^{(3)}$ nonlinearity in all three modes [9].

This paper is organized as follows. In Sec. 2, we write the model Hamiltonian of the system and solve it analytically to study the time evaluation of the system. Sec. 3 is devoted to study the bipartite entanglement properties of the system. Finally, we concluded in Sec. 4.

2. THE SYSTEM HAMILTONIAN

In this three-mode BEC system, the modes are in a λ configuration as shown Fig. 1.

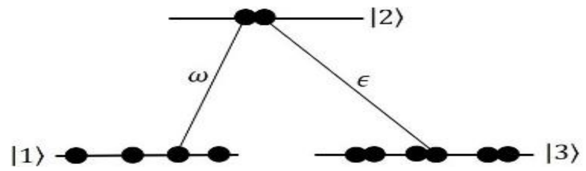


Fig. 1 – Energy level scheme of a three-mode atom-molecule BECs system

* sandipgiri26@gmail.com

The electronic states of atomic BEC, excited molecular BEC, and the stable molecular BEC are leveled as |1>, |2>, and |3>, respectively. The interaction Hamiltonian of this system can be written as [1, 9]

$$H = \delta b^\dagger b - \frac{\omega}{2}\left(a^{\dagger 2}b + a^2 b^\dagger\right) - \frac{\varepsilon}{2}\left(b^\dagger c + bc^\dagger\right) + \chi_a a^{\dagger 2}a^2 + \chi_b b^{\dagger 2}b^2 + \chi_c c^{\dagger 2}c^2, \tag{2.1}$$

where we have considered that the atomic BEC and the stable molecular BEC are in the same electronic state. The boson annihilation operators for ABEC mode |1>, excited MBEC mode |2>, and stable MBEC mode |3>, are a, b, and c, respectively. The intermediate detuning is δ, the effective strength of intermodal coupling between atomic and excited molecular modes is ω, and that between the excited MBEC and stable MBEC modes is ε. The strength of the intramodal interaction constants in the ABEC, excited MBEC, and the stable MBEC are χ_a, χ_b, and χ_c, respectively. Throughout the paper, we have taken $\hbar = 1$.

The time evolution of the field operators is given by the Heisenberg equations of motion, which are

$$\begin{aligned}
\dot{a}(t) &= i\omega a^\dagger(t)b(t) - 2i\chi_a a^\dagger(t)a^2(t),\\
\dot{b}(t) &= -i\delta b(t) + i\frac{\omega}{2}a^2(t) + i\frac{\varepsilon}{2}c(t) - 2i\chi_b b^\dagger(t)b^2(t),\\
\dot{c}(t) &= i\frac{\varepsilon}{2}b(t) - 2i\chi_c c^\dagger(t)c^2(t).
\end{aligned} \tag{2.2}$$

Above equations are the coupled nonlinear equations of the ladder operators associated with the three modes. The exact analytic solution for the time evaluation of the field operators is until unknown. We have to proceed through some approximations. Here we used a special approximation method [10]. In a previous work, we already established that this solution method gives more accurate result than the usual short time approximation [11]. Using this solution method, the time evaluation of the field operators in Heisenberg picture is derived as

$$\begin{aligned}
a(t) &= f_1 a(0) + f_2 a^\dagger(0)b(0) + f_3 a^\dagger(0)a^2(0) + f_4 a^\dagger(0)b(0) + f_5 a^\dagger(0)c(0)\\
&\quad + f_6 a^\dagger(0)a^2(0) + f_7 a(0)b^\dagger(0)b(0) + f_8 a^{\dagger 2}(0)a(0)b(0)\\
&\quad + f_9 a^\dagger(0)b^\dagger(0)b^2(0) + f_{10}a^3(0)b^\dagger(0) + f_{11}a^{\dagger 2}(0)a^3(0),\\
b(t) &= g_1 b(0) + g_2 a^2(0) + g_3 c(0) + g_4 b^\dagger(0)b^2(0) + g_5 b(0) + g_6 a^2(0)\\
&\quad + g_7 a^\dagger(0)a(0)b(0) + g_8 a^\dagger(0)a^3(0) + g_9 c^\dagger(0)c^2(0) + g_{10}a^{\dagger 2}(0)b^2(0)\\
&\quad + g_{11}a^2(0)b^\dagger(0)b(0) + g_{12}b^\dagger(0)b(0)c(0) + g_{13}b^2(0)c^\dagger(0)\\
&\quad + g_{14}b^\dagger(0)b^2(0) + g_{15}b^{\dagger 2}(0)b^3(0),\\
c(t) &= h_1 c(0) + h_2 b(0) + h_3 c^\dagger(0)c^2(0) + h_4 c(0) + h_5 a^2(0) \boxplus h_6 b^\dagger(0)b^2(0) +\\
&\quad + h_7 c^\dagger(0)c^2(0) + h_8 b^\dagger(0)c^2(0) + h_9 b(0)c^\dagger(0)c(0) + h_{10}c^{\dagger 2}(0)c^3(0),
\end{aligned} \tag{2.3}$$

where the parameters $f_i(t)$ are

$$\begin{aligned}
& f_1 = 1, f_2 = \frac{\omega}{\delta}G(t), f_3 = -2i\chi_a t,\\
& f_4 = -\frac{2\chi_a\omega}{\delta^2}\left[i\delta t - G(t)\right],\ G(t) = 1 - e^{-i\delta t},\\
& f_5 = \frac{\omega\varepsilon}{2\delta^2}\left[i\delta t - G(t)\right],\\
& f_6 = \frac{\omega^2}{2\delta^2}\left[i\delta t - G(t)\right] - 2\chi_a{}^2 t^2,\\
& f_7 = -2\left(f_6 + 2\chi_a{}^2 t^2\right),\\
& f_8 = \frac{2\chi_a\omega}{\delta^2}\left[3G(t) + i\delta t\left\{G(t) - 3\right\}\right],\\
& f_9 = \frac{2\chi_b\omega}{\delta^2}\left[i\delta t e^{-i\delta t} - G(t)\right],\\
& f_{10} = -\frac{2\chi_a\omega}{\delta^2}\left[i\delta t + G^*(t)\right], f_{11} = -2\chi_a{}^2 t^2,
\end{aligned} \tag{2.4}$$

the parameters $g_i(t)$ are

$$\begin{aligned}
& g_1 = e^{-i\delta t}, g_2 = \frac{f_2}{2}, g_3 = \frac{\varepsilon}{2\delta}G(t), g_4 = -2i\chi_b t e^{-i\delta t},\\
& g_5 = \frac{\left(2\omega^2 + \varepsilon^2\right)}{4\delta^2}\left[-i\delta t e^{-i\delta t} + G(t)\right], g_6 = \frac{f_4}{2},\\
& g_7 = \frac{\omega^2}{\delta^2}\left[-i\delta t e^{-i\delta t} + G(t)\right], g_8 = 2g_6,\\
& g_9 = \frac{\varepsilon\chi_c}{\delta^2}\left[-i\delta t + G(t)\right],\\
& g_{10} = \frac{\chi_b\omega}{\delta^2}e^{-i\delta t}\left[i\delta t - G(t)\right],\ g_{11} = f_9,\\
& g_{12} = \frac{2\varepsilon\chi_b}{\delta^2}\left[i\delta t e^{-i\delta t} - G(t)\right],\\
& g_{13} = \frac{\varepsilon\chi_b}{\delta^2}e^{-i\delta t}\left[i\delta t - G(t)\right],\\
& g_{14} = -2\chi_b{}^2 t^2 e^{-i\delta t}, g_{15} = g_{14},
\end{aligned} \tag{2.5}$$

and the parameters $h_i(t)$ are

$$
\begin{aligned}
h_1 &= 1,\\
h_2 &= \frac{\varepsilon}{2\delta}G(t),\\
h_3 &= -2i\chi_c t,\\
h_4 &= \frac{\varepsilon^2}{4\delta^2}\left[i\delta t - G(t)\right],\\
h_5 &= \frac{\omega\varepsilon}{4\delta^2}\left[i\delta t - G(t)\right],\\
h_6 &= \frac{\varepsilon\chi_b}{\delta^2}\left[i\delta t e^{-i\delta t} - G(t)\right],\\
h_7 &= h_{10} = -2\chi_c{}^2 t^2,\\
h_8 &= -\frac{\chi_c\varepsilon}{\delta^2}\left[i\delta t + G^*(t)\right],\\
h_9 &= \frac{2\chi_c\varepsilon}{\delta^2}\left[-i\delta t + G(t)\right].
\end{aligned}
\tag{2.6}
$$

In deriving the above equations, all the interaction constants are approximated up to the second order. The field operators satisfy the bosonic commutation relation and commute with each other's, i.e.

$$
\begin{aligned}
\left[a(0), a^\dagger(0)\right] &= 1,\\
\left[b(0), b^\dagger(0)\right] &= 1,\\
\left[c(0), c^\dagger(0)\right] &= 1,
\end{aligned}
\tag{2.7}
$$

and

$$
\begin{aligned}
\left[a(0), b(0)\right] &= 0,\\
\left[b(0), c(0)\right] &= 0,\\
\left[c(0), a(0)\right] &= 0.
\end{aligned}
\tag{2.8}
$$

In this system the total number of bosons is not conserved, since two bosons in the atomic mode can combine to form one boson in the molecular mode which can also decompose to two atomic bosons. But as the system is isolated from the environment, the total number of atoms (may be in atomic or molecular form) which is $a^\dagger(t)a(t) + 2b^\dagger(t)b(t) + 2c^\dagger(t)c(t)$ of the system is a conserved quantity. Also, the boson annihilation operators satisfy the equal time commutation relations (ETCR), which are

$$
\begin{aligned}
\left[a(t), a^\dagger(t)\right] &= 1,\\
\left[b(t), b^\dagger(t)\right] &= 1,\\
\left[c(t), c^\dagger(t)\right] &= 1.
\end{aligned}
\tag{2.9}
$$

Our solutions for the time evaluation of the field operators satisfy the conservation of particle numbers and ETCR at any instant. That establishes the correctness of our solutions.

3. TWO MODE ENTANGLEMENT

The two-mode entanglement properties of two-mode atom-molecule BECs system is well studied [12]. Such two-mode entangled state can be used as qubit in quantum computation and quantum communications [4]. There is no such work on the two-mode entanglement properties of a three-mode BECs system considering the intramodal interactions of all three modes. In the three-mode BEC system, the entangled states of any two modes can be a qubit. So, there is a possibility of getting three qubits from three entangled two-mode combinations (*ab*, *bc*, and *ca*).

To study the entanglement properties of the system, we consider the system initially in a composite coherent state [13]. So, the initial state can be written as

$$
\left|\psi(0)\right\rangle = \left|1\right\rangle \otimes \left|2\right\rangle \otimes \left|3\right\rangle . \tag{3.1}
$$

The eigen value equations for operations of a, b, and c on the state $\left|\psi(0)\right\rangle$ are

$$
\begin{aligned}
a(0)\left|\psi(0)\right\rangle &= \alpha\left|\psi(0)\right\rangle,\\
b(0)\left|\psi(0)\right\rangle &= \beta\left|\psi(0)\right\rangle,\\
c(0)\left|\psi(0)\right\rangle &= \gamma\left|\psi(0)\right\rangle,
\end{aligned}
\tag{3.2}
$$

where α, β and γ, are the eigen values for ABEC mode |1>, excited MBEC mode |2>, and stable MBEC mode |3>, respectively. There are several sufficient criteria to detect the quantum entanglement. Here we use one of the most useful inseparability criteria known as Hillery-Zubairy criterion-1 (HZ-1) [14, 15]. As per the HZ-1 criterion, two modes represented by their annihilation operators *i,* and *j* is entangled at an instant *t* if

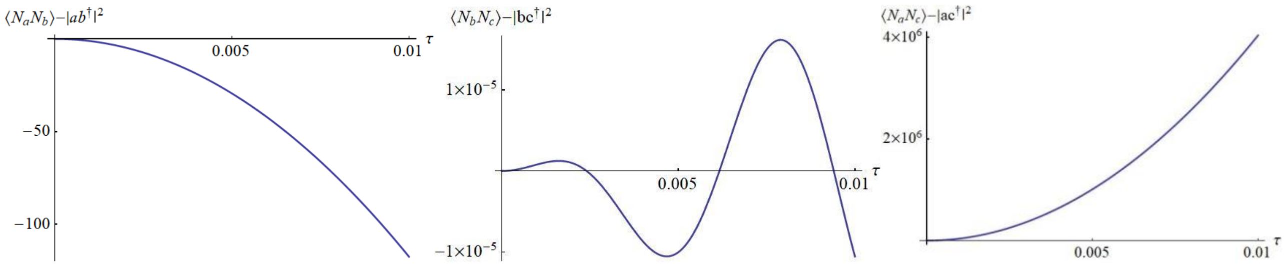
$$
\left\langle N_i(t) N_j(t)\right\rangle - \left|i(t) j^\dagger(t)\right|^2 < 0, \tag{3.3}
$$

where $N_i(t) = i^\dagger(t) i(t)$ is the number operators of the mode i. Now we check the above criterion for all three two-mode combinations, which are ABEC-excited MBEC, excited MBEC-stable MBEC, and ABEC-stable MBEC. After a rigorous calculation, we get

$$
\begin{aligned}
\left\langle N_a(t)N_b(t)\right\rangle - \left|a(t)b^\dagger(t)\right|^2 &= |f_2|^2\left(\frac{1}{2}|\alpha|^2 + |\beta|^2 + |\beta|^4 - |\alpha|^2|\beta|^2\right) + |f_3|^2|\alpha|^4|\beta|^2 + |g_4|^2|\alpha|^2|\beta|^4\\
&\quad + \begin{bmatrix} \left(f_2 f_3^* - 2g_1 g_{10}^* - 2g_1 g_{11}^* - f_2 g_1 g_4^*\right)|\beta|^2\alpha^{*2}\beta \\ + \left(f_{10}^* - f_2 f_3^*\right)|\alpha|^2\alpha^{*2}\beta - g_1 g_{11}^*|\alpha|^2|\beta|^2\alpha^{*2}\beta + c.c. \end{bmatrix}
\end{aligned}
\tag{3.4}
$$

$$\langle N_b(t)N_c(t)\rangle - \left|b(t)c^\dagger(t)\right|^2 = |h_3|^2|\beta|^2|\gamma|^4 + |g_4|^2|\beta|^4|\gamma|^2 + \\ +\left[\left(h_8 + g_1^* g_3 h_3\right)|\gamma|^2\beta^*\gamma + \left(g_1^* g_4 h_2 + g_1^* g_{13}\right)|\beta|^2\beta\gamma^* + c.c.\right], \tag{3.5}$$

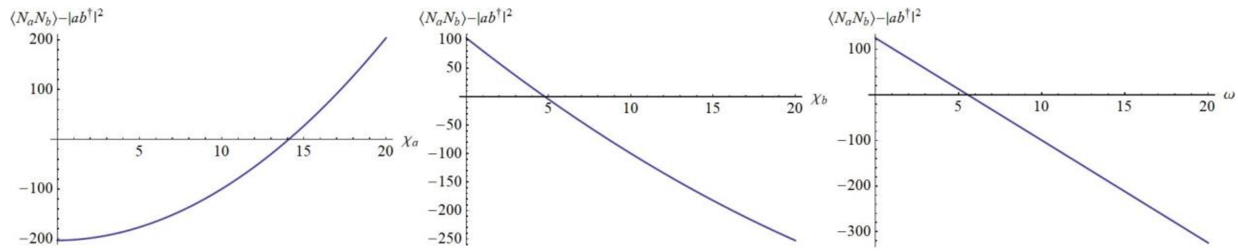
$$\langle N_a(t)N_c(t)\rangle - \left|a(t)c^\dagger(t)\right|^2 = |f_2|^2|\beta|^2|\gamma|^2 + |f_3|^2|\alpha|^4|\gamma|^2 + |h_3|^2|\alpha|^2|\gamma|^4 + \left[f_2 f_3^*|\gamma|^2\alpha^{*2}\beta + c.c.\right]. \tag{3.6}$$



a b c

Fig. 2 – Plot of $\langle N_i(t)N_j(t)\rangle - \left|i(t)j^\dagger(t)\right|^2$ with dimensionless time $\tau = \omega t$ for ABEC-excited MBEC mode for $\alpha = 10$, $\beta = 5$, $\gamma = 100$, $\delta = 1$ KHz (a), for exited MBEC-stable MBEC mode for $\alpha = 0.01$, $\beta = 0.001$, $\gamma = 10$, $\delta = 10$ KHz (b), for ABEC-stable MBEC mode for $\alpha = 10$, $\beta = 5$, $\gamma = 100$, $\delta = 1$ KHz (c)

To study the nature of the above equations, we plot them with rescaled time $\tau = \omega t$ in Fig. 2 for $\omega, \varepsilon, \chi_a, \chi_b, \chi_c = 10$ Hz considering the eigen values α, β, and, γ all are reals. These plots predict the entanglement between the atomic BEC-excited MBEC (Fig. 2a) and excited MBEC-stable MBEC modes (Fig. 2b), whereas the atomic BEC-stable MBEC mode is always separable (Fig. 2c). Stable entanglement is present between the atomic BEC and excited MBEC. The collapse and revival of entanglement with time is found between two molecular BECs.



a b c

Fig. 3– Plot of $\langle N_a(t)N_b(t)\rangle - \left|a(t)b^\dagger(t)\right|^2$ with χ_a for $\chi_b, \omega = 10$ Hz (a), with χ_b for $\chi_a, \omega = 10$ Hz (b), with ω for $\chi_a, \chi_b = 10$ Hz (c)

To study the dependence of the depth of entanglement and the requirements for signature of entanglement of two modes with the intramodal interaction constant of each mode and also on their intermodal coupling, we plot the entanglement properties between the ABEC and excited MBEC modes with χ_a (Fig. 3a), with χ_b (Fig. 3b), and with ω (Fig. 3c) for $\alpha = 10, \beta = 5, \gamma = 100, t = 0.001$ s, $\delta = 1$ KHz, $\chi_c, \varepsilon = 10$ Hz. All three plots show that there is a critical value of each interaction constant for signature of entanglement and the depth of entanglement depends on the magnitudes of χ_a, χ_b, and ω. It is interesting to note that the entanglement properties of two modes depend not only on the intermodal coupling between the modes but also on the intramodal interactions present in each mode. Fig. 3a shows that less the value of χ_a more the depth of entanglement which is just opposite for χ_b (Fig. 3b). So, the intramodal interactions in atomic BEC mode suppress the depth of entanglement whereas that of the molecular mode boosts the depth of entanglement. The intermodal coupling between the ABEC and excited molecular BEC modes enhances the depth of entanglement (Fig. 3c). Equation (3.4) shows that the entanglement properties of atomic BEC-excited MBEC modes is independent of χ_c and ε; i.e., the entanglement properties of any two modes are independent of the interaction and coupling constants related to other modes of the same system. A practically useful qubit for quantum communication and quantum computation requires sustain and deep entanglement between two BECs. To achieve this, we need to consider such a BEC system which has the desired values of the interaction and coupling constants.

4. CONCLUSION

We have considered a three-mode atom-molecule BEC system which can be prepared through the Bose-stimulated Raman adiabatic passage. The Hamiltonian of the system is constructed considering the intraspecies interactions in all three modes and the intermodal couplings in the ABEC-excited MBEC and excited MBEC-stable MBEC modes. The quantum mechanical Hamiltonian of the system is solved analytically approximating the interaction constants beyond the second order. The time evaluation of the system is studied starting from a coherent superposition of all three modes. The two-mode entanglement property of the system which is a key resource of practically realizable qubit is studied explicitly. Such qubit of BEC system has immense applications in quantum computation and quantum communication. The stable entanglement between the ABEC and the excited MBEC modes is reported. The time dependent collapse and revivals of entanglement between the excited MBEC and the stable MBEC modes is also reported. The ABEC and the stable MBEC modes are always separable. We reported that the signature and the depth of entanglement between two modes depend on the intermodal coupling and the intramodal interaction constants of the concerned modes.

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Властивості тримодових атом-молекул бозе-ейнштейнівських конденсатів: система, яка розглядає взаємодію внаслідок внутрішньомодової пружної s-хвилі

S.K. Giri

Department of Physics, Panskura Banamali College (Autonomous), 721152 Panskura, India

Ультрахолодні атоми в стані атомарного бозе-ейнштейнівського конденсату (ABEC) можуть утворювати молекулярний бозе-ейнштейнівський конденсат (MBEC) через фотоасоціацію. У атомно-молекулярних конденсатах Бозе-Ейнштейна (BEC) два або більше атомів можуть об'єднуватися, утворюючи молекулу в MBEC, і знову молекула з MBEC може розкладатися на атоми в ABEC. Стимульований Бозе адіабатичний прохід Рамана є ефективним механізмом для перетворення атомарного BEC у молекулярний BEC. Тримодову атомно-молекулну систему конденсатів Бозе-Ейнштейна можна отримати за допомогою фотоасоціативного адіабатичного переходу комбінаційного розсіювання, стимульованого Бозе. У нашій системі три режими: ABEC, збуджений MBEC і стабільний MBEC. Внутрішня трансмодальна взаємодія через нелінійність $\chi^{(3)}$ присутня у всіх трьох модах BEC разом із збудженими MBEC і MBEC-стабільними інтермодальними зв'язками MBEC. Квантово-механічний гамільтоніан системи побудовано з урахуванням усіх трьох внутрішньовидових взаємодій та інтермодальних зв'язків між модами. Гамільтоніан системи розв'язується аналітично за допомогою спеціального інтуїтивного підходу, який є більш загальним і дає більш точний результат, ніж добре відомий метод короткочасної апроксимації. Правильність розв'язку перевіряється через рівночасове комутаційне співвідношення. Виходячи з тримодового композитного когерентного стану, була обчислена часова еволюція операторів анігіляції поля всіх трьох мод за наявності всіх можливих взаємодій і зв'язків. Використовуючи ці рішення, ми досліджуємо властивості квантової заплутаності системи для всіх трьох двомодових комбінацій. Сплутаність виявлена для двох комбінацій режимів, де одна комбінація завжди роздільна. Також досліджено залежність властивостей заплутаності системи від констант взаємодії та зв'язку.

Ключові слова: Конденсати Бозе – Ейнштейна, Бозе-стимульований адіабатичний прохід Рамана, Квантова заплутаність, Кубіт.