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REGULAR ARTICLE

Quantum Statistical Properties of a Three-Mode Atom-Molecule Bose-Einstein Condensates System for Non-Zero Intraspecific Interactions: Antibunching

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Stable molecular Bose-Einstein condensate (MBEC) can be formed through the Bose-stimulated Raman adiabatic passage from atomic Bose-Einstein condensate (ABEC) via an intermediate excited MBEC state. Such a three-mode atom-molecule Bose-Einstein condensates (AMBECs) system is consists of three Bose-Einstein condensates (BECs) in which there are one ABEC, one excited MBEC, and one stable MBEC. Non-zero intermodal couplings and intramodal interactions are present in the system. The free-bound coupling is present between the ABEC and the excited MBEC, and the bound-bound coupling is present between the excited MBEC and the stable MBEC. The intraspecies interactions are due to the third order nonlinearity of the system. The quantum mechanical Hamiltonian of the system is constructed considering the intermodal couplings and intramodal interactions. We solve the system Hamiltonian analytically to find out the time evaluation of the field operators associated with all three BEC modes. In our solution process we have considered up to the second order of all coupling and interaction constants. Employing these solutions and starting from an initial composite coherent state of three BEC modes, we study the second order quantum coherence function of three pure BEC modes and three coupled modes (ABEC-exited MBEC, excited MBEC-stable MBEC, and ABEC-stable MBEC). Quantum antibunching is reported in all three pure BEC modes and in ABEC-excited MBEC, excited MBEC-stable MBEC coupled modes whereas the joint mode of ABEC and stable MBEC is always coherent. We also study the quantum statistical properties of the system by introducing an initial phase to the coherent states of pure BEC modes. The quantum statistical properties of different pure and coupled modes also depend on the initial phase angles of the pure BEC modes. A comprehensive study on this is also reported here.

Keywords: Bose-Einstein condensates, Bose-stimulated Raman adiabatic passage, Photoassociation, Quantum coherence, Quantum antibunching.

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1. INTRODUCTION

The formation of MBEC from the ABEC is possible through photoassocition [1] or through Feshbach resonance [2]. The Bose-stimulated Raman adiabatic passage is an efficient method for conversion of ABEC to MBEC [3]. The stimulated Raman adiabatic passage transfers the atoms from ABEC to molecule in the stable MBEC via an intermediate excited MBEC by the coherent two-colour photoassociation. Such system is a three level BECs system with one ABEC and two MBECs.

From an ordinary light source, photons emit in the form of bunches. So, the incoherent or chaotic light has the photon bunching property. The photons emitted from a coherent source have random distribution. The laser light interacting with a nonlinear medium shows a different type of photon distribution which has no classical explanation, known as photon antibunching. For manipulation of photons in quantum optics the basic requirement is the photon antibunching [4]. The antibunching effect of light can generate single photon source which has immense application in quantum information processing [5-7].

A BEC is equivalent to a laser beam in many ways. The atoms or molecules in BEC have the similar properties of photons in laser beam in many cases with the only exception that the atoms or molecules in BEC can interact with each other. Using extremely dilute condensates we can suppress the interactions. The gaseous BEC can be used as a coherent matter wave sources which have various applications in atom optics, beam splitters, diffraction study, high precession interferometry, and realization of axicon [8-10].

In the recent year the study of quantum statistical properties of BEC has become an important topic. For this we need to define some correlation functions which play a fundamental role in the coherence phenomenon. The first-order coherence in an atom-molecule BEC system is related to the formation of the molecular condensates from the atomic BEC [11]. The second-order quantum coherence properties of the system opens up a completely new property called antibunching which has no classical counterpart. The quantum second order coherence function at zero time delay of mode j defined as $g_j^{(2)}(0)$ which determines the quantum statistical properties of that mode [12]. For $0 \le g_j^{(2)}(0) < 1$, the particle

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distribution of mode j will be sub-Poissonian which generally exhibits the nonclassical antibunching property [13, 14].

The experimental measurement of the first-order temporal coherence in AMBECs system was performed by the Weiman's group [15]. Jin et al. calculated the second-order correlation function for a two-mode AMBECs system [11]. In some previous works we have reported the higher order antibunching of the two-mode BEC system [13, 16]. The sus-Poissonian statistical property of the stable MBEC of a three-mode AMBECs system is reported in Ref. [17]. But no such complete and details work was performed to study the quantum statistical properties of all three pure BEC modes and their coupled modes of a three-mode AMBECs system. Here we study the quantum statistical properties of a three-mode AMBECs system by deriving the second order correlation function in pure and coupled modes. The ABEC and the MBECs are coupled via two-colour photoassociation. The intraspecies interactions are due to the elastic s-wave scattering which introduce the $\chi^{(3)}$ nonlinearity in the system [17].

We organize the paper as follows. Sec. 2 is dedicated for the analytical solution of the Hamiltonian of the system and hence to find out the time evaluation of the field operators. In Sec. 3 we study the quantum statistical properties of the system by evaluating the second-order coherence function. Finally we concluded in Sec. 4.

2. TIME EVALUATION OF THE SYSTEM

The energy level diagram of the three-mode AMBECs system is shown in Fig. 1. The ABEC and the stable MBEC modes are in the same electronic state. The interspecies coupling strength between the ABEC and the excited MBEC is ω and that of the excited MBEC and stable MBEC is ε.

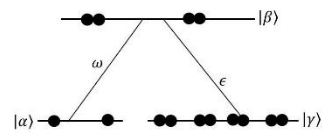


Fig. 1– Scheme of a three-mode AMBECs system in λ configuration

Here $|\alpha\rangle, |\beta\rangle$, and $|\gamma\rangle$ are the electronic states of ABEC, excited MBEC, and the stable MBEC modes, respectively. The three-mode bosonic Hamiltonian for exact two-photon resonance can be written as (taking $\hbar$=1) [3, 18]

$$H = \delta b^\dagger b - \frac{\omega}{2}\left(a^{\dagger 2}b + a^2 b^\dagger\right) - \frac{\varepsilon}{2}\left(b^\dagger c + bc^\dagger\right) + \chi_a a^{\dagger 2}a^2 + \chi_b b^{\dagger 2}b^2 + \chi_c c^{\dagger 2}c^2, \tag{2.1}$$

where δ is the intermediate detuning. The strength of the nonlinear intraspecies interactions in the ABEC, excited MBEC, and the stable MBEC are χ_a, χ_b, and χ_c, respectively. The bosonic annihilation operators for ABEC, excited MBEC, and stable MBEC states are a, b, and c, respectively.

The Heisenberg equations of motion for the field operators are

$$\begin{aligned} \dot{a} &= i\,\omega a^\dagger b - 2i\,\chi_a\, a^\dagger a^2, \\ \dot{b} &= -i\,\delta\, b + i\frac{\omega}{2}\, a^2 + i\frac{\varepsilon}{2}c - 2i\,\chi_b\, b^\dagger b^2, \\ \dot{c} &= i\frac{\varepsilon}{2}b - 2i\,\chi_c\, c^\dagger c^2. \end{aligned} \tag{2.2}$$

These are nonlinear and coupled operator's equations and hence it's difficult to find out the exact analytical solutions in closed-form for the time evaluation of the field operators. But we can proceed through some short of approximated solution method. We solved this using a special approximation method [19]. This method has an excellent degree of accuracy that we have established earlier in a previous work [13]. Solutions for the time evaluation of the field operators considering up to the second order of the coupling and the interaction constants are (In the rest of the paper we have written $a(0) = a$, $b(0) = b$, and $c(0) = c$)

$$\begin{aligned} a(t) &= f_1 a + f_2 a^\dagger b + f_3 a^\dagger a^2 + f_4 a^\dagger b + f_5 a^\dagger c + f_6 a^\dagger a^2 + f_7 a b^\dagger b + f_8 a^{\dagger 2} a b + f_9 a^\dagger b^\dagger b^2 + f_{10} a^3 b^\dagger + f_{11} a^{\dagger 2} a^3, \\ b(t) &= g_1 b + g_2 a^2 + g_3 c + g_4 b^\dagger b^2 + g_5 b + g_6 a^2 + g_7 a^\dagger a b + g_8 a^\dagger a^3 + g_9 c^\dagger c^2 + g_{10} a^{\dagger 2} b^2 + g_{11} a^2 b^\dagger b \\ &\quad + g_{12} b^\dagger b c + g_{13} b^2 c^\dagger + g_{14} b^\dagger b^2 + g_{15} b^{\dagger 2} b^3, \\ c(t) &= h_1 c + h_2 b + h_3 c^\dagger c^2 + h_4 c + h_5 a^2 \boxplus h_6 b^\dagger b^2 + h_7 c^\dagger c^2 + h_8 b^\dagger c^2 + h_9 b c^\dagger c + h_{10} c^{\dagger 2} c^3, \end{aligned} \tag{2.3}$$

where the time dependent parameters $f_i, i \in \{1-11\}$; $g_i, i \in \{1-15\}$; and $h_i, i \in \{1-10\}$ are given in APPENDIX A. The parameters have the properties that at $t = 0$ the first parameters ($i = 1$), i.e., $f_1 = g_1 = h_1 = 1$ and all other parameters ($i \geq 2$) are zero. As time evolves the $i \geq 2$ parameters come out due the presence of the intermodal couplings and intramodal interactions. The operators a, b, and c always satisfy the bosonic commutation relations. In the Bose-stimulated Raman adiabatic passage, two atoms in the ABEC can combine to form a molecule in the excited MBEC state and then the excited molecule can de-excites to a stable molecule to occupy the stable MBEC state. The total number of atoms in all three modes (considering two atoms for one molecule in the MBECs) is a constant of motion. If $N_a(t)$ is the atomic population and $N_b(t)$, $N_c(t)$ are the molecular populations in the ABEC, excited MBEC, and the stable MBEC modes, respectively at any instant t, then

$$N_a(t) + 2N_b(t) + 2N_c(t) = \text{Const.} \tag{2.4}$$

The time evaluation of the annihilation operators have to satisfy the equal time commutation relations (ETCR), which are $\left[a(t),a^{\dagger}(t)\right]=1, \left[b(t),b^{\dagger}(t)\right]=1,$ and $\left[c(t),c^{\dagger}(t)\right]=1$. The solutions (Equation (2.3)) satisfy the both (Equation (2.4) and the ETCR). Hence, the correctness of our solution is verified.

3. SECOND ORDER COHERENCE

The second order correlation is the correlation of intensities. The first experimental measurement of the second order coherence was performed by Hanbury Brown and Twiss [20]. The equal time second order quantum correlation function for mode *j* is defined as [11, 13]

$$g_j^{(2)}(0)=\frac{\left\langle n_j(n_j-1)\right\rangle}{\left\langle n_j\right\rangle^2}, \tag{3.1}$$

where n_j is the number operator of mode *j*. For $g_j^{(2)}(0)>1,$ the particle distribution in mode *j* will be super-Poissonian which exhibits the bunching phenomenon. For coherent state $g_j^{(2)}(0)=1$ and the coherent state always exhibits the Poissonian statistical property. Quantum mechanically $g_j^{(2)}(0)<1$ is also possible that opens up a completely new distribution property which can't be explain by any classical theory. This is known as quantum antibunching effect which also exhibits the sub-Poissonian statistical distribution. In the alternative form $g_j^{(2)}(0)$ can also be written as

$$g_j^{(2)}(0)=1+\frac{\left\langle\left(\Delta n_j\right)^2\right\rangle-\left\langle n_j\right\rangle}{\left\langle n_j\right\rangle^2}=1+\frac{D_j}{\left\langle n_j\right\rangle^2}. \tag{3.2}$$

The term $D_j=\left\langle\left(\Delta n_j\right)^2\right\rangle-\left\langle n_j\right\rangle$ determines the properties of the second order coherence of mode *j*. Here $D_j>0$, $D_j=0$, and $D_j<0$ correspond to super-Poissonian, Poissonian, and sub-Poissonian quantum statistical correlation properties of the system. Now the quantum second order coherence function for the joint mode *ij* is defined as [11, 13]

$$g_{ij}^{(2)}(0)=1+\frac{D_{ij}}{\left\langle n_i\right\rangle\left\langle n_j\right\rangle}. \tag{3.3}$$

For the coupled mode the term $D_{ij}=\left\langle n_i n_j\right\rangle-\left\langle n_i\right\rangle\left\langle n_j\right\rangle$ determines the quantum statistical properties of the mode *ij*. We derive the values of D_j for three pure BEC modes *a*, *b* and *c*, and the values of D_{ij} for the coupled modes *ab*, *bc*, and *ac*.

To study the quantum coherence properties of the system we consider all the BEC modes are initially coherent. Then the composite wavefunction of the system can be written as the direct product of all three coherent states [13, 21]. So, at $t=0$

$$\left|\psi(0)\right\rangle=\left|\alpha\right\rangle\otimes\left|\beta\right\rangle\otimes\left|\gamma\right\rangle. \tag{3.4}$$

Here $\left|\alpha\right\rangle$, $\left|\beta\right\rangle$ and $\left|\gamma\right\rangle$ are the coherent state eigen functions with corresponding eigen values α, β, and γ of modes *a*, *b*, and *c*, respectively. The eigen values may be real or complex. The eigen value equations are

$$\begin{aligned} a\left|\psi(0)\right\rangle&=\alpha\left|\psi(0)\right\rangle,\\ b\left|\psi(0)\right\rangle&=\beta\left|\psi(0)\right\rangle,\\ c\left|\psi(0)\right\rangle&=\gamma\left|\psi(0)\right\rangle. \end{aligned} \tag{3.5}$$

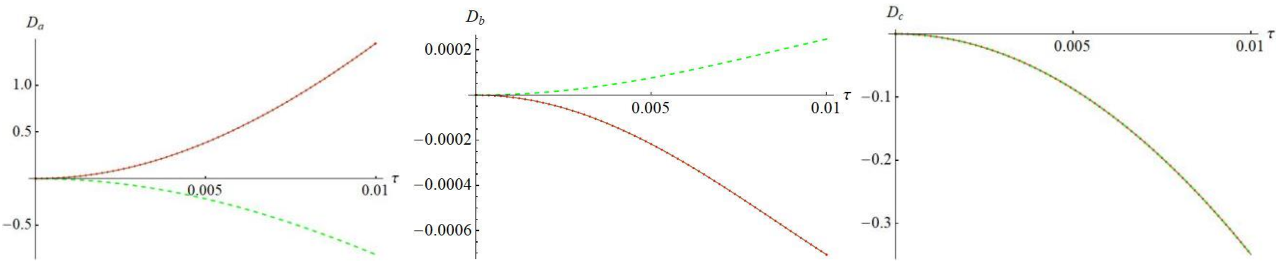
We study the time evaluation of this interacting system in the Heisenberg picture where the operators will be evolved as per Equation (2.3). We derive the values of D_j for three pure BEC modes. They are

$$D_a=\left|f_2\right|^2\left(\left|\beta\right|^2-\frac{1}{2}\left|\alpha\right|^4+6\left|\alpha\right|^2\left|\beta\right|^2\right)+\left[\left(f_2+f_9-f_2f_3\right)\alpha^{*2}\beta+f_5\alpha^{*2}\gamma+\left(2f_8-4f_2f_3\right)\left|\alpha\right|^2\alpha^{*2}\beta+f_9\left|\beta\right|^2\alpha^{*2}\beta+c.c.\right], \tag{3.6}$$

$$D_b=\left[\left(g_1^*g_{11}+2g_2g_4^*\right)\left|\beta\right|^2\alpha^2\beta^*+\left(g_1^*g_{12}+2g_3g_4^*\right)\left|\beta\right|^2\beta^*\gamma+c.c.\right], \tag{3.7}$$

$$D_c=-\left|h_3\right|^2\left|\gamma\right|^6\left(3+2\left|\gamma\right|^2\right)+\left[\left(h_9+2h_2h_3^*\right)\left|\gamma\right|^2\beta\gamma^*+c.c.\right]. \tag{3.8}$$

To study the quantum statistical properties of three pure BEC modes, we plot the above equations in Fig. 2 with dimensionless time $\tau(=\omega t)$ for $\delta=10^5$ Hz, ω, $\varepsilon=10^3$ Hz, $\chi_a=\chi_b=\chi_c=10^{-3}\omega$, $\left|\alpha\right|=\left|\beta\right|=5, \left|\gamma\right|=12$. In the present investigation we have taken the eigen value corresponding to the ABEC mode is complex and hence we can write $\alpha=\left|\alpha\right|e^{-i\theta}$, where θ is the initial phase angle of the ABEC mode. In Fig. 2 the eigen values for other two MBECs are real. Antibunching property is found in all three pure BEC modes and it depends on the initial phase angle of the ABEC mode. Where the ABEC mode exhibits the sub-Poissonian statistical properties for $\theta=\pi/2$ (Fig. 2a), the excited MBEC mode shows the super-Poissonian statistical properties for that value of θ (Fig. 2b). Antibunching and hence the sub-Poissonian statistical property is found in ABEC mode for $\theta=\pi/2$ (Fig. 2a), in excited MBEC mode for $\theta=0, \pi$ (Fig. 2b), and in stable MBEC mode for $\theta=0, \pi/2$, and π (Fig. 2c). Equation (3.8) shows that the statistical property of the stable MBEC mode is independent of the eigen value α and hence the initial phase angle of ABEC mode (Fig. 2c).



a b c

Fig. 2 – Plot of D_j with dimensionless time τ for ABEC mode (a), for exited MBEC mode (b), and for stable MBEC (c). The smooth line, dashed line, and the dotted line are for phase angles $\theta = 0, \pi/2$ and π, respectively

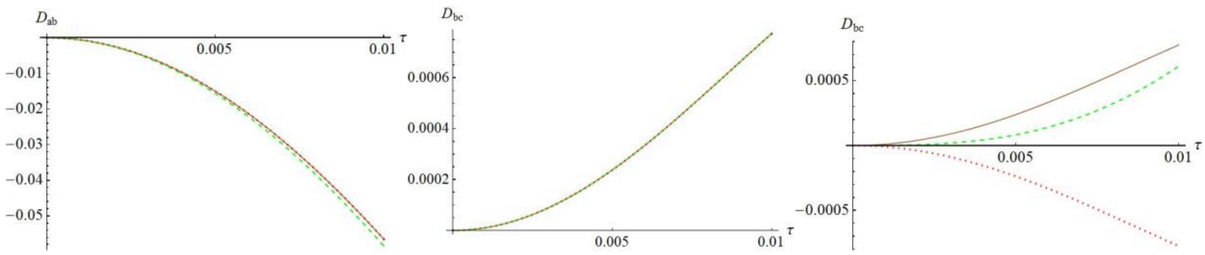
Now, we evaluate the values of $D_{ij} = \langle n_i n_j \rangle - \langle n_i \rangle \langle n_j \rangle$ for three coupled modes which are ABEC-excited MBEC, excited MBEC-stable MBEC, and ABEC-stable MBEC. They are

$$D_{ab} = -4|g_2|^2 |\alpha|^2 |\beta|^2 + \left[g_1^* g_8 |\alpha|^2 \alpha^2 \beta^* + f_9 |\beta|^2 \alpha^{*2} \beta + c.c. \right], \tag{3.9}$$

$$D_{bc} = \left[h_6 |\beta|^2 \beta \gamma^* + g_1^* g_9 |\gamma|^2 \beta^* \gamma + c.c. \right], \tag{3.10}$$

$$D_{ac} = 0. \tag{3.11}$$

We plot the Equations (3.9) and (3.10) in Fig. 3 with rescaled time $\tau (= \omega t)$ taking the same values of the interaction constants, intermediate detuning, coupling constants, and particle numbers as in Fig. 2. Antibunching is reported in the coupled mode of ABEC-excited MBEC irrespective of the initial phase angle of the ABEC mode (Fig. 3a). The coupled mode of two MBECs shows the super-Poissonian statistical behavior and it independent of α and hence the initial phase angle of the ABEC mode. Antibunching is possible in the joint mode of excited MBEC and stable MBEC if we vary the initial phase angle of any MBEC mode (Fig. 3c).



a b c

Fig. 3 – Plot of D_{ij} with rescaled time τ for ab mode (a), and for bc mode (b) & (c). In (a) & (b) the smooth line, dashed line, and the dotted line are for phase angles $\theta = 0, \pi/2$ and π, respectively. In (c) the smooth line, dashed line, and the dotted line are for phase angles $\psi = 0, \pi/2$ and π, respectively

Table 1 – Quantum statistical properties of different modes

Mode	$\phi = 0, \psi = 0$			$\theta = 0, \psi = 0$			$\theta = 0, \phi = 0$		
	$\theta = 0$	$\theta = \pi/2$	$\theta = \pi$	$\phi = 0$	$\phi = \pi/2$	$\phi = \pi$	$\psi = 0$	$\psi = \pi/2$	$\psi = \pi$
a	Super-Poissonian	Sub-Poissonian	Super-Poissonian	Super-Poissonian	Super-Poissonian	Sub-Poissonian	Super-Poissonian	Super-Poissonian	Super-Poissonian
b	Sub-Poissonian	Super-Poissonian	Sub-Poissonian	Sub-Poissonian	Super-Poissonian	Super-Poissonian	Sub-Poissonian	Sub-Poissonian	Sub-Poissonian
c	Sub-Poissonian	Sub-Poissonian	Sub-Poissonian	Sub-Poissonian	Sub-Poissonian	Sub-Poissonian	Sub-Poissonian	Sub-Poissonian	Sub-Poissonian
ab	Sub-Poissonian	Sub-Poissonian	Sub-Poissonian	Sub-Poissonian	Sub-Poissonian	Sub-Poissonian	Sub-Poissonian	Sub-Poissonian	Sub-Poissonian
bc	Super-Poissonian	Super-Poissonian	Super-Poissonian	Super-Poissonian	Sub-Poissonian	Sub-Poissonian	Super-Poissonian	Super-Poissonian	Sub-Poissonian
ac	Poissonian	Poissonian	Poissonian	Poissonian	Poissonian	Poissonian	Poissonian	Poissonian	Poissonian

Now we also consider the complex eigen values of the excited MBEC and stable MBEC modes as $\beta = |\beta| e^{-i\phi}$ and $\gamma = |\gamma| e^{-i\psi}$, respectively. Here ϕ and ψ are the initial phase angles of the excited MBEC and stable MBEC modes, respectively. Fig. 3c shows that the joint mode of two MBECs is sub-Poissonian for $\psi = \pi$. Equation (3.11) shows that the joint mode of ABEC and stable MBEC is remain coherent with time and always exhibit the Poissonian statistical property. The quantum statistical properties of the pure BEC modes and their joint modes generally depend on the initial phase angles of all three pure BEC modes. A details study on this is summarized in Table 1 where we have reported the quantum statistical properties of three pure BEC modes (*a*, *b*, and *c*) and three joint modes (*ab*, *bc*, and *ac*) for $\delta = 10^5$ Hz, $\omega, \varepsilon = 10^3$ Hz, $\chi_a = \chi_b = \chi_c = 10^{-3}\omega$, $|\alpha| = |\beta| = 5$, and $|\gamma| = 12$. Table-1 shows that the modes *c* and *ab* are exhibit the quantum antibunching property for any choice of the initial phases of all three BEC modes. So, the sustainable antibunching properties of stable MBEC and the joint mode of ABEC and excited MBEC may be useful in practical applications. All the pure BEC modes and the joint modes except *ac* can be antibunched for proper combinations of initial phases of the pure BEC modes. The joint mode *ac* shows the Poissonian statistical property irrespective of the initial phases, the coupling constants, and interaction constants.

4. CONCLUSION

We have considered a three-mode AMBECs system which is prepared in Bose-stimulated Raman adiabatic passage. Intermodal couplings are present in between the ABEC and excited MBEC and also in between the excited MBEC and stable MBEC. The intramodal interactions due to the $\chi^{(3)}$ nonlinearity are considered in all three pure BEC modes. The system Hamiltonian is written quantum mechanically considering the intermodal couplings and the intraspecies interactions. The Hamiltonian is solved analytically to derive the time evaluation of the field operators considering up to the second order of coupling and interaction constants. Considering the system initially in a composite coherent state and employing the solutions the second order correlation function is computed for three pure BEC modes (*a*, *b*, and *c*) and three joint modes (*ab*, *bc*, and *ac*). The antibunching property is reported in three pure BEC modes and two joint modes (*ab* and *bc*). The joint mode *ac* is remain coherent with time and hence always exhibits the Poissonian statistical property. We also reported the dependence of the quantum statistical properties with the initial phase angles of the pure BEC modes. A comprehensive study on this of three pure BEC modes and three coupled modes varying the initial phase angles of the pure BEC modes is summarized in Table-1. The BEC-light analogy is expected to find applications of the antibunching properties of the three-mode AMBECs system in quantum information processing. Our solutions can be employed to study the other nonclassical properties of this system.

APPENDIX A
PARAMETERS OF EQUATION (2.3)

The parameters $f_i, i \in \{1-11\}$ are

$$\begin{aligned}
&f_1 = 1,\ f_2 = \frac{\omega}{\delta}G(t),\ f_3 = -2i\chi_a t,\\
&f_4 = -\frac{2\chi_a\omega}{\delta^2}\left[i\delta t - G(t)\right],\\
&f_5 = \frac{\omega\varepsilon}{2\delta^2}\left[i\delta t - G(t)\right],\\
&f_6 = \frac{\omega^2}{2\delta^2}\left[i\delta t - G(t)\right] - 2\chi_a{}^2t^2,\\
&f_7 = -2\left(f_6 + 2\chi_a{}^2t^2\right),\\
&f_8 = \frac{2\chi_a\omega}{\delta^2}\left[3G(t) + i\delta t\left\{G(t) - 3\right\}\right],\\
&f_9 = \frac{2\chi_b\omega}{\delta^2}\left[i\delta t e^{-i\delta t} - G(t)\right],\\
&f_{10} = -\frac{2\chi_a\omega}{\delta^2}\left[i\delta t + G^*(t)\right],\ f_{11} = -2\chi_a{}^2t^2,
\end{aligned} \tag{A1}$$

where $G(t) = 1 - e^{-i\delta t}$. The parameters $g_i, i \in \{1-15\}$ are

$$\begin{aligned}
&g_1 = e^{-i\delta t},\ g_2 = \frac{f_2}{2},\ g_3 = \frac{\varepsilon}{2\delta}G(t),\\
&g_4 = -2i\chi_b t e^{-i\delta t},\\
&g_5 = \frac{\left(2\omega^2 + \varepsilon^2\right)}{4\delta^2}\left[-i\delta t e^{-i\delta t} + G(t)\right],\\
&g_6 = \frac{f_4}{2},\ g_7 = \frac{\omega^2}{\delta^2}\left[-i\delta t e^{-i\delta t} + G(t)\right],\\
&g_8 = 2g_6,\ g_9 = \frac{\varepsilon\chi_c}{\delta^2}\left[-i\delta t + G(t)\right],\\
&g_{10} = \frac{\chi_b\omega}{\delta^2}e^{-i\delta t}\left[i\delta t - G(t)\right],\ g_{11} = f_9,\\
&g_{12} = \frac{2\varepsilon\chi_b}{\delta^2}\left[i\delta t e^{-i\delta t} - G(t)\right],\\
&g_{13} = \frac{\varepsilon\chi_b}{\delta^2}e^{-i\delta t}\left[i\delta t - G(t)\right],\\
&g_{14} = -2\chi_b{}^2t^2e^{-i\delta t},\ g_{15} = g_{14},
\end{aligned} \tag{A2}$$

and the parameters $h_i, i \in \{1-10\}$ are

$$\begin{aligned}
&h_1 = 1,\ h_2 = \frac{\varepsilon}{2\delta}G(t),\\
&h_3 = -2i\chi_c t, h_4 = \frac{\varepsilon^2}{4\delta^2}\left[i\delta t - G(t)\right],\\
&h_5 = \frac{\omega\varepsilon}{4\delta^2}\left[i\delta t - G(t)\right],\\
&h_6 = \frac{\varepsilon\chi_b}{\delta^2}\left[i\delta t e^{-i\delta t} - G(t)\right],\\
&h_7 = -2\chi_c{}^2t^2,\ h_8 = -\frac{\chi_c\varepsilon}{\delta^2}\left[i\delta t + G^*(t)\right],\\
&h_9 = \frac{2\chi_c\varepsilon}{\delta^2}\left[-i\delta t + G(t)\right], h_{10} = -2\chi_c{}^2t^2,
\end{aligned} \tag{A3}$$

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Квантово-статистичні властивості тримодової системи конденсатів Бозе-Ейнштейна атом-молекула для ненульових внутрішньовидових взаємодій: антигрупування

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Стабільний молекулярний конденсат Бозе-Ейнштейна (MBEC) може бути утворений через адіабатичне проходження комбінаційного розсіювання, стимульоване Бозе, з атомарного конденсату Бозе-Ейнштейна (ABEC) через проміжний збуджений стан MBEC. Така тримодова система атом-молекула Бозе-Ейнштейнового конденсату (AMBEC) складається з трьох Бозе-Ейнштейнових конденсатів (BEC), в яких є один ABEC, один збуджений MBEC і один стабільний MBEC. У системі присутні ненульові інтермодальні зв'язки та внутрішньомодальні взаємодії. Вільний зв'язок присутній між ABEC і збудженим MBEC, а зв'язаний зв'язок присутній між збудженим MBEC і стабільним MBEC. Внутрішньовидові взаємодії зумовлені нелінійністю третього порядку системи. Квантово-механічний гамільтоніан системи побудовано з урахуванням міжмодальних зв'язків і внутрішньомодальних взаємодій. Ми розв'язуємо системний гамільтоніан аналітично, щоб знайти часову оцінку операторів поля, пов'язаних з усіма трьома режимами BEC. У нашому процесі вирішення ми врахували до другого порядку всіх констант зв'язку та взаємодії. Використовуючи ці рішення та починаючи з початкового складеного когерентного стану трьох мод BEC, ми досліджуємо функцію квантової когерентності другого порядку трьох чистих мод BEC і трьох пов'язаних мод (MBEC із збудженням ABEC, MBEC із збудженням MBEC та MBEC із стабільним ABEC). Повідомляється про квантове антигрупування у всіх трьох чистих модах BEC і в збудженому ABEC MBEC, збудженому MBEC-стабільному зв'язаному режимі MBEC, тоді як спільний режим ABEC і стабільного MBEC завжди когерентний. Ми також вивчаємо квантові статистичні властивості системи, вводячи початкову фазу до когерентних станів чистих мод BEC. Квантово-статистичні властивості різних чистих і пов'язаних мод також залежать від початкових фазових кутів чистих мод BEC.

Ключові слова: Бозе-ейнштейнівські конденсати, Бозе-стимульований адіабатичний прохід комбінаційного розсіювання, Фотоасоціація, Квантова когерентність, Квантове антигрупування.