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Features of a High-Stiffness Tire Interaction with a Bearing Surface During the Starting Period of Motion

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Abstract. The article emphasizes the importance and necessity of studying the behavior of automobile tires during operation in the starting mode, from the beginning of driving on “cold” tires to stabilizing its temperature and internal pressure. During this period, the main performance characteristics of the tire can change in a relatively wide range. Therefore, the main focus was on the initial period of driving as the most dangerous from the point of view of predicting the behavior of automobile tires. This article presents the results of analyzing a car tire’s condition and behavior during the starting movement. Features of the main parameter for assessing the stiffness characteristics of the tires were investigated. The research was conducted under conditions of low ambient temperatures during the operation of automobile tires. A numerical-analytical approach was used to estimate the stiffness parameters. Simultaneously, the initial data required for a correct analysis were obtained from the experimental results in actual road conditions. The obtained results allow for providing recommendations on the peculiarities of the automobile tires’ operation under adverse conditions, such as low ambient temperatures.

Keywords: adverse conditions, internal pressure, temperature, stiffness coefficient, damping coefficient, industry, innovation and infrastructure.

1 Introduction

Recently, the tire, one of the main components of a vehicle in direct contact with the bearing surface, has undergone significant development, and current trends point to further improvements. For the most part, the development takes place in two main directions: improving the tire design and using the latest materials.

Manufacturers usually provide recommendations on the optimum values of the main characteristics of automobile tires that should be maintained to ensure their efficiency and road safety. However, these recommendations are correct for tire operation only in certain environmental conditions and under certain bearing surface conditions, i.e., they are not universal.

Most of the time, the operating conditions are almost constant and predictable, but there is still a possibility of unfavorable environmental conditions (temperature and humidity) for the operation of automobile tires. There are also certain regions where the ambient air temperature can fluctuate in quite wide ranges, from -40°C to +50°C. Operation of tires in such conditions requires additional attention, as their behavior is somewhat unpredictable [1].

Stiffness is one of the leading performance characteristics of a car tire [2]. It largely determines the behavior of the tire during operation. In turn, stiffness is significantly affected by essential tire characteristics such as internal pressure and temperature, which can vary over a wide range during operation, especially when the vehicle is driven in the starting mode [3].

Accordingly, the stiffness will also not be constant and may differ slightly from the optimal values for safe and efficient operation.

2 Literature Review

The analysis of the main performance characteristics of automobile tires is presented in the works of many scientists. It is worth noting that most of the reviewed works do not study the starting period of driving, i.e., they do not consider the peculiarities of the behavior and condition of the tire. In previous scientific studies by the authors [1, 2], the concept of starting period of movement was first introduced, and the results of experimental studies of some tire characteristics were briefly presented. The presented dependences of the internal pressure and temperature of the tire filler indicate their change during the period of operation at the beginning of driving. An indirect assessment of the tire's rolling resistance was also performed, which showed that the rolling resistance coefficient can vary by 20–25 %, i.e., it may differ from the standard value. In this work, the main parameter for assessing the tire's condition will be its stiffness, namely indicators such as the stiffness and damping coefficients.

Paper [2] investigates the effect of tire stiffness on a car's suspension system. The authors analyze the influence of tire pressure, camber angle, and vehicle speed on tire stiffness and damping coefficient. In addition, an experimental formula (1) was proposed to determine the damping coefficient of a tire depending on the vehicle speed and excitation frequency [2]:

$$c = 1313.2V^{-1.196}\omega^{0.114\ln V-0.364}, \qquad (1)$$

where V – vehicle speed, m/s; ω – excitation frequency, Hz.

It should also be noted that dependence (1) is valid in the speed range of 1–25 m/s and frequencies of 5–100 Hz.

The influence of vehicle mass, moment of inertia, unsprung mass, and center of gravity on the amplitude and period of suspension oscillations was investigated. The body was assumed to be rigid, and the tires and other unsprung masses were considered elements of a system with four degrees of freedom. The equations of motion were solved analytically using the Laplace transform, and verification was performed using the Runge-Kutta numerical method. Based on the results obtained for the initial position, it was found that an increase in tire pressure leads to a redistribution of the load on the front and rear axles. The damping coefficient of the tire decreases on a logarithmic scale for a car with a speed of 1–25 m/s.

The authors of [4] presented the results of studies of tire stiffness parameters. The authors present models of vehicle tires and then experimentally investigate the effect of excitation frequency and filler pressure on the damping and stiffness coefficients of the proposed tires.

The car and motorcycle tire model parameters were obtained using a test bench. Based on the results, it was observed that both factors significantly impact the properties of the proposed tire models. In addition, a passenger car suspension model was investigated with the parameters of the modeled tires to illustrate the effect of tire characteristics on suspension performance.

The tire stiffness increases with the excitation frequency, while the tire damping is high for low frequencies but decreases sharply with increasing frequency. It is shown that the tire stiffness also increases with increasing pressure. At the same time, there is a slight change in tire damping. It has been shown that the change in damping and stiffness coefficients with frequency affects the vehicle's vertical dynamics. Thus, vertical models of cars should consider this effect to reproduce the car's motion correctly.

Papers [5–7] present a computational and experimental method for assessing the interaction features of a rigid bearing surface and an automobile tire. The influence of the surface condition on the stiffness and damping properties of automobile tires is investigated. The method is based on solving the tire-bearing surface contact problem by using finite element analysis. In particular, work [5] analyzes the effect of two types of bumps, their size and shape, and the pressure of the filler in the tire on the tire stiffness properties.

The study [5] showed that tire damping significantly depends on the shape of road irregularities and loading conditions (vertical load, tire pressure). The difference in tire deformation when interacting with a bearing surface of various shapes reaches 20 % at a tire pressure of 0.1 MPa (50 % of the rated pressure) and a wheel load of 3.5 kN (68 % of the rated load).

The study of the influence of the bump shape on the tire's radial stiffness showed that the most significant vertical deflection of the tire occurs in the contact area with triangular bumps with a length of about 20 % of the length of the contact area. This vertical deflection of the tire can reach 42 % of the tire deflection on a flat surface.

The calculations show that the damping coefficient is significantly affected by the tire pressure and the shape of the bumps, e.g., the air pressure in a pothole results in an 18 % reduction in the damping coefficient. For the peak (the same length), the damping coefficient decreases by 14 %. The developed method for assessing the envelope properties of a pneumatic tire allows obtaining new values of the tire's stiffness and damping properties, which can be used in the dynamic analysis of the mechanical system "car – tire – driver".

Paper [8] solves the problem of developing a mathematical model of the straight-line movement of a car, taking into account the elasticity and deformation of tires and, with the same radii of the car's wheels, plane-parallel movement at a constant speed. The generalized forces are the sum of the forces acting on the system and the forces due to deformation. After using these kinetic energies and generalized force expressions, the differential equations of the rectilinear motion of a car were obtained using Lagrange's equations of the 2nd kind.

Based on the obtained mathematical model, it is possible to check and analyze the dynamics and stability of the system in various specific cases, i.e., under the influence of non-potential forces in the tire materials and at relatively high vehicle speeds.

As a result of a brief analysis of scientific papers on the subject, we can conclude that the issues of assessing the stiffness properties of automobile tires are relevant and extremely important in terms of traffic safety research because the behavior of the tire and, accordingly, its stiffness, has a significant impact on the condition of the vehicle and road safety. However, it is worth noting that changes in stiffness indicators require a more detailed study. Especially during the starting movement, when it is almost impossible to predict the behavior of the tire. One of the extreme cases in which the car owner may find himself is operating under unfavorable conditions, for example, due to low ambient temperature.

The primary purpose of this research is to study the parameters of tire stiffness under such operating conditions.

3 Research Methodology

3.1 Problem statement

As mentioned, situations that are unfavorable for the optimal and safe operation of vehicles may sometimes arise. Such situations include, for example, environmental conditions. For example, in some regions, the ambient temperature can drop to relatively low values and sometimes vice versa to high values. There may also be a situation where environmental conditions can fluctuate in a wide range over a relatively short time.

Such phenomena can be observed in Eastern Europe. For example, in winter, the ambient temperature during the day is up to +10 °C, and it can drop to –15 °C during 8–12 hours. That is, it is quite possible that a car, for example, was left outside for a long time under the same environmental conditions (favorable), and then there is a significant change in weather conditions and, accordingly, the characteristics of the tire. The tires were initially brought to their optimum performance (internal pressure) under normal conditions, i.e., on warm premises, and then the car was left outside for a long time.

Another example of vehicle operation under unfavorable conditions for the state and behavior of tires is when the vehicle is operated in areas with significant ambient temperature fluctuations (e.g., mountainous terrain). Operation in such regions is exceptionally unpredictable (unforeseen) regarding environmental conditions. It is also worth noting that in such an operation, it is not always possible to leave the car for a long time in a garage or on another premises with normal conditions. This usually applies to tourists who prefer long-term storage of cars in open-air parking lots.

Therefore, it is worth highlighting such situations and separately investigating the behavior of the tire during a sharp change in operating conditions because this ultimately impacts driving safety during the starting period. Let's consider one of the possible extreme situations when tires are exposed to low temperatures.

Experimental studies were conducted to assess the state and behavior of automobile tires during operation under environmental conditions unfavorable to their safe operation. The purpose of the experiment was to determine the main performance characteristics of the tire. The research was conducted on a Toyota Hilux equipped with 265/70R16 tires. Before the experiment, the internal tire inflation pressure was set at 0.24 MPa, corresponding to the standard values range. The air temperature on the premises where the pressure was set was +20 °C, and the humidity was 50 %. Then, the car was left outside at a temperature of –30°C, remaining stationary for 10 hours. In the morning, after a long parking time, the internal pressure and temperature of the tires were measured, and the interaction between the tires and the bearing surface was studied.

Figure 1 shows the 265/70 R16 car tire under test.


a b

Figure 1 – The tire under study after overnight parking at –30°C: a – in the initial state; b – after starting to drive.

As a result of measuring the internal pressure in the morning, after parking, a 20 % decrease was recorded, which can also be seen in Figure 1. It shows that after parking at a relatively low temperature, the tire was deformed, i.e., due to the decrease in internal pressure, the contact area increased. At the beginning of the movement, the tire in this place retained the same shape (Figure 1 b); this phenomenon of tire hardening occurred due to environmental conditions because the tire material, when exposed to such unfavorable conditions for a sufficiently long time, has somewhat lost its elastic properties.

Therefore, it is necessary to apply some additional measures to bring the tire to its optimal condition. Some peculiarities of tire behavior during the starting movement have already been highlighted in [1, 3]. These studies present the peculiarities of tire behavior during this period under unfavorable environmental conditions in low temperatures.

To evaluate the stiffness, it is necessary to determine the main parameters that will be used to assess the tire's condition. These parameters are the stiffness and damping coefficients.

During operation, the tire rolls and vibrates vertically. Therefore, the coefficient depends on the deformation rate of the tire material.

There are various methods for determining these coefficients. The simplest method is to determine these parameters under static tire loading. In this case, the total resistance force of the deformed tire consists of elastic and inelastic resistance forces. As mentioned above, the inelastic resistance force depends on the deformation rate, so loading and unloading should be done slowly.

In addition to the component that depends on the strain rate, the inelastic drag force also includes a component of internal friction. Under slow loading and unloading, the type of hysteresis loop determines the static damping coefficient (in a stationary state). Radial stiffness determines the elastic resistance force.

The methodology for determining dynamic coefficients is described in detail in [9, 10]. It is known that the frequency of damped free oscillations determines the damping coefficient, which is determined by the logarithm decrement.

A special stands were developed at the Department of Machine Parts and Theory of Mechanisms and Machines of Kharkiv National Automobile and Highway University. The principle of operation and the results of determining the parameters characterizing the stiffness properties of tires are described in detail in [10].

Determining the coefficients using the above method is complicated and time-consuming when conducting relevant experiments.

Currently, when studying the operational properties of a car, methods of numerical modeling of the interaction processes with the bearing surface are often used. For this purpose, programs based on the finite element method (FEM) are used; the results of such studies are presented in [11–15].

As a result of experimental studies [2], the behavior of an automobile tire under the influence of changes in internal pressure was evaluated. By performing the study using the analytical method, it will be possible to compare the results obtained.

3.2 Numerical and analytical approaches

A method based on solving the linear differential equations of a mechanical motion system was applied to estimate the stiffness of a tire.

It should be understood that tire stiffness consists of several main components, such as the stiffness component of the tire material (carcass and rim) and the component due to internal filler pressure. This article considers the combined (total) tire stiffness without separating the individual components. Using a simplified mass-spring-damper model of vehicle suspension (Figure 2), it is possible to model the behavior of a car tire and suspension system with sufficiently high accuracy.

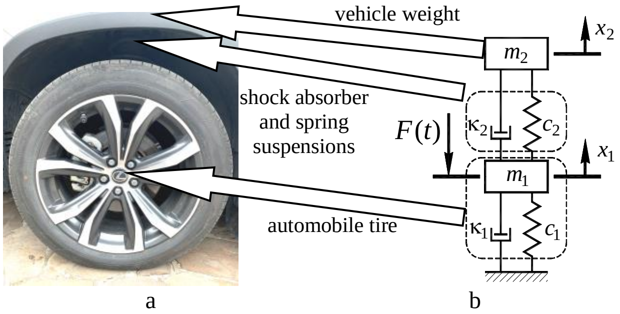

Figure 2 – Automobile wheel (a) and its mass-spring-damper model (b)

Studies of similar models of a car wheel are presented in [16, 17].

It is advisable to use the Laplace transform to solve the equations effectively. When studying transients in dynamic systems, it is convenient to use not a direct method of integrating differential equations of motion but to use the operational calculus, which is based on the Laplace transform [18], which is used to solve problems in many areas of deformable solid mechanics [19]. When using this method, initial conditions are taken into account automatically, and accordingly, there is no need to calculate arbitrary constants. Another advantage of the Laplace transform is the ability to replace the rather complex operation of differentiating functions with simpler multiplication and division operations. That is, solving differential equations is reduced to solving a system of algebraic equations.

The following system of differential equations describes the system’s motion (Figure 2):

$$\begin{cases} m_1 \dfrac{d^2 x_1(t)}{dt^2} + (\kappa_1 + \kappa_2) \dfrac{dx_1(t)}{dt} + (c_1 + c_2) x_1(t) - \kappa_2 \dfrac{dx_2(t)}{dt} - c_2 x_2(t) = F(t); \\ m_2 \dfrac{d^2 x_2(t)}{dt^2} + \kappa_2 \dfrac{dx_2(t)}{dt} + c_2 x_2(t) - \kappa_2 \dfrac{dx_1(t)}{dt} - c_2 x_1(t) = 0, \end{cases} \tag{2}$$

where c_1, c_2 – stiffness coefficients of the car wheel and suspension, respectively, N/m; κ_1, κ_2 – wheel and suspension damping coefficients, N·s/m; x_1 – the coordinate describing the movement of the car wheel, m; x_2 – coordinate of the vehicle’s movement, m.

To evaluate the behavior of a car tire and the car suspension as a whole, it is necessary to obtain displacement functions x_1 and x_2.

The initial conditions are as follows:

$$x_1(0) = 0;\ x_1(0) = 0;\ \dot{x}_1(0) = 0;\ x_2(0) = 0;\ \dot{x}_2(0) = 0. \tag{3}$$

The direct integral Laplace transform was applied to expression (2), considering the zero initial conditions (3).

As a result, the system of equations was obtained:

$$\begin{cases} m_1 s^2 x_1(s) + (\kappa_1 + \kappa_2) s \cdot x_1(s) + (c_1 + c_2) \cdot x_1(s) - \kappa_2 s \cdot x_2(s) - c_2 x_2(s) = F(s); \\ m_2 s^2 x_2(s) + \kappa_2 s \cdot x_2(s) + c_2 x_2(t) - \kappa_2 s \cdot x_1(s) - c_2 \cdot x_1(s) = 0, \end{cases} \tag{4}$$

where *s* – complex variable.

The solution of this system of linear algebraic equations is as follows:

$$x_1(s) = \frac{m_2 s^2 + \kappa_2 s + c_2}{(m_1 s^2 + (\kappa_1 + \kappa_2)s + c_1 + c_2)(m_2 s^2 + \kappa_2 s + c_2) - (\kappa_2 s + c_2)^2} F(s); \tag{5}$$

$$x_2(s) = \frac{\kappa_2 s + c_2}{(m_1 s^2 + (\kappa_1 + \kappa_2)s + c_1 + c_2)(m_2 s^2 + \kappa_2 s + c_2) - (\kappa_2 s + c_2)^2} F(s). \tag{6}$$

To obtain the original functions x_1 and x_2, the inverse transformation can be performed using the convolution theorem and the 2nd decomposition theorem, then the functions will take the general form:

$$x_i(t) = \int_0^t K_i(t-\tau)F(\tau)d\tau \tag{7}$$

where $K_i(t) = \sum_{j=1}^{4} \frac{A_i(s_j)}{B'(s_j)} \cdot e^{s_j \cdot t}$ – the finite-difference kernel of a convolution-type integral; $A_i(s_j)$ – the polynomial of the numerator of expressions (5)–(6); $B'(s_j)$ – the derivative of the polynomial of the expressions in the denominator (5)–(6).

By discretization using the method of average rectangles [20], the integral equation (7) is transformed into a system of the following linear algebraic equations:

$$\boldsymbol{A}^*_{Fj}\boldsymbol{F} = \boldsymbol{x}_{Fj}, \tag{8}$$

where vector $\boldsymbol{F}$ corresponds to the change in force over time $F(t)$, and vector $\boldsymbol{x}_{Fj}$ corresponds to a change in time coordinates x_1 and x_2.

4 Results

Let’s consider the behavior of the car suspension system under specific operating conditions, namely, the unfavorable conditions under which the above experiment was carried out. These results can be compared with modeling the tire’s behavior under normal operating conditions.

The stiffness and damping coefficients characterize the tire’s condition under low ambient temperature conditions.

Information on these parameters can be found in the scientific literature, where the results of their research are presented, and through experimental studies. In particular, such parameters as the vehicle’s mass per wheel m_2 and the mass of the vehicle wheel m_1 were measured during the experimental study of the tire condition described in the previous section.

Table 1 shows an automobile tire’s stiffness characteristics and the suspension system’s parameters under normal operating conditions [1, 3]. Such parameters as unsprung weight (wheel weight) and sprung weight (weight of the suspension and vehicle per wheel) are the same in both cases.

Also, the characteristics of the suspension itself are considered the same in both cases, although it is quite clear that they undergo specific changes when the operating conditions (ambient temperature) change, which will be considered in subsequent studies.

Table 1 – Suspension system parameters
under normal operating conditions

Designation	Value	Units of measurement
m_1	25	kg
m_2	700	kg
c_1	$2.0\cdot10^5$	N/m
c_2	$2.2\cdot10^4$	N/m
κ_1	53	N·s/m
κ_2	$2.0\cdot10^3$	N·s/m

Table 2 shows a car tire’s stiffness characteristics and the suspension system’s parameters under unfavorable operating conditions [1, 3].

Table 2 – Suspension system parameters
under adverse operating conditions

Designation	Value	Units of measurement
m_1	25	kg
m_2	700	kg
c_1	$1.7\cdot10^6$	N/m
c_2	$2.2\cdot10^4$	N/m
κ_1	12	N·s/m
κ_2	$2.0\cdot10^3$	N·s/m

An external force $F = H(t)$ is also applied to the system (Figure 2), which is represented by the Heaviside function (a piecewise constant function $H(t)$ equal to zero for negative values of the argument and one for positive values):

$$H(t) = \begin{cases} 0, & t < 0; \\ 1, & t \geq 0. \end{cases}$$

Figure 3 shows the results of modeling the operation of the car suspension under normal conditions.

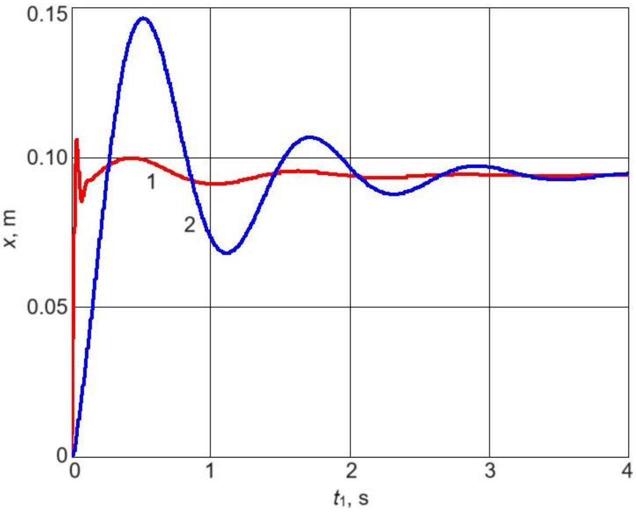

Figure 3 – Moving coordinates for the model of the vehicle suspension under normal operating conditions:
1 (red) – x_1; 2 (blue) – x_2

Figure 4 separately shows the results of moving the coordinate x_1 (in an enlarged scale along the abscissa axis) under normal operating conditions.

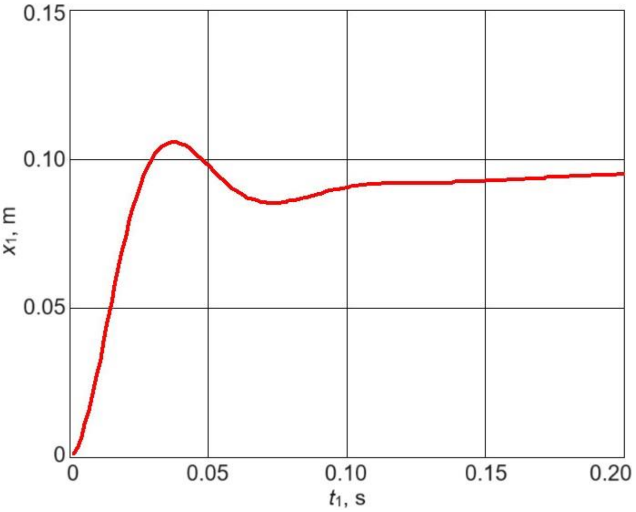

Figure 4 – Moving coordinate x_1 for the vehicle suspension model under normal operating conditions

Figure 5 shows the results of modeling the operation of the car suspension under unfavorable operating conditions.

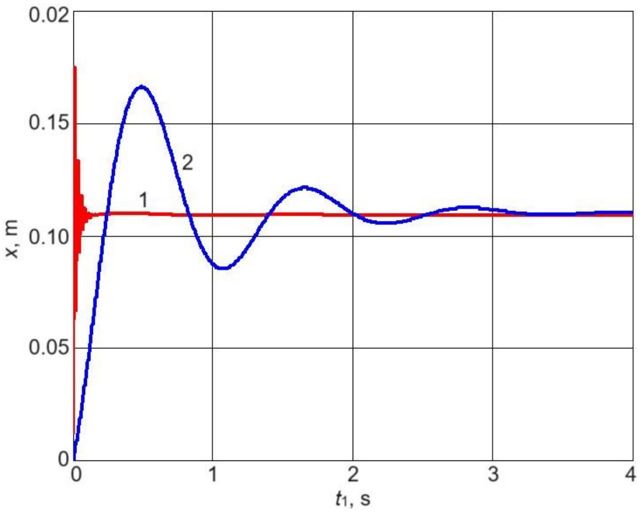

Figure 5 – Moving coordinates for the suspension model of the car under unfavorable operating conditions:
1 (red) – x_1; 2 (blue) – x_2

Figure 6 separately shows the results of coordinate displacement (in an enlarged scale along the abscissa axis) under unfavorable operating conditions.

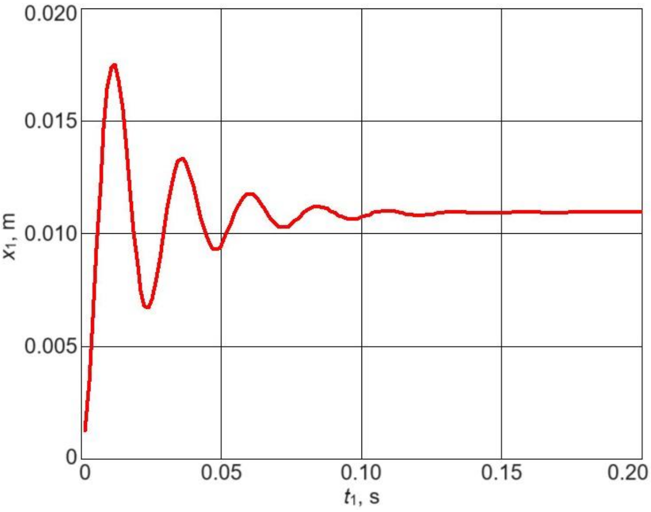

Figure 6 – Moving coordinates x_1 for the car suspension model under unfavorable operating conditions

5 Discussion

After comparing the results, the difference between the periods and amplitudes of oscillations of the car wheel (unsprung mass) and the car (sprung mass) can be seen.

Also, the period of fluctuations changes insignificantly while the amplitude varies greatly. Under unfavorable operating conditions, the operation of the tire and suspension differs from their behavior under normal (optimal) conditions. In other words, the suspension is generally more rigid, significantly affecting comfort and driving safety during the starting period.

Based on the results of modeling the interaction of an automobile tire with a bearing surface, we can say that the statement about the increased stiffness of an automobile tire is correct.

From Figures 3–6, the peculiarities of the tire behavior under different operating conditions can be observed (Tables 1–2), particularly those corresponding to unfavorable conditions under which the effect of increased stiffness may occur.

Therefore, the results of modeling the operation of a car tire and suspension as a whole show that the method of numerical and analytical solution of the problem used in this paper is reliable. This conclusion can be made by analyzing the results of other scientific studies [4, 5].

However, the peculiarity of this work is that it is the starting period of a car's movement that is studied, and the numerical-analytical solution of the equations of motion is used to assess the behavior of a car tire.

6 Conclusions

After analyzing scientific publications closely related to this research, it can be said that studying the condition and behavior of automobile tires is an essential factor in predicting the entire vehicle's behavior during operation, especially during the starting mode of movement.

The influence of the stiffness characteristics of an automobile tire is described in many works. However, they usually consider an automobile tire with optimal performance properties, which is rarely observed during operation in actual conditions at the starting mode. Indeed, as it was established in previous works, the tire undergoes significant changes in its performance during the starting period.

In addition, automobile tires can be used in regions where unfavorable conditions are often observed, such as mountainous areas. Therefore, studying the influence of tire parameters, particularly stiffness characteristics, is essential to this type of research.

This paper presents the results of an experimental study of the condition and behavior of an automobile tire under unfavorable operating conditions, i.e., at low ambient temperatures. Such conditions are expected in Eastern Europe, where temperatures vary relatively quickly from +20 °C to –30 °C. The study found that a tire exposed to the environment is unsafe, which can negatively impact the vehicle's performance, including driving comfort.

The presented model of the car suspension in the form of a two-mass system "mass – spring– damper" allows for simulating the suspension operation depending on the tire characteristics, accounting for the operating conditions. The studies were performed using the numerical-analytical method of the Laplace transform to solve the equations of the proposed system motion.

The results indicate a significant influence of the tire condition on the behavior of the car suspension. Under unfavorable operating conditions, suspension oscillations differ in amplitude and period from those under optimal parameters by 10–15 %.

With the standard tire stiffness parameters, the car's suspension has better absorption properties and is more adapted to the impact of external force factors.

Therefore, when studying the performance properties of a car, it is necessary to consider the tire's behavior and changes in its characteristics depending on the operating conditions.

Overall, tires significantly impact all vehicle performance properties and driving safety, especially during the initial period of operation.

References

1. Karpenko, V, Voropay, O., Neskreba, E. (2022). Indirect assessment of the rolling resistance of a car tire in the starting mode of motion. *Motor Vehicles*, Vol. 50, pp. 5–13. https://doi.org/10.30977/AT.2019-8342.2022.50.0.01
2. Rostami, H. T., Najafabadi, M. F., Ganji, D. D. (2023). Investigation of tire stiffness and damping coefficients effects on automobile suspension system. *Proceedings of the Institution of Mechanical Engineers, Part D: Journal of Automobile Engineering*, Vol. 237(14), pp. 3313–3325. https://doi.org/10.1177/09544070231151860
3. Berntorp, K., Di Cairano, S. (2024). Tire-stiffness and vehicle-state estimation based on noise-adaptive particle filtering. *IEEE Transactions on Control Systems Technology*, Vol. 27(3), pp. 1100–1114. https://doi.org/10.1109/TCST.2018.2790397
4. Acosta, E. C., Aguilar, J. J. C., Carrillo, J. A. C., García, J. M. V., Fernández, J. P., Vargas, M. G. A. (2020). Modeling of tire vertical behavior using a test bench. *IEEE Access*, Vol. 8, pp. 106531–106541. https://doi.org/10.1109/ACCESS.2020.3000533
5. Czapla, T., Pawlak, M. (2022). Simulation of the wheel-surface interaction dynamics for all-terrain vehicles. Applied Mechanics, Vol. 3(2), pp. 360–374. https://doi.org/10.3390/applmech3020022
6. El-Sayegh, Z., El-Gindy, M. (2020). Rolling resistance prediction of off-road tire using advanced simulation and analytical techniques. *SN Applied Sciences*, Vol. 2, 1620. https://doi.org/10.1007/s42452-020-03444-0
7. Li, B., Li, N., Yang, X., Yang, J. (2014). In-plane rigid ring-based tire model: Parameter identification, sensitivity analyses, and effect on ride comfort. *In: International Design Engineering Technical Conferences and Computers and Information in Engineering Conference*, Vol. 46346, V003T01A042. https://doi.org/10.1115/DETC2014-34333
8. Dechkova, S. (2023). Elastic properties of tyres affecting car comfort, driving and riding. *International Journal of Applied Mechanics and Engineering*, Vol. 28(4), pp. 54–68. https://doi.org/10.59441/ijame/173020

9. Fabra-Rodriguez, M., Abellán-López, D., Simón-Portillo, F. J., Campello-Vicente, H., Campillo-Davo, N., Peral-Orts, R. (2024). Numerical model for vibro-acoustics analysis of tyre-road noise generation caused by speed bumps. *Applied Acoustics*, Vol. 216, 109830. https://doi.org/10.1016/j.apacoust.2023.109830
10. Cong, N. T., Do, C. K. D., Truong, D. C. (2023). Structural and thermal investigations of rolling tires in a flat road. *Transport and Communications Science Journal*, Vol. 74(1), pp. 47–57. https://doi.org/10.47869/tcsj.74.1.5
11. Nakajima, Y. (2019). *Advanced Tire Mechanics*. Springer Nature, Singapore. https://doi.org/10.1007/978-981-13-5799-2
12. Fathi, H., Khosravi, M., El-Sayegh, Z., El-Gindy, M. (2023). An advancement in truck-tire–road interaction using the finite element analysis. *Mathematics*, Vol. 11(11), 2462. https://doi.org/10.3390/math11112462
13. Karpenko, M., Skačkauskas, P., Prentkovskis, O. (2023). Methodology for the composite tire numerical simulation based on the frequency response analysis. *Maintenance and Reliability*, Vol. 25(2), 163289. https://doi.org/10.17531/ein/163289
14. Nguyen, T. C., Cong, K. D. D., Dinh, C. T. (2023). Rolling tires on the flat road: thermo-investigation with changing conditions through numerical simulation. *Applied Sciences*, Vol. 13(8), 4834. https://doi.org/10.3390/app13084834
15. Adamek, V., Vales, F., Tikal, B. (2009). Non-stationary vibrations of a thin viscoelastic orthotropic beam. *Nonlinear Analysis: Theory, Methods & Applications*, Vol. 71(12), pp. e2569–e2576. https://doi.org/10.1016/j.na.2009.05.068
16. Cuong, D. M., Ngoc, N. T., Ran, M., Sihong, Z. (2018). The use of the semi-empirical method to establish a damping model for tire-soil system. *Coupled Systems Mechanics*, Vol. 7(4), pp. 395–406. https://doi.org/10.12989/csm.2018.7.4.395
17. Mihon, L., Lontiş, N., Deac, S. (2018). The behaviour of a vehicle's suspension system on dynamic testing conditions. *IOP Conference Series: Materials Science and Engineering*, Vol. 294, 012083. https://doi.org/10.1088/1757-899X/294/1/012083
18. Beerends, R. J., ter Morsche, H. G., van den Berg, J. C., van de Vrie, E. M. (2003). *Fourier and Laplace Transforms*. Cambridge University Press, Cambridge, UK. https://doi.org/10.1017/CBO9780511806834
19. Li, J., Keshavarzi, A., Omran, A. H., Ahmad, N., Alkhafaji, M. A., Nasajpour-Esfahani, N., Mousavian, S. H. B. (2023). Simulation the effect of capply layer length on the longitudinal stiffness and lateral stiffness of the tire and the stability of the car. *Ain Shams Engineering Journal*, Vol. 2, 102354. https://doi.org/10.1016/j.asej.2023.102354
20. Yanyutin, E. G., Voropay, A. V. (2004) Controlling nonstationary vibrations of a plate by means of additional loads. *International Journal of Solids and Structures*, Vol. 41(18–19), pp. 4919–4926. https://doi.org/10.1016/j.ijsolstr.2004.04.022