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Determination of Chatter-Free Cutting Mode in End Milling

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Abstract. Chatter accompanies the cutting process and is the main obstacle to achieving precision and productivity in milling operations. To reduce the amplitude of vibrations, it was proposed to use a stability lobes diagram (SLD) when assigning cutting modes. The machining system in end milling was represented by a two-mass dynamic model in which each mass has two degrees of freedom. The behavior of such a system was described by a structure with two inputs, in-depth and cutting feed, and a delay in positive feedback on these inputs. A new criterion was applied to design the SLD based on an analysis of the location of the machining system Nyquist diagram on the complex plane. The algorithm for designing a stability chart was developed into an application program, a tool for the technologist-programmer when assigning cutting modes. A method for parameter identification necessary for designing the dynamic system “tool – workpiece” was proposed. The effectiveness of the developed method was proven experimentally when the choice of spindle speed during end milling allows one to reduce the roughness parameter Ra from 3.2 μm to 0.64 μm at the same feed rate of 650 mm/min.

Keywords: end milling, simulation, process innovation, stability lobes diagram.

1 Introduction

Any cutting process is accompanied by vibrations, which lead to negative consequences on the machined surface, increased roughness, waviness, deformation, and premature wear of the tool and machine components [1]. The importance of studying vibrations during machining is explained by the fact that they become the main limitation of process productivity.

Since vibrations are caused by the very physics of the cutting process, which always occurs in an elastic processing system closed by natural feedback, it is fundamentally impossible to eliminate them. However, increasing the vibration level to an acceptable value is always possible under certain restrictions on processing quality. The effectiveness of measures to control vibrations is entirely related to identifying the causes of their occurrence and creating adequate models of such processes.

According to the research of many authors, such reasons consist of nonlinearities of dynamic models of machining systems and the cutting processes themselves, cross-kinematic connections of elastic systems of machine tools, and insufficient rigidity. However, most believe that the most reasonable assumption is the occurrence of regenerative oscillations in the machining system, caused by the process of machining along the traces when the allowance is cut off at each subsequent pass, distorted by the oscillations of the system in the previous pass.

Moreover, the system oscillations in maximum amplitude occur along the main harmonic of the elastic dynamic system. Therefore, the geometric distortion of the allowance on the next pass during its cutting will cause a disturbance in the system precisely with the frequency of the main harmonic. Thus, the elastic system will sway under the influence of such a disturbance.

All measures to control chatter and reduce the amplitude of vibrations can be divided into three directions. Firstly, these are passive chatter control (PCC), which consists of changing the dynamic properties of the original dynamic structure by attaching additional masses with damping devices. Passive methods also include software control of the spindle speed, for example, spindle speed variation according to the harmonic law, which indirectly confirms the main reason for the occurrence of regenerative oscillations – "self-swinging" of the system when machining along the traces. In this case, a change in the rotation speed leads to a change in the disturbance frequency, which will already differ from the frequency of the main harmonic of the dynamic machining system.

Secondly, active chatter control (ACC) consists of the forced introduction into a dynamic system of oscillations with a phase opposite to the regenerative oscillations that need to be damped.

All such methods require specific, sometimes very costly, measures to modernize existing equipment. At the same time, studies of the causes of vibrations during cutting have led to the opinion that changing the cutting mode affects the stability of the process. Therefore, cutting mode control methods have emerged, which consist of designing the so-called stability lobes diagram (SLD), which defines the stability zones of the machining system.

In general, when studying the vibration processes for an elastic closed-loop machining system and determining the stability conditions, one should focus on its structure, not on the form and type of disturbing external influences, which are most often considered cutting forces. Therefore, the stability of a machining system, like any dynamic system with feedback, should be considered according to its mathematical model using universal stability criteria, such as the Nyquist criterion.

So then, why would the stability of the machining system depend on the cutting conditions? The answer to this question lies in the nonlinear dependence of the cutting force coefficients on the mode, and it is such coefficients that are included in the dynamic structure of the machining system.

Notably, according to the diagram, a cutting mode that falls into the instability zone will not mean a loss of stability and entering a destructive mode in the classical sense. This is also related to the adequacy of the mathematical model of the system, which most often appears to be single-mass with one degree of freedom.

Therefore, increasing the adequacy of the model and bringing it closer to actual processes, the spectrum of which is not limited to one natural frequency, is an urgent scientific and practical problem, the solution of which will significantly increase the efficiency of such control.

2 Literature Review

Chatter in machining has been studied by scientists over the past decades because vibration significantly impacts surface quality and productivity. Most scientists believe that the main reason for the occurrence of chatter is regenerative vibrations caused by machining along the wake. The model of the machining system is presented in the form of a block diagram with positive feedback through a delay link [2]. The study of the stability of the milling process is carried out based on an SLD, which represents the zones of stable and unstable cutting in the coordinates "spindle speed – feed".

However, the solution to the mathematical model is proposed based on an approximate method through the roots of the characteristic equation when interpreted based on the frequency transfer function of the system. Next, a proposed diagram design algorithm involves sequential calculations at each spindle speed to create SLD.

After identifying the frequency transfer function of the machining system and the cutting force coefficients for a given cutter, workpiece material, and cutting width, a design algorithm is proposed, which consists of 5 steps [3]:

1) select the vibration frequency from the transfer functions around the dominant mode;
2) find the roots of the characteristic equation;
3) calculate the critical cutting depth;
4) calculate the spindle speed using the formula for each stability lobe $k = 0, 1, 2, \ldots$;
5) repeat the procedure for all oscillation frequencies around all dominant modes of the structure.

To analyze the stability of the end milling process, an approach is proposed based on the representation of the wavy thickness of the chips formed during the passage of the previous cutter tooth and the trajectory of the tool cutting edge during oscillations in the flow pass [2]. This approach leads to a loss of adequacy of the process representation due to the lack of a general model considering feedback in an elastic machining system. Also, the diagram design procedure is based on algebraic stability criteria, which are determined from the characteristic equation and consist of using a well-known multi-step algorithm, which hinders its use in practice.

A stability lobe diagram based on regenerative vibration theory effectively predicts vibration in-process research and parallel kinematics machining systems [4]. A vibration model of the machine is derived, which considers the dynamic behavior of the spindle/holder/tool system. The regenerative cutting dynamics are combined with the vibration model, and stability analysis is performed. The SLD step-by-step design procedure also consists of five steps and is not programmable. However, a single-mass system is proposed. Despite the complex kinematics of the system with a parallel structure, the stability condition, as in previous studies, is based on the characteristic equation. The dynamic system does not consider closure and positive feedback through the delay function.

The stability lobe diagram is an effective tool that helps the operator select specific spindle speeds during production to avoid vibration in the machine [5]. Control programs for CNC machines are designed using computer-aided manufacturing (CAM) systems, but the cutting mode is assigned by a programmer-technologist who finds it challenging to consider the machining system's dynamic properties. Therefore, given the ability to select cutting conditions over a wide range, SLD's recommendations are essential, especially when milling thin-walled parts. When using experimental SLD design methods, milling is performed at a certain speed, and the axial depth of the cut is increased due to the conical geometry of the workpiece [5]. Vibration is detected if the energy of the measured signal from the microphone exceeds a particular threshold value.

The experimental method for determining lobes requires a lot of time and resources, and in order to reduce the labor intensity of experiments, a semi-analytical method is proposed [5]. This method calculates most of the parameters to obtain the stability lobes analytically. The modal parameters of the spindle/tool holder/tool system are obtained experimentally using impact testing.

To obtain data on the transition from a stable state to a vibration mode, some experiments also continuously increase the depth of cut using a wedge-shaped workpiece [6]. Chatter diagnostics are performed using various methods of time-frequency analysis, which makes it possible to analyze the frequency components of nonlinear and time-varying signals. Research challenges and prospects were discussed and summarized as simultaneous monitoring of tools and machining conditions using multiple sensors and information fusion. The experimental data is proposed to develop a robust monitoring system that can operate under various machining conditions and provide accurate and reliable vibration detection [6].

Modern signal processing techniques enable a digital twin machining system to detect and prevent vibration in real-time or implement adaptive control. Such methods can also provide much information about the surface formation process in the presence of vibrations in the machining system, although methods for assessing vibrations in milling based on analysis of the machined surface profile are not covered at all in this extensive review.

The study [7] presents an analysis of the reflection of chatter of the machining system on the surface topography during peripheral milling. It is shown that in a 3D image, vibrations during cutting are displayed on the surface in the form of waves in the location of such trajectories along the axial coordinate of the cutter. Therefore, developing research to consider the influence of dynamic phenomena on the formation of roughness is essential for increasing the adequacy of models and controlling the end milling process.

In the aviation, aerospace, and energy industries, thin-walled parts such as aircraft structural parts, impellers, and turbine blades are becoming increasingly important [8]. Some difficult-to-cut superalloys, such as titanium and nickel-based alloys, are increasingly used as workpiece materials. The dual superposition of difficult-to-cut material and thin-walled structure makes high-throughput machining of thin-walled parts difficult. Chatter during machining is the main obstacle to improving the machining accuracy and efficiency [8].

Typically, the dynamics of a machining process, considering the regenerative effect, can be mathematically modeled using differential equations with delay and time-dependent coefficients. Experimental modal analysis is used to dynamically identify a part under machining, usually based on hammer impact tests and sometimes modal vibration tests. Frequency response functions (FRF) and modal parameters can be extracted and set by the data acquisition and machining system [8].

It is noted in [9] that during machining, when one of the machine modes is self-exciting, regenerative vibration occurs at frequencies close to, but not equal to, the dominant frequencies of the machine. Further, the main reason for the occurrence of regenerative vibrations is considered to be the periodic action of the cutting force. The geometric features of the location of the chip thickness and the phase difference between its two successive waves are considered. It is stated that the machining system experiences forced vibration, the frequency of which is equal to the frequency of the cutting force. Therefore, the cutting force is considered an external disturbance that can cause instability in the cutting process. However, the stability of any system, including machining, must be explained by internal processes in its structure and determined through classical stability criteria, such as the Nyquist criterion.

In the aircraft industry, parts sections are becoming much thinner due to weight requirements; there is an urgent need to predict stable cutting conditions during machining [10] more accurately. Improvements to the well-known stability model are proposed by considering the nonlinearity of the cutting force coefficients, the axial plunge angle, and their dependence on the axial depth of the cut. A dynamic system consisting of a two-mass model with two degrees of freedom is also considered.

Waviness on the machined surface is considered the leading cause of cutting instability, and each tooth removes the waviness created by the previous tooth, resulting in modulated chip thickness. Next, a single-frequency solution and an identified transfer function in the contact zone between the tool and the workpiece are used, which leads to an algorithm for calculating the SLD, similar to the well-known ones [3], consisting of five steps. However, the proposed algorithm, although more detailed than the known ones, does not provide for the possibility of design automation since some steps require intervention to decide depending on the result of the process evaluation.

Milling machine performance is limited by vibration, and SLD allows the selection of appropriate process parameters to maximize productivity. However, calculating SLD is very labor-intensive and requires complex experiments. Therefore, a new online learning method that allows calculating diagrams during production is noteworthy [11]. The algorithm combines reinforcement learning and nearest neighbor classification and can estimate the stability boundary based on measured vibration signals during machining. The algorithm is validated using analytical reference functions and 2DOF milling stability simulations.

An essential place in the study of chatter during milling is occupied by vibration indication and the selection of criteria for detecting instability [12]. A promising experimental method of continuously increasing the cutting depth and continuously varying the spindle speed is proposed to obtain SLD quickly and automatically. For this purpose, a real-time vibration detection algorithm based on the Poincaré section was designed and implemented to detect instability as well as stability. This approach can be used to create automatic vibration control systems during cutting.

SLD design involves identifying the dynamic parameters of the machining system, which is why much attention has been paid to this issue [13]. Experimentally based methods for collecting features and theoretical formulation of methodologies for determining dynamic parameters are outlined and compared. Comments and discussions are carried out through in-depth consideration of the relevant methods for obtaining the FRF of the tool-holder-spindle system, the parameters of the process damping properties, and for the thin-walled workpiece.

Of several vibration suppression and prevention methods, processing modes selected using an SLD are the most reliable method [14]. This paper examines a new approach to estimating tool tip FRF during machining operations using cutting tool vibration signals. A non-contact laser vibration meter is used to measure the vibration of the end mill tip at various spindle speeds.

The end milling of workpieces with curved guides differs from machining flat surfaces. A study [15] proposes a new dynamic model of the tool-workpiece system that considers the dynamic behavior of the tool and workpiece, as well as the influence of the interaction and tool feed direction. An effective method based on a structural dynamic modification scheme is developed to characterize the effect of material removal on the dynamics of a part during machining.

To form the SLD, an experimental methodology is proposed based on tests in which a tapered workpiece is allowed to gradually increase the axial depth of cut in the feed direction, which represents the Y-coordinate of the SLD, while the spindle speed is increased for each pass, which represents the X-coordinate [16]. Vibrations are recorded using an acoustic sensor. This operation will allow the operator to select a combination of spindle speed and depth of cut for chatter-free milling.

In [17], an algorithm was proposed for automatically constructing an SLD in the "spindle speed – feed" coordinates according to the stability criterion, using the frequency characteristics of the dynamic system in the form of a Nyquist diagram. The algorithm is built into a computer program that automatically designs a given stability diagram, determining its practical usefulness.

The presented analysis of literature sources shows that, despite the large number of publications on the research topic, most of them are devoted to controlling peripheral milling processes through the assignment of cutting modes according to the stability diagram. However, the lack of a reliable criterion for the stability of a machining system makes it challenging to design a stability diagram that is calculated using a multi-step algorithm. Moreover, the proposed approach is based on algebraic criteria associated with the roots of the characteristic equation, which is obtained from a differential equation with delay and which fundamentally does not have an analytical solution. They use approximate methods that cannot be presented as an algorithm suitable for computer use. In addition, it remains problematic to determine the consequences of chatter during milling, which would naturally be associated with the topography of the machined surface. Mathematical models are presented as single-mass systems with one or two degrees of freedom, although at least two masses take part in cutting: "tool – workpiece". Therefore, we should expect an increase in adequacy when considering such interaction. The solution to these problems remains relevant in scientific and practical terms.

This work aims to create a new method for determining the chatter-free cutting mode during end milling based on constructing an SLD, which will allow optimizing the process on machines with CNC. To achieve the aim, it is necessary to solve the following tasks:

– develop a mathematical model of the end milling taking into account a two-mass dynamic system, feedback of elastic displacement of the machining system, and machining along the traces;

– identify the dynamic parameters of the tool and workpiece system;

– create a program for automatic design of SLD in the coordinates "spindle speed – feed";

– conduct experimental testing of theoretical results.

The object of the study is the process of milling with end mills, and the subject is the vibrations that arise in the elastic system during cutting. To analyze stability, a frequency criterion was used for systems whose behavior is described by differential retarded equations.

3 Research Methodology

3.1 Mathematical model

A systematic approach was used to develop the model, which allows the representation of the machining system as a connection of individual blocks with certain transfer functions. This allows for acquiring a mathematical model as a system of differential equations composed of variable states. The new stability criterion can also be used in the algorithm and numerical method for constructing SLD for end milling. The results of full-scale experiments on the topography of the machined surface confirm the adequacy of the proposed solutions and evaluation of the results.

The mathematical model of the milling process should reproduce the actual processes of the elastic system, considering the machining system's closedness, cross-links along two coordinates, and machining along the trace. In addition, the modern approach to creating mathematical models requires its presentation in the form of software, the operation of which: firstly, the adequacy (quality) of the created model is checked, and secondly, it becomes possible to obtain modeling results in digital form, suitable for use in solving practical problems mentioned above.

The dynamic model of the machining system during end milling can be represented as a two-mass system with elastic connections that determine the rigidity of the tool system (k_{mX}, k_{mY}) and the rigidity of the workpiece system (k_{wX}, k_{wY}) in XOY coordinates (Figure 1).

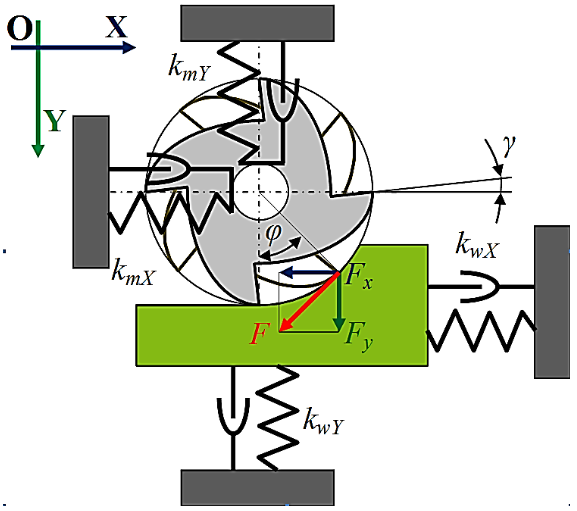


Figure 1 – Dynamic model of the machining system

During milling, the machining system is subject to force disturbance by the cutting force F, which determines the components acting along the axes:

$$F_x = F\cos(\phi - \gamma - \beta);\ F_y = F\sin(\phi - \gamma - \beta), \quad (1)$$

where φ – the cutting angle; $\gamma = 10°$ – the rake angle.

The angle β is determined by the ratio of the normal and tangential components ($26°30'$) [3].

A generalized mathematical model of the cutting force during end milling must be built based on a mechanistic approach to calculating the cutting force based on the geometric parameters of undeformed chips cut by each tooth along its length. This dependence is nonlinear [17] and can be linearized with a sufficient degree of accuracy as a function of the components of the cutting mode:

$$F = k_f (f_t)_a + k_H H_a, \quad (2)$$

where $(f_t)_a$, H_a – actual feed and cutting depth.

Linearization coefficients are defined as the values of partial derivatives of the cutting force at the values of the variables at the linearization point:

$$k_f = \left(\frac{\partial F}{\partial f}\right)_0 = C_F \alpha (f_t)_a^{\alpha-1} H_a^{\alpha} B;$$

$$k_H = \left(\frac{\partial F}{\partial H}\right)_0 = C_F (f_t)_a^{\alpha} \alpha H_a^{\alpha-1} B.$$

The adequacy of the model is ensured by the fact that during the modeling process, at each point, the actual depth of cut and feed per tooth are substituted into formula (2). In this way, the current state of the processing system is taken into account, which changes during dynamic phenomena according to the elastic movements of each dynamic system (δ_{mX}, δ_{mY}, δ_{wX}, δ_{wY}) along the coordinate axes:

$$H_a = H_c + \delta_{mY} + \delta_{wY}; (f_t)_a = (f_t)_c + \delta_{mX} + \delta_{mY}, \quad (3)$$

where H_c, $(f_t)_c$ – the commanded depth and feed per tooth.

These connections will determine the shape of the allowance layer (chips) that is cut by each tooth of the cutter. Therefore, on the passage of the next cutter tooth, the allowance layer will be curved due to vibrations of the dynamic machining system during cutting. This process is reflected through the delay argument function in positive feedback for each coordinate and is called "trace machining". As shown in [2, 3, 17], machining along the trace is the main reason for the occurrence of regenerative oscillations in the machining system. Considering the closedness of the cutting process in an elastic dynamic system, represented by a two-mass model with two degrees of freedom, the model of the entire system may be presented in the form of a block diagram in Figure 2.

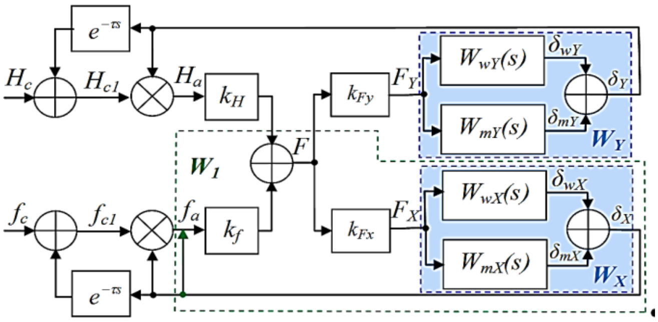


Figure 2 – Block diagram of the machining system

In the block diagram, the two-mass dynamic system is represented by the following transfer functions:

$$\begin{aligned} W_{wX}(s) &= \frac{1/k_{wX}}{T_{wX}^2 s^2 + 2\xi T_{wX} s + 1}; \\ W_{wY}(s) &= \frac{1/k_{wY}}{T_{wY}^2 s^2 + 2\xi T_{wY} s + 1}; \\ W_{wX}(s) &= \frac{1/k_{wX}}{T_{wX}^2 s^2 + 2\xi T_{wX} s + 1}; \\ W_{wY}(s) &= \frac{1/k_{wY}}{T_{wY}^2 s^2 + 2\xi T_{wY} s + 1}, \end{aligned} \quad (4)$$

where T_{wX}, T_{wY} – the period of natural oscillations of the mass of the workpiece system along the X and Y axes; T_{mX}, T_{mY} – the period of natural oscillations for the mill system along the X and Y axes; s – the Laplace operator.

The parallel connection of blocks in Figure 2, using the rules for transforming block diagrams, can be replaced by one block, the transfer function of which is reduced to a standard form.

For Y coordinate:

$$\begin{gathered} W_Y = \frac{\delta_Y(s)}{F_Y(s)} = \frac{a_1 s^2 + a_2 s + a_3}{b_1 s^4 + b_2 s^3 + b_3 s^2 + b_4 s + 1}; \\ a_1 = \frac{T_{mY}^2}{k_{wY}} + \frac{T_{wY}^2}{k_{mY}};\ a_2 = 2\xi\left(\frac{T_{mY}}{k_{wY}} + \frac{T_{wY}}{k_{mY}}\right); \\ a_3 = \frac{k_{mY} + k_{wY}}{k_{mY} k_{wY}};\ b_1 = T_{wY}^2 T_{mY}^2; \\ b_2 = 2\xi(T_{wY} T_{mY}^2 + T_{wY}^2 T_{mY}); \\ b_3 = T_{mY}^2 + 4\xi^2 T_{wY} T_{mY} + T_{wY}^2; \\ b_4 = 2\xi(T_{mY} + T_{wY}). \end{gathered} \quad (5)$$

Similarly, for the X coordinate:

$$W_X = \frac{\delta_X(s)}{F_X(s)} = \frac{a_{11}s^2 + a_{12}s + a_{13}}{b_{11}s^4 + b_{12}s^3 + b_{13}s^2 + b_{14}s + 1};$$
$$a_{11} = \frac{T_{mX}^2}{k_{wX}} + \frac{T_{wX}^2}{k_{mX}};\ a_{12} = 2\xi\left(\frac{T_{mX}}{k_{wX}} + \frac{T_{wX}}{k_{mX}}\right);$$
$$a_{13} = \frac{k_{mX} + k_{wX}}{k_{mX}k_{wX}};\ b_{11} = T_{wX}^2 T_{mX}^2; \quad (6)$$
$$b_{12} = 2\xi(T_{wX}T_{mX}^2 + T_{wX}^2 T_{mX});$$
$$b_{13} = T_{mX}^2 + 4\xi^2 T_{wX}T_{mX} + T_{wX}^2;$$
$$b_{14} = 2\xi(T_{mX} + T_{wX}).$$

The main influence on the formation of the thickness of the allowance layer, which will determine the delay effect and machining along the trace, is exerted by elastic displacement along the Y coordinate. To determine the transfer function of the entire system at input H, it first needed to take the feed $f = 0$ and find the transfer function $W_1(s)$:

$$W_1(s) = \frac{1}{1 + k_{FX}W_X k_f} =$$
$$= \frac{c_1 s^4 + c_2 s^3 + c_3 s^2 + c_4 s + c_5}{d_1 s^4 + d_2 s^3 + d_3 s^2 + d_4 s + 1};$$
$$c_1 = \frac{T_{wX}^2 T_{mX}^2}{z_1};\ c_2 = \frac{2\xi(T_{wX}T_{mX}^2 + T_{wX}^2 T_{mX})}{z_1};$$
$$c_3 = \frac{T_{mX}^2 + 4\xi^2 T_{wX}T_{mX} + T_{wX}^2}{z_1};\ c_4 = \frac{2\xi(T_{mX} + T_{wX})}{z_1};$$
$$c_5 = \frac{1}{z_1};\ d_1 = \frac{T_{wX}^2 T_{mX}^2}{z_1}; \quad (7)$$
$$d_2 = \frac{2\xi(T_{wX}T_{mX}^2 + T_{wX}^2 T_{mX})}{z_1};$$
$$d_3 = \frac{T_{mX}^2 + 4\xi^2 T_{wX}T_{mX} + T_{wX}^2 + k_f k_{FX}\left(\frac{T_{mX}^2}{k_{wX}} + \frac{T_{wX}^2}{k_{wX}}\right)}{z_1};$$
$$d_4 = \frac{2\xi(T_{mX} + T_{wX}) + 2\xi k_f k_{FX}\left(\frac{T_{mX}}{k_{wX}} + \frac{T_{wX}}{k_{wX}}\right)}{z_1};$$
$$z_1 = \frac{k_{mX}k_{wX} + k_f k_{FX}(k_{mX} + k_{wX})}{k_{mX}k_{wX}}.$$

The transfer function of the entire system can be determined as follows:

$$\frac{\delta_Y(s)}{H(s)} = \frac{k_H W_1(s) k_{FY} W_Y(s)}{1 + k_H W_1(s) k_{FY} W_Y(s)} =$$
$$= \frac{A_1 s^6 + A_2 s^5 + A_3 s^4 + A_4 s^3 + A_5 s^2 + A_6 s + A_7}{B_1 s^8 + B_2 s^7 + B_3 s^6 + B_4 s^5 + B_5 s^4 + B_6 s^3 + B_7 s^2 + B_8 s + 1}; \quad (8)$$

with the following coefficients:

$$A_1 = (k_H k_{FY} c_1 a_1)/z;\ A_2 = k_H k_{FY}(c_1 a_2 + c_2 a_1)/z;$$
$$A_3 = k_H k_{FY}(c_1 a_3 + c_2 a_2 + c_3 a_1)/z;$$
$$A_4 = k_H k_{FY}(c_2 a_3 + c_3 a_2 + c_4 a_1)/z;$$
$$A_5 = k_H k_{FY}(c_3 a_3 + c_4 a_2 + c_5 a_1)/z;$$
$$A_6 = k_H k_{FY}(c_4 a_3 + c_5 a_2)/z;\ A_7 = k_H k_{FY} a_3/z;$$
$$B_1 = (d_1 b_1)/z;\ B_2 = (d_2 b_1 + d_1 b_2)/z;$$
$$B_3 = (d_3 b_1 + d_2 b_2 + d_1 b_3 + k_H k_{FY} c_1 a_2)/z;$$
$$B_4 = (d_4 b_1 + d_3 b_2 + d_2 b_3 + d_1 b_4 + k_H k_{FY} c_2 a_1 + k_H k_{FY} c_1 a_2)/z;$$
$$B_5 = (b_1 + d_4 b_2 + d_3 b_3 + d_2 b_4 + d_1 + k_H k_{FY} c_3 a_1 +$$
$$+ k_H k_{FY} c_2 a_2 + k_H k_{FY} c_1 a_3)/z;$$
$$B_6 = (b_2 + d_4 b_3 + d_3 b_4 + d_2 + k_H k_{FY} c_4 a_1 + k_H k_{FY} c_3 a_2 +$$
$$+ k_H k_{FY} c_2 a_3)/z;$$
$$B_7 = (b_3 + d_4 b_4 + d_3 + k_H k_{FY} c_5 a_1 + k_H k_{FY} c_4 a_2 + k_H k_{FY} c_3 a_3)/z;$$
$$B_8 = (b_4 + d_4 + k_H k_{FY} c_5 a_2 + k_H k_{FY} c_4 a_3)/z;$$
$$z = 1 + k_H k_{FY} c_5 a_3.$$

For numerical modeling using standard integration procedures to obtain the transient response and the procedure for calculating the amplitude-frequency response, it is necessary to add coupling equations to the resulting mathematical model:

$$\begin{cases} H_a = H_c - \delta_Y(1 - e^{-\tau s}); \\ f_a = f_c - \delta_X(1 - e^{-\tau s}), \end{cases} \quad (9)$$

where $\tau = 2\pi/(n\omega_s)$ – time delay, s; n – number of cutter teeth; ω_s – cutter rotation speed, rad/s.

3.2 Parameter identification of the dynamic system

Modeling requires preliminary determination of the dynamic parameters of the machining system. Such parameters include the frequency of natural vibrations and the rigidity of the "tool" system in two directions along the coordinate axes, the frequency of natural vibrations, and the rigidity of the "workpiece" system in two directions. These parameters must be identified in the machining system where the experiment will be conducted.

The stiffness was determined according to the scheme described in [17] when the machining system was loaded by the drives of a CNC machine using a manually adjusted encoder. In this case, a dynamometer controlled the force, and the elastic movements in the required directions were controlled by indicators.

Dynamic parameters are identified using modal analysis methods when the machining system's frequency response function (FRF) is determined by the impulse response function (IRF) obtained by impacting the system with an impact hammer. The experimental design is also like in [17], with the difference that a dual-beam storage oscilloscope model XDS3202E was used, making it possible to simultaneously record two IRF from the accelerometer and the impact hammer (Figure 3).

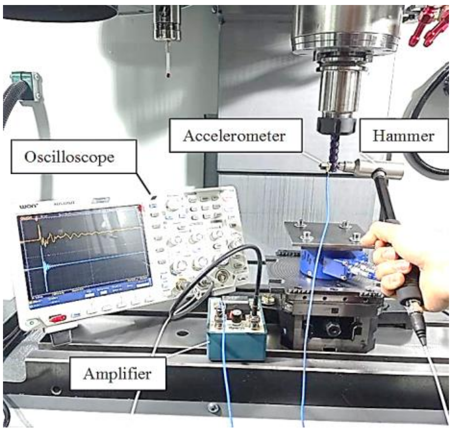


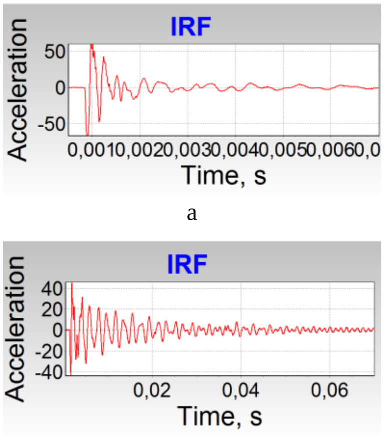
Figure 3 – Determination of the impulse response function (IRF) of the "tool" system

The IRF of the impact hammer allows one to evaluate the sufficiency of its frequency characteristics for the required excitation of the “tool” system.

A similar scheme was used to determine the FRF of the “workpiece” system. In this case, the piezoaccelerometer was installed on the workpiece, which was struck with the impact hammer.

The files of the obtained impulse characteristics were transferred to a special program in which, using Fast Fourier Transforms (FFT), the frequency responses of the system were determined.

Figure 4 shows the program interface with the downloaded file of the IRF obtained from the accelerometer when hitting the mill with a hammer.

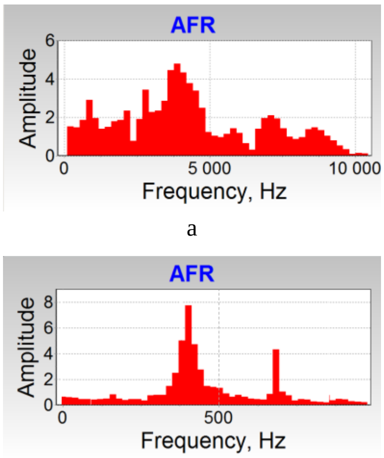


b

Figure 4 – IRF for the “tool” (a) and “workpiece” (b) systems

The file was saved in the storage oscilloscope and converted to *.txt format.

Next, the IRF is converted into a signal spectrum using fast Fourier transform (FFT) as shown in Figure 5.



b

Figure 5 – Spectrums for the “tool” (a) and “workpiece” (b) systems

The solution was adopted to calculate the natural frequency of the substituting mass in the model for the energy equivalence of the spectrum and mass signals. Therefore, the natural frequency of the replacement mass was calculated using the formula:

$$(\omega_m)_Y = \sqrt{\frac{2E}{A_a^2}}; \tag{10}$$

where E – the energy of the experimental spectrum; A_a – the average amplitude.

The energy of the experimental spectrum and the average amplitude are calculated using the following formulas:

$$E = \frac{1}{2}\sum_{i=1}^{kn} A_i^2 \omega_i^2 \,;\; A_a = \frac{1}{kn}\sum_{i=1}^{kn} A_i, \tag{11}$$

where ω_i, A_i – the frequency and amplitude of each harmonic of the discrete spectrum; kn – the number of discrete ranges (selected harmonics).

To increase the reliability, the natural frequency of the replacement mass was calculated as the average between the frequency according to formula (11) and the frequency calculated from the spectral energy density of the signal [18]. Thus, the natural frequencies of oscillations of each mass necessary for modeling were determined according to the schema in Fig. 1. Oscillation damping coefficients are also calculated using the well-known formula [18]:

$$\xi = \frac{1}{T} ln\left[\frac{A(t)}{A(t+T)}\right]; \tag{12}$$

where T – the period; $A(t)$ – the amplitude of the transition function at time t; $A(t + T)$ – the amplitude of the transition function at time $(t + T)$.

4 Results

4.1 Program modeling and SLD design

In accordance with the aim of this study, the developed methodology should provide for automatic determination of the SLD. To solve such a problem, it is necessary to use a new technique based on the analysis of the frequency responses of systems described by differential equations with delay [19]. The solution is based on a new criterion for the stability of such systems with a delayed argument in positive feedback. The system will be stable if the graph of its Nyquist diagram on the complex plane does not cover the point with coordinates [+1, 0].

The mathematical model of the machining system (9) is nonlinear and eighth order, so a special application program has been created for digital modeling (Figure 6).

Modeling of the transient response is carried out by integration using the standard 4th order Runge-Kutta procedure in according of initial data in windows of interface. The simulation results are displayed in the virtual oscilloscope window. The transition to the frequency domain of modeling is performed by pressing the button “Nykuist chart” on interface. An additional interface appears where possibility is select the option of constructing a Nyquist plot or a SLD. The Nyquist diagram is calculated for a system open to loop positive feedback, and the SLD is constructed automatically using a special algorithm [19].

When modeling, the dynamic parameters of the machining system, which were identified in full-scale experiments, are entered into the program (Table 1).

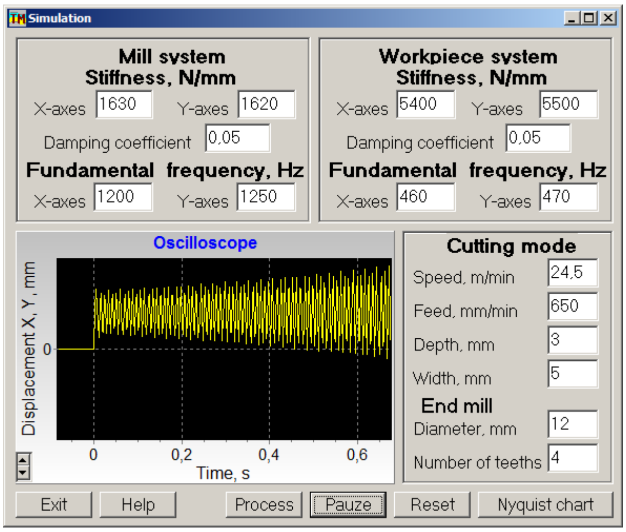


Figure 6 – Interface of the program created

Table 1 – Dynamic parameters of machining system

System	Stiffness, N/mm		Eigenfrequency, Hz	
	Axis *Y*	Axis *X*	Axis *Y*	Axis *X*
"tool"	1620	1630	1250	1200
"workpiece"	5500	5400	470	460

In addition, the vibration damping coefficients for all systems are assumed to be 0.05, and the empirical coefficient and exponent in the cutting force model are adopted according to reference data for milling a workpiece made of carbon steel C45.

The algorithm for constructing an SLD search for a digital array "speed – feed" using a stability criterion for systems with a delay in positive feedback. At each algorithm step, a Nyquist diagram is calculated for the initial values of cutter speed and feed within a given range. Next, for a phase angle of $360°\cdot i$, where $i = 0, 1, 2, \ldots, n$, and the maximum amplitude A_{max} of the diagram is determined.

If $A_{max} > 1$, then the feed is reduced by a given step; if $A_{max} < 1$, then the feed is increased, and the Nyquist diagram is calculated again. Such iterations are performed until $1 - \delta < A_{max} < 1 + \delta$, where δ – the specified calculation accuracy. This determines one point on the SLD.

The process is repeated with a given speed change step for the entire range of cutter speed changes, and the saved data array forms a stability diagram (Figure 7).

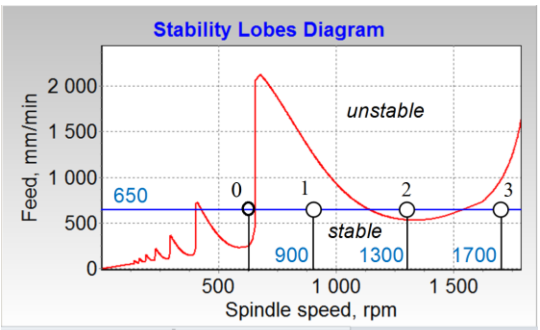


Figure 7 – Stability lobes diagram (SLD)

The stability diagram divides the entire range of possible values into two zones: stable and unstable cutting process. Figure 6 shows the transient response of the machining system when selecting the cutting mode at point "0" in the diagram. The data enters an instability zone, as evidenced by the transient response of the machining system in the time domain.

The importance of the results obtained for assigning a chatter-free cutting mode is obvious: depending on the spindle speed at the same feed (650 mm/min), both stable and unstable machining modes can be obtained. Compliance with the new stability criterion for machining systems with a delay in positive feedback is confirmed by analysis of the process at the points.

Figure 8 presents the results of modeling the machining system with dynamic parameters obtained experimentally (Table 1).

Simulation in the created program makes it possible to obtain both the transient and frequency characteristics of the system for a given cutting mode.

The cutting mode corresponding to points "1" (900 rpm) and "3" (1700 rpm) of the diagram ensures rapid damping of system oscillations along the Y axis; the Nyquist diagram does not cover the critical point [+1, 0] on the complex plane (Figure 8 a, c). The cutting mode corresponding to point "2" of the diagram (1300 rpm) provokes a process that diverges in time, and the Nyquist diagram covers the critical point (Figure 8 b).

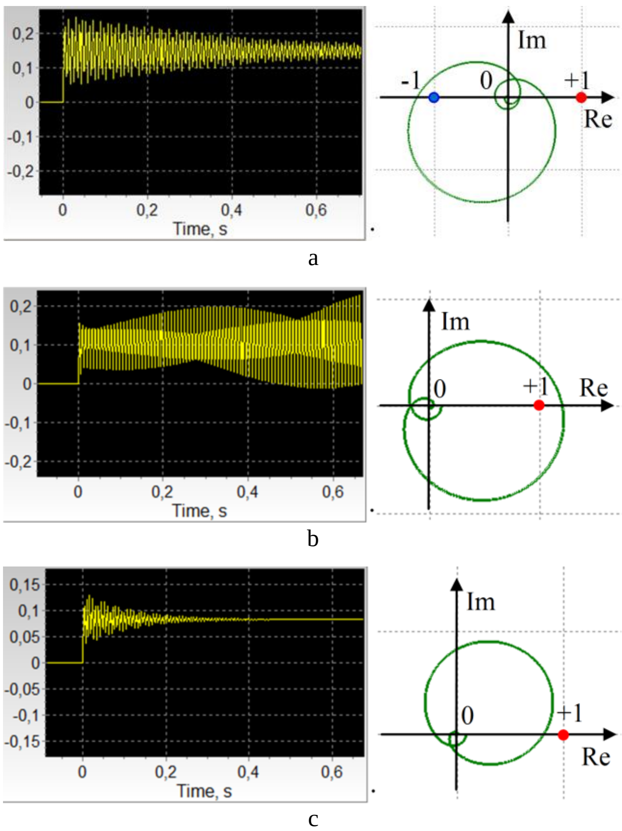


Figure 8 – Simulation results for $f = 650$ mm/min:
a – $n = 900$ rpm; b – $n = 1300$ rpm; c – $n = 1700$ rpm

The transient response of the machining system when selecting the cutting mode at point “0” in the diagram. The data enters an instability zone, as evidenced by the transient response of the machining system in the time domain.

The attention should be paid to the location of the Nyquist diagram for the cutting mode corresponding to point “1” of the SLD. The graph of the Nyquist diagram in this mode covers a point with coordinates [–1, 0] on the complex plane, which for systems with negative feedback means a loss of stability. This once again demonstrates the uniqueness of the proposed criterion for constructing a SLD.

4.2 Experimental testing

A square-shaped workpiece was milled during the experiments and mounted on a dynamometer table (Figure 3). The dynamometer is installed on a CNC milling machine table from HAAS TM0p. This workpiece arrangement allows milling all four sides of the workpiece when changing cutting conditions without removing it from the machine. Milling was carried out at cutting modes according to the points “1”, “2” and “3” of the stability diagram (Figure 7).

Since vibrations in the machining system during cutting form the surface relief of the part, the roughness of the machined surface assessed the vibration level. It is known that surface roughness consists of a deterministic and random component. The deterministic component when milling a flat surface depends only on the geometric interaction of the cutter teeth with the workpiece and can be calculated according to:

$$Rz = R_m - \sqrt{R_m^2 - \left(\frac{f_t}{2}\right)^2}, \tag{14}$$

where R_m – the radius of the mill; $f_t = f/(zn_s)$ – the feed per tooth; f – the feed, mm/min; z – the number of cutter teeth; n_s – the spindle rotation speed, rpm.

Thus, the level of vibration, estimated by amplitude, can be defined as the amplitude of the random component of the surface relief:

$$\Delta = Ra - Rz; \tag{15}$$

where *Ra* – the arithmetic mean deviation of the profile; *Rz* – the height of irregularities according to formula (14).

During the experiments, 3 parts were machined with different cutting modes on 12 surfaces of each part.

Figure 9 shows profilograms of surfaces machined at cutting modes corresponding to points 1, 2, and 3 on the stability diagram (Figure 7).

The results of measurements of the roughness of treated surfaces generally confirm the adequacy of the developed methodology. Thus, the amplitude of the random component of the surface profile machined under the cutting modes that correspond to points 1 and 3 is 1.4 µm and 0.64 µm, while the surface machined under the cutting mode of point 2 is 3.2 µm.

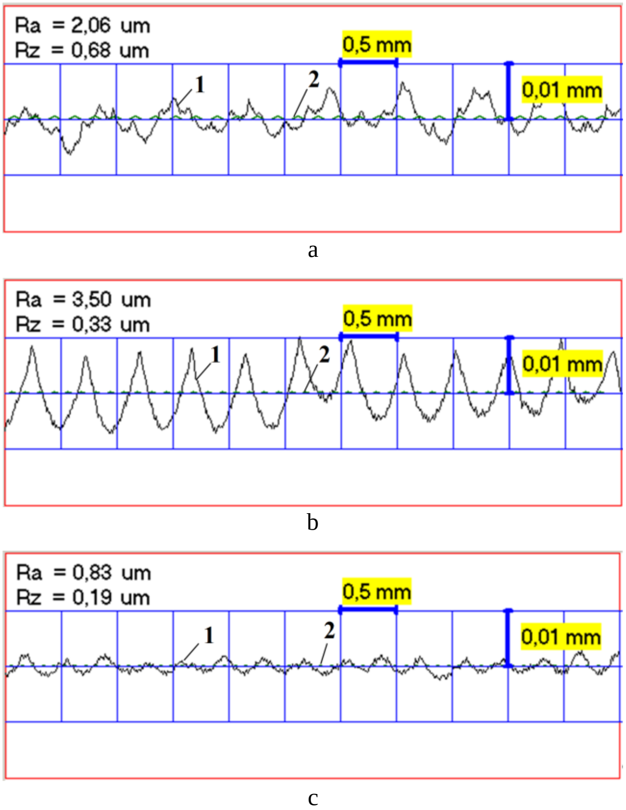


Figure 9 – Profilograms of part surfaces for f = 650 mm/min:
a – n = 900 rpm; b – n = 1300 rpm; c – n = 1700 rpm;
1 – profile line, 2 – deterministic component

If the linear dependence of the roughness parameter *Ra* on the spindle speed is accepted, then when machined with point 2 mode, *Ra* = 1.45 µm is expected.

Remarkably, *Ra* under this cutting mode is 2.2 times greater, indicating a process stability violation. Similar results were observed in all experiments performed.

Thus, the effectiveness of the developed method for determining the vibration-free cutting mode in end milling, based on a new method for constructing an SLD, has been experimentally proven.

5 Discussion

It is shown that vibration accompanies the cutting process and is the main obstacle to achieving the accuracy and productivity of milling operations. Passive or active control methods are used to reduce the chatter level, but the easiest to implement is to assign a cutting mode based on the results of the SLD design. This mode is easy to implement when machining on CNC machines, where it is possible to continuously adjust the feed and spindle speed over a wide range.

However, the proposed SLD design methods [2, 3] focus on the approximate solution of the system's characteristic equation, which is described by a differential equation with a delay in positive feedback. Design is carried out according to a special algorithm, which provides five steps that are difficult to automate for computer use.

This paper continues research into the stability of end milling processes using a previously developed stability criterion. Considering the uniqueness of the proposed criterion, it was essential to investigate the effectiveness of its application for systems with a new structure in the form of interaction between the elastic systems of the tool and the workpiece through the cutting process.

In this study, a new criterion was used to design the SLD based on the analysis of the location of the Nyquist diagram of an open-loop feedback lag system [19]. This criterion was first applied to a machining system in the form of end milling, the dynamics of which are represented by two masses, each of which has two degrees of freedom. The dynamic model was developed when presented as a structure with two inputs: depth of cut and feed (Figure 2).

Differential equations describe the mathematical model with a delay argument, have an eighth order, and are presented as state variables. This form is most convenient for numerical modeling and allows the design of the system's transient and frequency characteristics. The delay function is implemented as a recurrence relation in numerical simulation. This automatically considers the change in the thickness of the chips cut by each tooth, which does not require additional geometric calculations, unlike works [1, 3], where such calculations determine some conditions for the system's stability. In general, in our opinion, external disturbance – in the form of changes in chip thickness - cannot be the reason for the system's stability. Its stability is determined only by internal processes and can be studied by well-known methods of automatic control theory.

The developed algorithm for the automatic design of the stability diagram has confirmed its effectiveness and ability to function for such high-order models. The created software allows for generating a digital array and constructing a stability diagram in the "spindle speed – feed" coordinates (Figure 7). Practical testing was carried out using measurements of the roughness of machined surfaces (Figure 9), which fully confirmed the adequacy of the SLD design.

Thus, the developed methodology and software have significantly simplified the practical application of assigning cutting modes according to the stability diagram. However, preliminary identification of the dynamic parameters of the machining system remains problematic, and special measuring equipment is required.

It is expected that the practical use of the results obtained directly on a machine tool, which is equipped with a vibration and cutting force sensor following the example of Okuma machines [20], will be promising. In this case, the stability diagram can be designed during the cutting process, and the search for a vibration-free mode can be carried out based on the level of vibrations from the feedback sensor toward the stable region of the resulting diagram.

6 Conclusions

A mathematical model of the end milling process has been developed in the form of a two-mass dynamic system with two degrees of freedom for each mass and two inputs for depth and cutting feed. The mass of the "tool" system interacts with the mass of the "workpiece" system through the cutting process, which ensures the closedness of the machining system. In addition, the model considers positive feedback through the delay argument function; the delay time is equal to the mill's rotation time by one tooth.

Identification of the dynamic parameters of the machining system was carried out using the impulse response obtained from an accelerometer mounted on the system under test. The natural frequency for the substitute mass in the model was determined from the signal's spectrum as frequency according to the criterion of equality of power and spectral density of the experimental signal spectrum.

It is shown that a new criterion for the stability of the cutting process as a closed system with a delay in the positive feedback circuit, based on the analysis of frequency characteristics in the form of a Nyquist diagram, can be used to diagnose chatter for systems described by a high order of retarded differential equations.

The validity of the new criterion for the stability of machining systems is confirmed by a full-scale experiment during end milling based on an analysis of the roughness of the machined surface. It is shown that changing the spindle speed from 1300 rpm to 1700 rpm with the same feed rate of 650 mm/min leads to a change in the *Ra* parameter range from 3.2 μm to 0.64 μm.

Therefore, assigning a cutting mode using a stability diagram allows for improvoing the machining quality even with the process's intensification.

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