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REGULAR ARTICLE

Novel Approach Based Minimization of Geometric Action for Predicting Rare and Extreme Events in Non-Equilibrium Systems

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Identifying and quantifying unexpected events in non-equilibrium systems is critical work that is necessary for systems managers to make well-informed decisions, particularly when forecasting rare and extreme events. In this paper neural networks are integrated to increase the predictive capacity of information theory. Two information theory techniques, “Information Length (IL) and Information Flow (IF)”, are being examined for their sensitivity to rapid changes. To simulate the first occurrence of extreme and rare events, we utilize a non-autonomous Kramer model to introduce a perturbation. we introduced a Dynamic Osprey Long Short-Term Memory (DOLSTM) for predicting rare and extreme events in non-equilibrium systems. Our results show that IL performs better than IF in accurately forecasting unexpected occurrences when combined with a neural network. This study highlights a novel integration between information theory & neural networks, giving an effective strategy for forecasting rare & extreme events in non-equilibrium environments. An effective methodology for identifying and forecasting the behavior of dynamic systems is established by combining information-length diagnostics with neural network predictions, especially in situations involving rare and extreme events. This novel method illustrates that the theory of information and neural networks can be used to provide robust predictions for dynamic systems, when encountering rare and extreme events.

Keywords: Rare and extreme events, Information length, Information flow.

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1. INTRODUCTION

Extreme Precipitation Event (EPE) prediction is an essential scientific problem that is crucial to society for water management optimization and civil protection. EPEs can arise as a result of a variety of atmospheric and geographic conditions. Moisture availability and movement are essential components, as they are required to attain excessive daily accumulations [1]. Due to their catastrophic effects and sporadic occurrence, extreme events provide significant challenges to several scientific communities in interdisciplinary research. A wide range of processes, from environmental catastrophes to call dropouts in cellular networks, exhibit the emergent behavior of extreme events [2]. It is common in physics, chemistry and many other scientific fields to examine systems whose dynamics are changeable or fluctuating and where crucial information is found in "rare events," or unique occurrences of the dynamics that deviate from the usual pattern [3]. Equilibrium statistical mechanics, which is based on the assumption that systems would eventually attain a stable state, is inadequate in the context of non-equilibrium systems. Alternatively, non-equilibrium systems exchange matter and energy with their environment, resulting in complex behaviors that require new methods of analysis [4]. Sophisticated approaches that capture the fundamental factors, fluctuations and effects of external perturbations are necessary for the prediction of uncommon occurrences in such systems [5].

The intricacy of non-equilibrium systems necessitates the application of sophisticated computer models and computational techniques. Machine learning techniques, stochastic calculus as well as dynamic systems theory are used to interpret the complex relationships and patterns that provide shape to unusual events [6]. The impact of outside disturbances and fluctuations in pushing a system toward critical states is considered into factoring when predicting uncommon events. Recognizing the ways that

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internal dynamics interact with external factors is crucial for predicting unusual occurrences as even small variations have the ability to set off a chain reaction that deviates from the general pattern [7]. Non-equilibrium systems are unpredictable and exhibit emergent behaviors because they are defined by dynamic, changing states that are remote from thermodynamic equilibrium. In these systems, occurrences with low frequency but important ramifications, like big fluctuations, abrupt transitions, or catastrophic events, are referred as rare and extreme events [8].

The following categories can be used to classify the remaining research: Section 2 provided an explanation of related works. Section 3 outlines our recommended course of action. Section 4 presents the study's result, while section 5 concludes the paper.

2. RELATED WORK

Study [9] examined the effectiveness of deep learning techniques in complex chaotic dynamical systems for predicting severe events. Deep neural networks have proven effective in solving a variety of big data image processing issues and had demonstrated promise in the research of dynamical systems. Study [10] introduced a HybridNet framework that combines model-driven computation with data-driven deep learning to forecast the spatiotemporal evolution of dynamical systems, even if the parameters do not exist with clarity. Study [11] presented a deep neural network method that enables event cameras to be used for a difficult motion-estimation task: predicting the steering angle of a moving vehicle. Study [12] utilized the dehumidification technique to reduce damage caused by condensation flow that was not in equilibrium. The Eulerian-Eulerian technique and sensitive grid size test were used for the first modeling & validation of the phenomena.

Study [13] developed a thermodynamic and dynamical non-equilibrium framework for generic complex systems. An analogy to the classical statistic mechanic method for dealing with the phases of equilibrium transitions was employed in the technique to capture and reflect the character of the non-equilibrium dynamics. Study [14] presented a novel paradigm for handling multiscale non-equilibrium flows using machine learning that was based on physical constraints and inspiration. The resulting model, "coarse-grained deep operator networks (CG-DeepONet)", utilized a hierarchical architecture inspired by physics to train integral response operators for multi-fidelity coarse-textured master equations. Its central component, PI-DeepONet, was physics-informed. Study [15] constructed an effective method that uses machine learning techniques to estimate the rate of entropy formation over time. The algorithm validated the numerical estimates in relevant parameter regimes utilizing tractable Langevin models. Study [16] employed the non-equilibrium statistical mechanics' landscape-flux theory as a general framework to measure the ecological systems' overall stability and provide alerts for crucial transitions.

Study [17] presented a multiscale model that describes how three-dimensional strain affects non-equilibrium wall-bounded turbulence. That was achieved utilizing direct numerical simulation of transient, three-dimensional turbulent channels up to 1,000 frictions Reynolds numbers that were subjected to a lateral pressure gradient.

3. PROPOSED WORK

3.1 The Statistics of Non-autonomous Linear Stochastic Procedures

The Gaussian stochastic noises, denoted as $\Gamma \in R^n$, are produced by n dimension vectors of δ-correlated Gaussian noise, denoted as Γ_i (i = 1,2,3,..n) & so on. The consistent real matrices A & B are $n \times n$ and $n \times 1$, respectively.

"A linear non-autonomous procedure" is given by,

$$\dot{w}(s) = Bw\,(s) + Av(s) + \Gamma(s) \tag{1}$$

$$\langle \Gamma_j(s) \rangle = 0, \langle \Gamma_j(s)\Gamma_i(s_1) \rangle = 2C_{ji}(s)\,\delta(s-s_1)\,, c_{ji}(s)\,, \forall j, i = 1,2,..,m \tag{2}$$

Here, the average over Γ_i is indicated by the angle brackets. The Gaussian "probability density function (PDF)" is assumed at the outset and it stays Gaussian throughout time. Therefore, the following is true.

At every given time t, the value of the combined PDF of systems (1) and (2) is provided by,

$$o(w;s) = \frac{1}{\sqrt{\det(2\pi\Sigma)}} f^{-\frac{1}{2}(w-\langle w(s)\rangle)^S \Sigma^{-1}(w-\langle w(s)\rangle)} \tag{3}$$

$$\langle W(s) \rangle = f^{Bs}\langle w(0) \rangle + \int_0^s f^{B(s-\tau)} Av(\tau) d\tau \tag{4}$$

$$\Sigma(s) = f^{Bs}\langle \delta W(0)\delta w(0)^S \rangle f^{B^S s} + 2\int_0^s f^{B(s-\tau)}\, C f^{B^S(s-\tau)} d\tau \tag{5}$$

Where the matrix $D \in R^{n \times n}$ has the entries $c_{ji}\,(s)$. The covariance matrix is denoted by Σ and the mean value for W(s) is represented by ⟨W(s)⟩.

The following outcome can be used to calculate the exponential matrix f^{Bs}.

$$f^{Bs} = \ell^{-1}[(tJ - B)^{-1}] \tag{6}$$

The complex variable t inverse Laplace transform is represented by l^{-1} in this case.

3.2 Diagnostics for Information Length (IL)

We determine the IL "L"of the system provided by the joint PDF $o(w;s)$.

$$\mathcal{L}(s) = \int_0^s ds_1 \sqrt{\int_{-\infty}^{\infty} dw \frac{[\partial_{s_1} o(w;s_1)]^2}{o(w;s_1)}} = \int_0^s ds_1 \sqrt{\varepsilon} \tag{7}$$

Where the information velocity squared is represented by $\mathcal{E} = \int_{-\infty}^{\infty} dw \frac{[\partial_{s_1} o(w;s_1)]^2}{o(w;s_1)}$.

A dimension of $1\sqrt{\varepsilon} \equiv \tau$ provides a dynamic measure of time for information change. This means that by combining √ε from times 0 and *t*, every data change in the specified interval can be found.

The combined PDF of the system's IL is given by,

$$\mathcal{L}(s) = \int_0^s ds_1 \sqrt{\varepsilon(s_1)} \quad (8)$$

$$\varepsilon(s_1) = (\partial_{s_1}\langle w(s_1)\rangle^s)\Sigma^{-1}(\partial_{s_1}\langle w(s_1)\rangle) + \frac{1}{2} tr \left((\Sigma^{-1}\partial_{s_1}\Sigma)^2\right) \quad (9)$$

Eqs. (4) and (5) can be used to determine ⟨W(s)⟩ and Σ(t), respectively, which we need to compute Eq. (9). For $\partial_s\langle W(s)\rangle i$ in particular, we have:

$$\partial_s\langle w(s)\rangle = B\langle w(s)\rangle + Av(s) \quad (10)$$

It is helpful to establish $\varepsilon_n(s)$ as follows for a linear process of order n (1) with an unknown variable $x \epsilon R^n = [x_1, x_2, \dots, x_n]^T$.

$$\mathcal{E}_n(s) = \sum_{j=1}^m \varepsilon_j(s) = \sum_{j=1}^m \frac{(\partial_s\langle w_j\rangle)^2}{\Sigma_{w_j w_j}} + \sum_{j=1}^m \frac{\left(\partial_s \Sigma_{w_j w_j}\right)^2}{2\Sigma^2_{w_j w_j}} \quad (11)$$

Where a marginal PDF of $o(w;s)$ is used to compute $\mathcal{E}_i$. Keep in mind that whenever the n random factors are independent, E in Equation (9) is the same as $\mathcal{E}_n$ in Eq. (11).

$\mathcal{E} = \mathcal{E}_n$ can be used to introduce independent variables.

$$\varepsilon(s) - \varepsilon_n(s) \quad (12)$$

3.3 Information Flow Based on Entropy

An important information theory metric that has been examined in terms of causes, knowledge development and predictability transmission is “information flow (IF)”, known as information transfer.It shows the exact way the various states of the system to the others. Having the consideration of a pair of Brownian particles with coordinate $x = (x_1, x_2)$, subjected to a potential $G(w)$, given by the Langevin equation, interacting with two separate thermal baths at temperatures T_1 and T_2, respectively.

$$p = -\partial w_j G(w) - \Gamma j w^{\cdot} j(s) + vj(s) + \eta j(s),$$

$$\langle \eta_j(s)\eta_i(s_1)\rangle = 2\Gamma_j S_j \delta_{ji}\delta\,(s - s_1),\ j, i = 1,2 \quad (13)$$

In this case, the bounded input is denoted by $vj(s)$, Kronecker's symbol is represented by δ_{ji} and the particle-environment interaction is characterized by the lowering constants Γ_j (based on the temperature S_j).

Next, data flows S from 1 → 2 and 2 → 1 are provided by,

$$S_{2\to1} = \frac{1}{\Gamma_1}\int dwO(w;s)\left[\partial_{w_1} G(w) + S_1\partial_{w_1}\ln O(w;s)\right]\partial_{w_1}\ln\frac{O_{w_1}(w_1;s)}{O(w;s)}, \quad (14)$$

$$S_{1\to2} = \frac{1}{\Gamma_2}\int dwO(w;s)\left[\partial_{w_2} G(w) + S_2\partial_{w_2}\ln O(w;s)\right]\partial_{w_2}\ln\frac{O_{w_2}(w_2;s)}{O(w;s)}, \quad (15)$$

Recalling that Eqs. (14) and (15) can be used to interpret the actual meaning of IF in the context of shared understanding or entropy S is helpful.

$$S_{2\to1} = \partial_s T[w_1(s)] - \partial_{s_1} T[w_1(s + s_1)|w_2(s)]|_{s_1\to0} \quad (16)$$

Where the notation $T[w_1(s + s_1)|w_2(s)]$ represents the variance of $w_1(s + s_1)$ at the time $s + s_1$, conditioned by w_2(s)at the previous time and t. According to Eq. (16), IF is the amount of change in the entropy conditional of w_1, w_2 that remains frozen between the times $(s, s + s_1)$ and the final entropy of w_1. Put otherwise, $S_{2\to1}$ represents the portion of the entropy variation in w_1 (between $(s, s + s_1)$ that results from variations in w_2.

There are a few important things to note. In the event where $S_{2\to1}$ (T_1) is negative, w_2 operates to decrease the least entropy as x_1. $S_{2\to1}$ & $S_{1\to2}$ might be both positive and negative. Second, the only variable utilized to determine causality is the absolute value of IF. Eq. (14) has one last advantage over Eq. (16): Rather than employing two-point time PDFs, equal-time joint/marginal PDFs can be used to construct it. Finally, although Eqs. (15) and (16) do not immediately show this, the IF depends on the (equal-time) correlation matrix.

4. RESULT

To assess the suggested method's effectiveness using the following metrics: detection time (sec), F1-score (%), accuracy (%), precision (%), recall (%), false positives and negatives. Some existing methods are “Random Forest (RF) (20), Support Vector Machine (SVM) (20) and Logistic Regression (LR) (20)”. Table 1 presents the dataset.

Table 1 – Dataset description

Date	Temperature (°C)	Pressure (hPa)	Velocity (m/s)	Abrupt Change
2022-01-01	25.3	1013.2	10.5	0
2022-01-02	24.8	1012.8	11.2	0
2022-01-03	23.5	1011.4	12.1	0
2023-01-01	22.0	1014.3	9.2	1
2023-01-02	22.3	1014.4	9.3	1
2023-01-03	22.5	1014.6	9.5	1
2023-12-31	23.8	1014.9	11.3	0

Note*: 1 = changes, 0 = no changes.

An important parameter for evaluating the detection system's overall correctness is its accuracy, which involves identifying rare & extreme events in non-equilibrium systems. We assess and contrast the innovative detection approach's accuracy with existing approaches. Fig. 1 shows the accuracy rates of proposed and existing approaches. Attained accuracy rates of LR (80%, 85%), SVM (82%, 89%) and RF (83%, 90%) in the IF and IL, respectively. In contrast to existing approaches, the suggested strategy (DOLSTM) has an accuracy rate of (85%, 92%).It demonstrates the superiority of our suggested method over existing techniques.

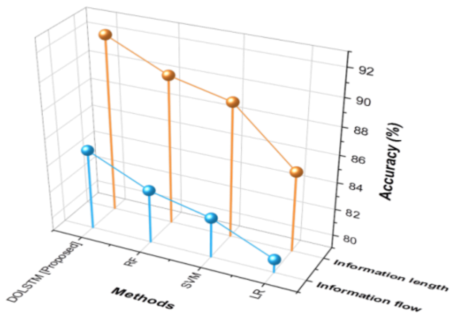

Fig. 1 – Result of Accuracy (Source: Author)

The detection system's precision is important since it measures how well the system recognizes rare & extreme events from the anticipated positive cases. We compare the accuracy of the suggested strategy with existing approaches. The precision rates for existing and suggested techniques are displayed in Fig. 2. LR obtained (71%, 83%), SVM (73%, 85%) and RF (77%, 89%) precision values in the IF and IL, respectively. Compared with existing methods, the proposed strategy (DOLSTM) has a precision value of 78%, 91%. It demonstrates the superiority of our suggested method over existing techniques.

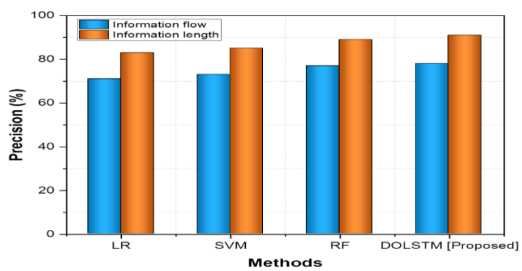

Fig. 2 – Result of Precision (Source: Author)

Evaluating recall is essential for recognizing the sensitivity of the detection system to rare & extreme events. This study examines and compares the proposed approach's recall performance to that of the existing methods. Fig. 3 shows the recall rates for the proposed and existing approaches. Acquired recall values for the IF and IL of LR (83%, 85%), SVM (87%, 89%) and RF (88%, 91%), respectively. When compared to existing methods, the suggested approach (DOLSTM) achieves 90% and 94% recall rates; respectively. It demonstrates the superiority of our suggested method over existing techniques.

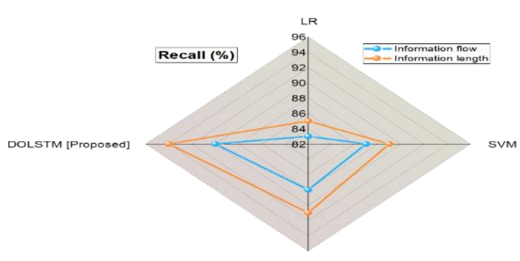

Fig. 3 – Result of Recall (Source: Author)

The F1-score combines precision and recall as factors to provide a thorough assessment of the detection system's performance. The F1 scores of the suggested strategy and existing approaches are contrasted in this analysis. The f1-score values for the suggested and existing approaches are displayed in Fig. 4. Obtained f1-score values for LR (75%, 83%), SVM (80%, 88%) and RF (81%, 90%), correspondingly, for the IF as well as IL. The proposed method (DOLSTM) achieves 84% and 92% f1-score values, respectively, when compared to existing approaches. It demonstrates the superiority of our suggested method over existing techniques.

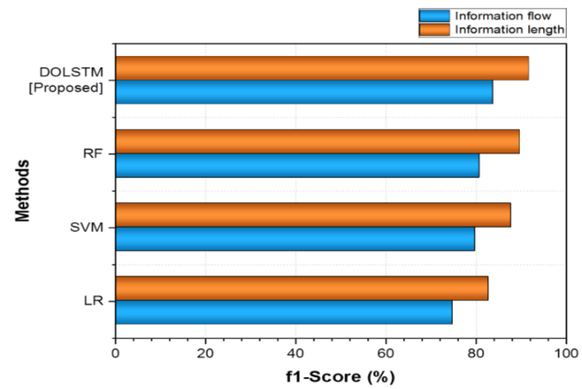

Fig. 4 – Result of F1-score (Source: Author)

Detection time is an important factor to consider as well because it shows the speed at which the system is able to respond to rare and extreme events. This study compares and examines the detection times attained by proposed and existing strategies.

5. CONCLUSION

Rare & extreme occurrences in non-equilibrium systems investigate infrequent and unusual events in dynamic systems, offering light on their fundamental causes and effects on complex phenomena. The aim of this study is to improve the identification and estimation of abrupt changes in dynamic structures, which is important for managing systems as well as forecasting uncommon and extreme events. We investigate the possibility of mimicking abrupt changes with a non-autonomous Kramer equation and introducing an LSTM tailored for Dynamic Osprey. The findings show that when combined with neural networks, IL produces better accurate predictions of abrupt occurrences than IF. This novel method illustrates that the theory of information and neural networks can be used to provide robust predictions for dynamic systems, when encountering rare and extreme events. The model's ability to accurately forecast rare and extreme events is impacted by these limitations. Future work needs to address data limits, improve extrapolation skills and create more adaptable hyperparameter techniques to increase accuracy in forecasting for rare as well as extreme events to improve LSTM stability in non-equilibrium systems in addition to overcome these limitations.

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Мінімізація геометричної дії на основі нового підходу для прогнозування рідкісних та екстремальних подій у нерівноважних системах

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Виявлення та кількісна оцінка неочікуваних подій у нерівноважних системах є критично важливою роботою, яка необхідна системним менеджерам для прийняття обґрунтованих рішень, особливо при прогнозуванні рідкісних та екстремальних подій. У цій статті нейронні мережі об'єднані для підвищення прогнозної здатності теорії інформації. Дві методики теорії інформації, «Довжина інформації» (IL) і «Потік інформації» (IF)», вивчаються на предмет їх чутливості до швидких змін. Щоб змоделювати перше виникнення екстремальних і рідкісних подій, ми використовуємо неавтономну модель Крамера, щоб ввести збурення. ми представили довгострокову пам'ять Dynamic Osprey (DOLSTM) для передбачення рідкісних і екстремальних подій у нерівноважних системах. Наші результати показують, що IL працює краще, ніж IF, у точному прогнозуванні несподіваних подій у поєднанні з нейронною мережею. Це дослідження підкреслює нову інтеграцію між теорією інформації та нейронними мережами, що дає ефективну стратегію для прогнозування рідкісних та екстремальних подій у нерівноважних середовищах. Ефективна методологія ідентифікації та прогнозування поведінки динамічних систем створена шляхом поєднання діагностики довжини інформації з прогнозуванням нейронної мережі, особливо в ситуаціях, пов'язаних із рідкісними та екстремальними подіями. Цей новий метод показує, що теорія інформації та нейронні мережі можуть бути використані для забезпечення надійних прогнозів для динамічних систем, коли вони стикаються з рідкісними та екстремальними подіями.

Ключові слова: Рідкісні та екстремальні події, Довжина інформації, Потік інформації.