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Substantiation of the Operating Parameters of Walnut Drying Equipment

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Abstract. The most important operation in walnut processing technology is drying to achieve the required moisture content. It is the most energy-consuming, and therefore, of course, there is an attempt to increase the energy efficiency of this process. Recent research and publications have shown many ways and means to dry nuts. According to the authors' developments, the most effective are convective dryers, which can intensify the process by vibrating the material layer, which increases the heat transfer ratio. To determine the rational parameters of the drying process, a mathematical model was developed in which the nut is represented by a complex geometric shape – a sphere in a sphere, where the kernel is the inner sphere, and the shell is the outer sphere. The presence of harmful internal heat sources modeled the process of moisture removal. An equation was obtained in the criterion form that allowed temperature determination at an arbitrary point of the inner sphere at an arbitrary time depending on specific initial and boundary conditions, as well as the operating parameters of the drying process. The obtained mathematical model made it possible to reliably simulate the process of drying walnuts in a convective-vibration dryer and control the change in material temperature to ensure the high quality of the final product.

Keywords: drying process, process innovation, energy efficiency, humidity, boundary conditions, kernel temperature.

1. Introduction

It is difficult to overestimate the benefits of nuts consumed in pure form and as components for food and medicines. The global industrial production of walnuts is almost 2.1 million tons [1]. To bring walnuts to consumer quality, they should be dried to a conditioned moisture content of 10 % [2].

Given that the walnuts collected in the garden have an average moisture content of 35 %, and the heat consumption for evaporation of 1 kg of moisture in modern dryers is at least 5 MJ/kg [3], a considerable amount of energy is required to dry all this volume of nuts - about $3\cdot10^9$ MJ, which is equivalent to about $1\cdot10^5$ tons of conventional fuel.

Therefore, the problem of increasing the energy efficiency of the walnut drying process arises. Of course, the efficiency of the drying process depends on several factors, such as the drying method, the means for implementing this process, rational constructive, and technological parameters.

In this regard, rational process parameters and intensification methods are currently relevant for global walnut producers. This article is devoted to this issue.

2. Literature Review

There are many ways and means for drying nuts - from conventional outdoor drying devices to high-performance industrial dryers of various constructions.

In [4], drying in dryers and fresh air was investigated. The physiological index and other quality indicators of walnuts were determined. Drying in dryers is recommended, as they produce higher quality nuts after them. In [5], a single-layer nut drying in the shell at 43–75 °C of air was studied. To determine the value of moisture diffusion efficiency for nut shells and kernels. In [6], it was investigated that a hopper dryer with an upper screw working body was used for drying to reduce the layer of nuts.

Of course, industrial production of nuts requires high-performance dryers that use various energy sources and devices that intensify the drying process. Convective dryers with a fixed (or limitedly moving) material layer can be considered the most suitable for implementing the nut drying process [7]. Other advantages of convective dryers include intensifying the process according to the constructive conditions and organization of the technological process. In particular, one of the promising areas of intensification of the drying process is using vibration to reduce the nut layer's aerodynamic resistance and, accordingly, increase the heat transfer coefficient between the material and the drying agent.

Many materials are devoted to various aspects and options for drying nuts, including the operating parameters of the technological process. In particular, work [8] describes drying walnuts in the shell and nut kernels under different convection conditions. The drying process was carried out by supplying a coolant with a temperature of 24.5 °C and a relative humidity of 41 % at a speed of 1.2 m/s. The nuts were dried to a final moisture content of 6.8 %.

Another approach to drying nuts was implemented in the research work [9], in which the authors combined radiofrequency irradiation with simultaneous exposure to a coolant at a temperature of 50 °C. This made it possible to reduce the moisture content of nuts from 20 % to 8% in 190 minutes compared to 240 minutes of drying with hot air alone. Unfortunately, the authors do not indicate the energy indicators of these processes for comparison and selection of a rational drying method.

It should be noted that this combined method of combining radiofrequency radiation with convective drying with hot air is gradually becoming widespread in the world [10]. The authors note that radio frequency radiation promotes rapid and volumetric heating, deep penetration, and the effect of moisture self-balancing. However, it has not yet become widespread due to high energy costs and little practical experience.

For the large-scale implementation of the drying process, convective drying with the possibility of using various methods of intensification, such as vibration, remains relevant.

The analysis of recent research and publications shows that the performers choose an original variant of the walnut drying process in each case depending on the specific production conditions. This also outlines the directions for further research to intensify the drying process, improve the quality of finished products, and reduce energy costs to implement the process. All this determines the relevance of the issue of studying the drying process of walnuts.

3. Research Methodology

The work aims to increase the walnut drying process efficiency at the theoretical and experimental research stage by determining the laws of heat and mass transfer in the material layer and its elements.

During the drying process of walnuts, it is important to choose the proper operating parameters, including the drying exposure, temperature, and speed of the drying agent, and, in the case of a convective-vibration dryer, the amount of vibration impact on the drying chamber and, accordingly, on the material layer in it. These factors affect the drying process intensity, i.e., its energy efficiency and the quality of the finished product. Each of these factors has specific characteristics that affect the drying process, but for walnuts, the most important is the temperature of the drying agent. After all, walnuts are rich in oils, which become rancid at high temperatures and negatively affect the taste. Therefore, a kernel temperature of about 20 °C is rational for nuts and should not exceed 28 °C [11]. Of course, these are recommendations for the temperature of the nut kernel; the temperature of the shell and the coolant itself can be higher.

While drying a wet material, there is an equilibrium between the amount of moisture coming from the inner layers of the material and the moisture evaporating from the material's surface. This regularity is described by the following equation [12].

$$a_m \rho_0 \left(\nabla U + \delta \nabla \vartheta\right) = \frac{\alpha}{r}\left(t - \vartheta\right), \qquad (1)$$

where a_m – moisture diffusion coefficient, $\mathrm{m^2/s}$; ρ_0 – density of the absolutely dry part of the material, $\mathrm{kg/m^3}$; δ – thermogradient coefficient; ∇U – moisture content gradient; $\nabla \vartheta$ – material temperature gradient; α – heat transfer coefficient, $\mathrm{W/(m^2 \cdot K)}$; r – latent heat of vaporization of moisture, kJ/kg; t – temperature of the drying agent, °C; ϑ – material surface temperature, °C.

The analysis of equation (1) shows two main ways to intensify the drying process - increasing the heat transfer coefficient and the temperature difference between the heat carrier and the material. The heat transfer coefficient between the drying agent and the material depends on many factors: the type of material, the design features of the drying equipment, the aerodynamics of the drying agent flow, and its physical characteristics.

At the same time, the concept of "material type" is quite multifaceted and, in turn, includes many factors, such as the geometric characteristics of the material, the form of moisture bonding with the dry base of the material, and its physical, mechanical, and thermal characteristics. An effective way to increase the heat transfer coefficient is to use vibration to change the rheological properties of the material, in our case, walnuts. Without delving into the theory of vibration displacements, dry friction between particles is transformed into viscous friction.

Simultaneously, the mass of bulk material (including walnuts) acquires the properties of a quasi-fluid, which significantly improves the conditions for vertical and horizontal movement of particles. One of the parameters characterizing the intensity of vibration impact on the material is the ratio of vibration acceleration to gravity acceleration – the following ratio:

$$\lambda = \mathrm{A}\frac{\omega^2}{g},$$

where vibration liquefaction can be observed at λ = 0.80–0.99, and vibratory boiling – at λ > 1.0.

Thus, it can be concluded that vibration is an auspicious direction for intensifying the drying process. It should be noted that vibration is quite challenging to understand, describe, and control as a physical and mechanical process. A detailed study of the vibration effect on walnuts' drying process is not within this article's scope. In it, we will consider another possible way to intensify the drying process – increasing the temperature difference between the drying agent and the material.

The easiest way to achieve this goal is to increase the temperature of the drying agent. Increasing the temperature of the drying agent is usually much easier to organize from a technical point of view than other intensifying factors. However, there are certain limitations, especially when drying thermolabile materials, which include walnuts. We have already discovered that when the walnut kernel is heated above 28 °C, processes occur that negatively affect the quality of the finished product. Therefore, the temperature of the drying agent can be increased only up to a specific limit and with a particular exposure. The theoretical determination of these parameters is the main objective of this work.

The temperature of the drying agent should characterize the drying process as always being higher than the temperature of the material. The difference between these temperatures is maximum at the beginning of the process and decreases with time.

In previous experimentations, as well as the studies of other authors [13, 14], allowed us to establish that the difference between the temperature of the drying agent and the temperature of the material at time τ can be expressed as follows:

$$\Delta t\tau = (t_0 - \vartheta_0)\, exp(-k_2\tau), \tag{2}$$

where Δt_τ – the temperature difference between the drying agent and the material at time τ; t_0 – the temperature of the drying agent at time $\tau = 0$; ϑ_0 – the temperature of the material at time $\tau = 0$; τ – the exposure of the drying process; k_2 – an empirical coefficient characterizing the material heating rate.

The coefficient k_2 characterizes the heating rate of the material. The higher its value, the faster the material heats up, and vice versa. This coefficient depends on the material's properties (the form of moisture bonding with the material, its physical, mechanical, and thermal characteristics) and process parameters. The coefficient k_2 is determined as a result of experimental studies.

The parameters of the drying process depend on many factors, including the geometric characteristics of the material, i.e., its size and shape. In the heat transfer theory, three classical shapes are considered: a plate, a cylinder, and a sphere. In the research, a body of any shape is considered one of these geometric shapes. Walnut can be considered as a sphere, but not an ordinary sphere, but folded as a sphere within a sphere, where the outer sphere is a shell with radius R and the inner sphere is a kernel with radius r_1. In the following, the ratio of these radii is used. According to morphological studies, most walnut varieties are as follows:

$$r^* = \frac{r_1}{R} = 0.9. \tag{3}$$

During the drying process, the thermal energy supplied to the material by the drying agent is used to warm the dry part of the material (the so-called material skeleton), moisture, and evaporation.

In many studies, for example [15], this heat is classified as a negative internal heat source, and it is noted that the intensity of this source decreases over time according to the exponential law.

Thus, we can say that a negative heat source with an intensity of

$$q = q_0 \cdot exp(-k_1\tau), \tag{4}$$

where q_0 – the intensity of the negative internal heat source at time $\tau = 0$; τ – the drying exposure; k_1 – an empirical coefficient characterizing the rate of change in the intensity of the negative internal heat source.

The kernel temperature must not exceed the maximum permissible temperature to obtain a high-quality product after dried walnut. For most walnut varieties, this value is 28–30 °C.

Therefore, from this point of view, we are interested in the distribution of the nut temperature in time and space during the drying process.

The initial expressions for solving this problem, i.e., finding the temperature distribution in the nut along the radius and over time, are the following conditions.

The equation of thermal conductivity is as follows:

$$\frac{\partial\vartheta(r,\tau)}{\partial\tau} = a\left[\frac{\partial^2\vartheta}{\vartheta r^2} + \frac{2}{r}\cdot\frac{\partial\vartheta}{\vartheta r}\right] - \frac{q_0}{c\gamma} exp(-k_1\tau), \tag{5}$$

with the initial condition

$$\vartheta(r,0) = \vartheta_0 \tag{6}$$

boundary condition

$$\left[\frac{\partial\vartheta(r,\tau)}{\partial r}\right]_{r=R} = H(t_0 - \vartheta_0)\, exp(-k_2\tau), \tag{7}$$

and symmetry condition:

$$\left[\frac{\partial\vartheta(r,\tau)}{\vartheta r}\right]_{r=0} = 0, \tag{8}$$

where $\vartheta(r,\tau)$ – the temperature of the nut kernel at any point on the radius r at time τ; a, c, γ – the thermal conductivity, heat capacity, and density of the material, respectively.

Also, the parameter $H = \frac{\alpha}{\lambda}$ is used, where α – the heat transfer coefficient between the material and the drying agent; λ – the coefficient of thermal conductivity for the material.

Equation (7) defines the peculiarities of the process at the boundaries of the body, i.e., the boundary conditions. According to the heat transfer theory, there are four types (more often called genera) of boundary conditions. In our case, these are boundary conditions of the third kind, which consist of setting the ambient temperature (i.e., the temperature of the drying agent) and the intensity of heat transfer on the body surface.

Equation (5) can be solved using the operational method under the following assumptions. First, the thermal characteristics of the material are constant throughout the drying process. They depend on the material's moisture content and temperature, but these parameters, in our case, vary within a relatively narrow range, so with a particular assumption, we will consider the thermal characteristics of the material to be constant.

Second, the heat transfer coefficient between the material and the drying agent remains constant throughout the drying process. The heat transfer coefficient also depends on the temperature of the drying agent and the material, but the range of temperature changes is quite small, and therefore, the assumption of a constant heat transfer coefficient can be made without significant error in the results.

Operation calculus is used to solve differential equations. The idea of the operational calculus is that a correspondence is established between the function *f*(*t*) of the real variable *t* (the original function) and the function *F*(*p*) of the complex variable $p = s + i\sigma$, which allows the operations of differentiation and integration over the original *f*(*t*) to be replaced by the image *F*(*p*).

In the considered case study, the original will be the function $\vartheta(r,\tau)$ characterizing the temperature distribution in the kernel of a nut by place and time, and the image function will be denoted by $\theta(r, S)$. There will be a relationship between the original and image functions:

$$\theta(r,s) = \int_0^\infty e^{-s\tau}\vartheta(r,\tau)d\tau. \tag{9}$$

Equation (5) for the image $\theta(r, S)$ according to the Laplace transform is as follows:

$$s\theta(r,s) - \vartheta_0 = a\left[\frac{d^2\theta(r,s)}{dr^2} + \frac{2}{r}\cdot\frac{d\theta(r,s)}{dr}\right] - \frac{q_0}{c\gamma}\cdot\frac{1}{s+k_1}. \tag{10}$$

After identical transformations, it can be obtained:

$$\begin{gathered}\frac{d^2\theta(r,s)}{dr^2} + \frac{2}{r}\cdot\frac{d\theta(r,s)}{dr} - \frac{q_0}{ac\gamma}\times \\ \times\frac{1}{s+k_1} - \frac{s}{a}\theta(r,s) + \frac{\vartheta_0}{a} = 0.\end{gathered} \tag{11}$$

The initial, boundary, and symmetry conditions for the image will be as follows:

$$\vartheta(r,0) = \vartheta_0; \tag{12}$$

$$\frac{d\theta(0,s)}{dr} = 0; \tag{13}$$

$$\frac{d\theta(R,s)}{dr} = H\left(\frac{t_0-\vartheta_0}{k_2+s}\right). \tag{14}$$

The homogeneous equation for expression (11) is

$$\frac{d^2\theta(r,s)}{dr^2} + \frac{2}{r}\cdot\frac{d\theta(r,s)}{dr} - \frac{s}{a}\theta(r,s) = 0. \tag{15}$$

It is a special case of the Bessel equation, the solution of which is as follows:

$$\theta(r,s) = \frac{1}{r}\sqrt{\frac{2}{\pi}}\left[c_1 ch\left(\sqrt{\frac{s}{a}}\cdot r\right) + c_2 i\cdot sh\left(\sqrt{\frac{s}{a}}\cdot r\right)\right] \tag{16}$$

Then, the solution of the inhomogeneous equation (11) is determined by the following expression:

$$\begin{gathered}\theta(r,s) = \frac{1}{r}\sqrt{\frac{2}{\pi}}\left[c_1 ch\left(\sqrt{\frac{s}{a}}\cdot r\right) + c_2 i\cdot sh\left(\sqrt{\frac{s}{a}}\cdot r\right)\right] + \\ +\frac{1}{s}\left(-\frac{q_0}{c\gamma}\cdot\frac{1}{k_1+s} + \vartheta_0\right).\end{gathered} \tag{17}$$

The constants c_1 and c_2 are determined from the boundary conditions for the image (12)–(14).

After the identical transformations, the solution of equation (11) for the image $\theta(h, s)$ takes the form:

$$\begin{gathered}\theta(r,s) = \frac{HR^2(t_0-\vartheta_0)\cdot\frac{1}{r}sh\left(\sqrt{\frac{s}{a}}\cdot r\right)}{\left[\sqrt{\frac{s}{a}}Rch\left(\sqrt{\frac{s}{a}}\cdot R\right) - sh\left(\sqrt{\frac{s}{a}}\cdot R\right)\right](k_2+s)} + \\ +\frac{1}{s}\left(-\frac{q_0}{c\gamma}\cdot\frac{1}{k_1+s} + \vartheta_0\right).\end{gathered} \tag{18}$$

The resulting equation (18) can be represented as follows:

$$\theta(r,s) - \frac{\vartheta_0}{s} = \frac{\varphi_1}{\psi_1} - \frac{\varphi_2}{\psi_2}, \tag{19}$$

where the following functions are introduced:

$$\begin{gathered}\varphi_1 = HR^2(t_0-\vartheta_0)\cdot\frac{1}{r}sh\left(\sqrt{\frac{s}{a}}\cdot r\right); \\ \psi_1 = \left[\sqrt{\frac{s}{a}}Rch\left(\sqrt{\frac{s}{a}}\cdot R\right) - sh\left(\sqrt{\frac{s}{a}}\cdot R\right)\right](k_2+s); \\ \psi_2 = \frac{q_0}{c\gamma}; \\ \psi_2 = s(s+k_1).\end{gathered} \tag{20}$$

The second decomposition theorem should be applied to find the original function of the right-hand side of the equation (19). According to it, the original is determined by using the roots of the denominator.

The polynomial ψ_1 has the roots $s_1 = k_2$ and the set of roots of the equation

$$\sqrt{\frac{s}{a}}Rch\left(\sqrt{\frac{s}{a}}R\right) - sh\left(\sqrt{\frac{s}{a}}R\right) = 0. \tag{21}$$

which has many roots:

$$s_n = -\frac{a}{R^2}\mu_n^2, \tag{22}$$

where μ_n – roots of the characteristic equation:

$$tg\mu_n = -\mu_n. \tag{23}$$

According to the second decomposition theorem, the original function φ_1/ψ_1 is given by the formula:

$$L^{-1}\left[\frac{\varphi_1}{\psi_1}\right] = \lim_{S\to k_2}\frac{\varphi_1(s)}{\psi_1(s)}e^{s\tau} + \sum_{n=1}^{\infty}\frac{\varphi_1(s_n)}{\psi_1(s_n)}e^{s_n\tau}. \tag{24}$$

After substituting and transforming the resulting expression, the original function φ_1/ψ_1 can be found:

$$\begin{gathered}L^{-1}\left[\frac{\varphi_1}{\psi_1}\right] = \frac{HR^2(t_0-\vartheta_0)\sin\left(\sqrt{\frac{k_2}{a}}r\right)}{r\left[\sqrt{\frac{k_2}{a}}R\cos\left(\sqrt{\frac{k_2}{a}}R\right) - \sin\left(\sqrt{\frac{k_2}{a}}R\right)\right]}e^{-k_2\tau} + \\ +2\frac{aH}{r}(t_0-\vartheta_0)\sum_{n=1}^{\infty}\frac{\sin\frac{\mu_n}{R}r}{\left(k_2-\frac{a}{R^2}\mu_n{}^2\right)\sin\mu_n}e^{-\frac{a}{R^2}\mu_n{}^2\tau}.\end{gathered} \tag{25}$$

Similarly, the original function φ_2/ψ_2 can also be found. For this case, the roots of the polynomial ψ_2 have the following values: $s = 0$; $s_2 = -k_1$. Then, the original φ_2/ψ_2 is defined as follows:

$$L^{-1}\left[\frac{\varphi_2}{\psi_2}\right] = \frac{q_0}{R_1 c\gamma} - \frac{q_0}{k_1 c\gamma} e^{-k_2\tau} = \\ = \frac{q_0}{k_1 c\gamma} - \left(1 - e^{-k_1\tau}\right) \tag{26}$$

After considering the above partial solutions and mathematical transformations, the solution of equation (5) in a dimensionless form can be represented as follows:

$$\frac{\vartheta(r,\tau)-\vartheta_0}{t_0-\vartheta_0} = \frac{HR^2 \sin\left(\sqrt{\frac{k_2}{a}}r\right) e^{-k_2\tau}}{r\left[\sqrt{\frac{k_2}{a}}R \cos\left(\sqrt{\frac{k_2}{a}}R\right) - \sin\left(\sqrt{\frac{k_2}{a}}R\right)\right]} + \\ +2\frac{aH}{r}\sum_{n=1}^{\infty}\left[\frac{\sin\left(\frac{\mu_n}{R}r\right)}{\left(k_2-\frac{a}{R^2}\mu_n{}^2\right)\sin\mu_n} e^{-\frac{a}{R^2}\mu_n{}^2\tau}\right] - \\ - \frac{q_0}{k_1 c\gamma(t_0-\vartheta_0)}\left(1 - e^{-k_1\tau}\right). \tag{27}$$

To simplify it, it is advisable to present it in a criterion form, which is a standard method in heat transfer theory. For a more complete perception of this method, it is necessary first to explain the physical essence of some criteria used in the research.

The Fourier criterion $Fo = a\tau/R^2$ has the meaning of generalized time. It is also called the number of thermal homochrony (homogeneity in time). The F_a criterion characterizes the relationship between the rate of change of the temperature field, physical properties, and geometric characteristics of the body.

Pomerantsev's criterion $Po = \frac{q_0 R^2}{\lambda(t_0-\vartheta_0)}$ makes sense of the generalized intensity of internal heat sources under a non-stationary temperature field.

The Bio criterion $Bi = HR$ – is the ratio between the amount of heat that can be supplied to the body and the heat that flows from the surface to the interior of the body.

The criterion $Pd = \left|\frac{dT_c}{dFo}\right|_{max}$ [16] in the considered case study is $Pd = \frac{kR^2}{a}$. It characterizes the intensity of the temperature change of a material.

From equation (27), we can distinguish another set of parameters, which we denote as

$$S = \frac{aH}{k_2 r}. \tag{28}$$

Parameter s characterizes the ratio of the development rates of unsteady-state thermal processes in the material and the drying agent (heat carrier). The rate of development of an unsteady-state thermal process is the ability of material particles to equalize the temperature in parts and throughout the entire volume when heated.

In addition, we introduce the following notations:

$$Pd_1 = \frac{k_1 R^2}{a};\ Pd_2 = \frac{k_2 R^2}{a};\ Pd_2^* = \frac{k_2 r^2}{a}; \\ \vartheta = \frac{\vartheta(r,\tau)-\vartheta_0}{t_0-\vartheta_0};\ r^* = \frac{r}{R}. \tag{29}$$

4. Results

After considering the notations (28) and (29) in the criterion form, equation (27) will be as follows:

$$\vartheta = \frac{Bi\cdot \sin\sqrt{Pd_2^*}\cdot \exp(-Pd_2 Fo)}{r^*\left(\sqrt{Pd_2}\cos\sqrt{Pd_2} - \sin\sqrt{Pd_2}\right)} + \\ +2S\sum_{n=1}^{\infty}\frac{\sin(\mu_n r^*)\exp\left(-\mu_n{}^2 Fo\right)}{\left(1-\frac{\mu_n{}^2}{Pd_2}\right)\sin\mu_n} - \\ - \frac{Po}{Pd_1}\left[1 - \exp(-Pd_1 Fo)\right]. \tag{30}$$

The roots of the characteristic equation (23) for engineering calculations can be determined graphically or by computer. In particular, the values of the first five non-zero roots of equation (23) are given: $\mu_1 = 2.029$, $\mu_2 = 4.913$, $\mu_3 = 7.979$, $\mu_4 = 11.085$, and $\mu_5 = 14.208$.

Equation (30) also includes the sum from one to infinity. Usually, the first two or three roots are sufficient for engineering calculations since the subsequent roots do not affect the calculation results.

Equation (30) allows you to determine the temperature at any point of the body at any time of the drying τ exposure at the given initial parameters of the drying process: temperature and speed of the coolant, humidity, and temperature of the material, parameters of vibration, mass, and thickness of the material layer in the drying chamber.

The temperature distribution inside the nut is of some interest due to the restrictions on reducing the oil content. With a constant supply of a drying agent with constant parameters, the temperature of the material in the drying chamber changes from ϑ_0 at the beginning of the drying process to its maximum value $\vartheta(\tau)_{max}$ at the end of the process. Consider the temperature distribution inside the nut at its constant temperature, i.e., when the left-hand side of equation (5) is zero. This problem is reduced to solving the following equation:

$$\frac{\partial^2\vartheta(r,\tau)}{\upsilon r^2} + \frac{2}{r}\frac{\partial\vartheta(r,\tau)}{\partial r} - \frac{q_0}{\lambda}e^{-k_1\tau} = 0 \tag{31}$$

The initial conditions will remain the same, i.e., equations (6)–(8) also characterize the initial conditions in this case.

First, the solution of the following homogeneous equation should be found:

$$\frac{\partial^2\vartheta(r,\tau)}{\partial r^2} + \frac{2}{r}\cdot\frac{\partial\vartheta(r,\tau)}{\partial r} = 0. \tag{32}$$

The solution to this equation is

$$\vartheta(r,\tau) = \frac{c_1}{r} + c_2. \tag{33}$$

where c_1 and c_2 – integration constants.

The integration constants c_1 and c_2 can be found using the method of variation of arbitrary constants. According to this method, in our case, we assume that c_1 and c_2 are functions of the argument, and then they must be the roots of the following system:

$$\begin{cases} \frac{c_1^*}{r} + c_2^* = 0; \\ -\frac{c_1^*}{r^2} = \frac{q_0}{\lambda} e^{-k_1\tau}, \end{cases} \tag{34}$$

where c_1^* and c_2^* – the derivatives of r, respectively, from c_1 and c_2.

In this case, (33) is also the equation (31) solution.

The solution of this system (34) is as follows:

$$\begin{cases} c_1 = -\frac{r^3}{3} \cdot \frac{q_0}{\lambda} \cdot e^{-k_1\tau} + A_1; \\ c_2 = \frac{r^2}{2} \cdot \frac{q_0}{\lambda} \cdot e^{-k_1\tau} + A_2. \end{cases} \tag{35}$$

The constants A_1 and A_2 can be found from the initial conditions (6)–(8).

The general solution of equation (31) will be in the form:

$$\vartheta(r,\tau) = \vartheta_0 + \left(\frac{r^2}{6} + \frac{R^3}{3r}\right)\frac{q_0}{\lambda}\left(e^{-k_1\tau} - 1\right) + \\ + HR\frac{r}{R}(t_0 - \vartheta_0)\left(1 - e^{-k_2\tau}\right). \tag{36}$$

The same equation can be written in a dimensionless form, similar to (27):

$$\vartheta = \left(\frac{r^2}{6R^2} + \frac{R}{3r}\right) Po[exp(-Pd_1Fo) - 1] + \\ + Bi \cdot \frac{r}{R}[1 - exp(-Pd_2Fo)]. \tag{37}$$

For the practical use of equation (37), it is necessary to know the dependence of the change in criteria on temperature and humidity, the dependence of heat transfer coefficients on process parameters, the value of the thermal and physical characteristics of the material, and the nature of the change in the intensity of the negative internal heat source.

The study of negative internal heat sources is of particular interest. This concept is not entirely typical for drying agricultural products, so we consider in more detail the theoretical aspects of changes in the intensity of these sources.

To determine other indicators included in equation (37), it is necessary to conduct experimental studies on special laboratory equipment. The laboratory installation and the equipment used to determine the parameters of the walnut drying process are presented in Figure 1.

Figure 1 – Laboratory installation and devices for determining the parameters of the walnut drying process: a – laboratory installation for drying walnuts

The thermal anemometer AM 831 with a remote sensor was used to measure movement speed and air temperature. The thermometer-moisture meter with a remote sensor was also used.

During the drying process, heat is used to warm up the dryer and material and evaporate moisture. When the process is in a steady state, i.e., has constant parameters, the amount of heat released to the environment with the initial coolant used to heat the dryer remains constant.

At the beginning of the drying process, heat is used to heat the dry matter of the nut, heating, and evaporating moisture. As the drying exposure increases, the heat absorbed by the material decreases. The amount of heat absorbed by the dry weight of the nut remains almost constant, while the amount of heat consumed by the evaporation of moisture from the material changes. It can be reasonably assumed that the power of negative internal heat sources is equivalent to the amount of moisture contained in the material.

Based on these considerations, the coefficient k_1, which can be determined by identifying the change in the intensity of internal heat sources. We assume that at the time τ_1 the intensity of negative internal heat sources is equal to $q_{\tau 1}$, and at the time τ_2 – respectively $q_{\tau 2}$.

Intensity of internal heat sources is proportional to the moisture content of the material, i.e.,

$$\frac{q\tau_1}{q\tau_2} = \frac{q_0\,exp(-k_1\tau_1)}{q_0\,exp(-k_1\tau_2)} = \frac{exp(-k_1\tau_1)}{exp(-k_1\tau_2)} = \frac{U_1}{U_2}. \tag{38}$$

From expression (38), we can obtain the equation for determining the following coefficient:

$$k_1 = \frac{ln\frac{U\tau_1}{U\tau_2}}{\tau_2 - \tau_1} \tag{39}$$

It depends on several factors, including the thermophysical properties of material, the process parameters, among which the most important are the temperature and speed of the drying agent and the parameters of vibration impact on the material layer.

The results of the change in the temperature of the material at an ambient temperature of 15 °C and the temperature of the drying agent t_0 = 34 °C are shown in Figure 2.

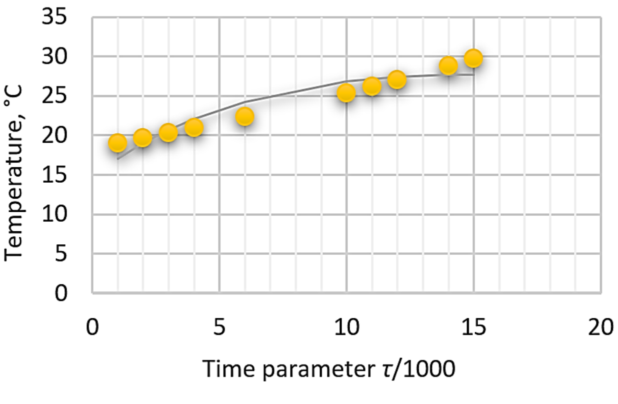
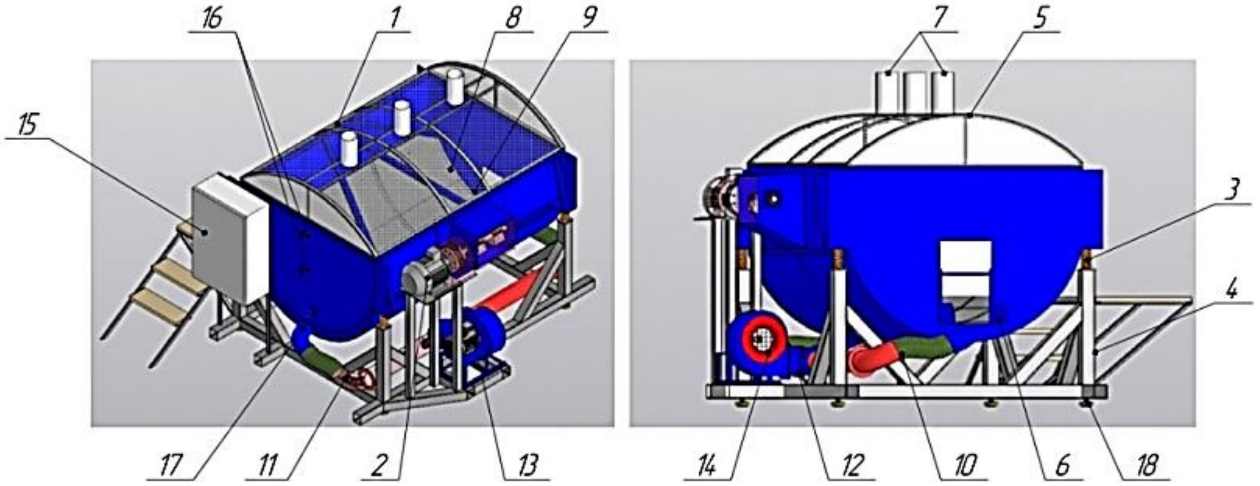


Figure 2 – Temperature of the material during the drying process at the temperature of the coolant t_0 = 34 °C

The dots show the experimental data, and the curve describes the temperature value according to (37).

A few experiments on drying walnuts were conducted on a laboratory and industrial convective-vibration dryer, the schematic diagram and detailed view of which are shown in Figure 3.

The main elements of the laboratory dryer are a chamber with a perforated bottom, a fan for supplying the drying agent, electric heating elements, and a device for creating a vibration effect on the layer of nuts in the chamber. Laboratory equipment (Figure 1) was used to conduct the experiments.



a

b

Figure 3 – 3D model of a laboratory and industrial convective vibration dryer (a – top view; b – side view): 1 – U-shaped drying chamber; 2 – inertial vibrator; 3 - springs; 4 – frame; 5 – lid; 6 – discharge tray; 7 – air ducts; 8 – perforated bottom; 9 – pallet; 10 – heated air supply line; 11 – heat-electric heater; 12 – air distribution device; 13 – electric motor; 14 – blower fan; 15 – automatic control system; 16 – three-level temperature and humidity sensors; 17 – temperature sensor; 18 – anti-vibration support

One of the experiments was conducted at a drying agent temperature of 38 °C; its speed at the entrance to the drying chamber was 9 m/s, and the amplitude of the chamber oscillations was 3 mm. The drying time for 300 kg of nuts with an initial moisture content of 28 % to a final moisture content of 10% was 7.5 hours. For this drying process with the specified operating parameters, the value of the coefficient characterizing the intensity of negative internal heat sources calculated by formula (39) is $k_1 = 5 \cdot 10^{-5}$ s^{-1}.

According to equation (4), the intensity of negative internal heat sources at a given time also depends on q_0 – the intensity of the source at time $\tau = 0$.

The dependence between the total and instantaneous intensity of negative internal heat sources is as follows:

$$q_\Sigma = \int_0^{\tau_K} q_0\, exp(-k_1\tau)\, d\tau = \frac{q_0}{k_1}\left[1 - exp\left(-k_1\tau_f\right)\right], \quad (40)$$

where τ_f – the total drying time, s.

From equation (40), we can find q_0:

$$q_0 = \frac{q_\Sigma k_1}{1 - exp(-k_1\tau_f)}. \quad (41)$$

The value q_0 as a k_1 depends on many factors: the initial amount of moisture in the material, its thermal and physical characteristics, and the operating parameters of the drying process.

The total intensity of negative internal heat sources also depends on the moisture that evaporates from the material during the entire drying period.

5. Discussion

In this article, we consider the process of drying walnuts from the initial moisture content to the final moisture content, which is equal to 10 %. The amount of moisture that evaporates from 1 kg of moisture material (walnuts) from the initial moisture W_n to the final 10 % moisture is equal to the difference in the mass of moisture in the material at the appropriate humidities and can be determined by the equation:

$$\Delta m = 0.0109 \cdot W_n - 0.1067, \quad (42)$$

where Δm – the amount of evaporated moisture from 1 kg of material, kg; W_n – initial moisture content of the material, %.

Notably, in equation (40), the value q_Σ має розмірність has the dimension of [J], which corresponds to the amount of heat energy consumed to evaporate all the moisture from W_n to 10 %.

Experiments on drying nuts carried out on a laboratory convective-vibration dryer (Figure 1) showed that the total energy consumption is 3.91 MJ per 1 kg of evaporated moisture: 3.3 MJ/kg is spent on heating the dry part of the material, heating, and evaporation of water, i.e., this is the energy that called the energy of negative internal heat sources.

The intensity of negative internal heat sources depends on the initial moisture content. The value calculated by equation (41) is 0.045 kW/kg for the experiment whose results are being described.

6. Conclusions

In the postharvest processing of walnuts, the most energy-intensive process is drying. World producers spend about $3 \cdot 10^9$ MJ of energy to obtain walnuts of conditioned moisture content. Therefore, increasing the efficiency of the drying process is an urgent issue for all global producers.

The analysis of previous theoretical and experimental studies suggests that the most effective way to intensify the drying process is to increase the temperature difference between the drying agent and the material. However, in the case of drying walnuts, this value should be limited because an uncontrolled increase in the temperature difference can lead to a deterioration in the quality of the final product.

To adequately describe the change in the amount of moisture in the material, the concept of negative internal heat sources was used, which is equivalent to the current moisture content of the material and depends on its initial moisture content and process parameters.

The solution of the equation of the non-stationary heat conduction process, taking into account the exponential change in the intensity of negative internal heat sources over time, makes it possible to obtain an equation that can determine the temperature of the material at any point in the volume depending on the drying exposure.

The obtained equation allows for reliable modeling of the walnut drying process in a convective-vibration dryer and controlling the change in material temperature to ensure the high quality of the final product.

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