Dynamic analysis of the locking automatic balancing device of the multistage centrifugal pump

IVAN PAVLENKO

The reliability of automatic balancing device of centrifugal pumps can be decreased in case of intensive wear of cylindrical throttle before and after the unloading device of the balancing disk. Therefore, the locking automatic balancing device is offered, operating as the axial hydrostatic bearing with hyper bearing capacity and, simultaneously, as the non-contact consolidation with self-adjustable leaking. The dynamic calculation is based on the equations of pump rotor and regulator rod axial movement and also on equations of fluid flow balance through throttles taking into consideration compression and displacement flow. The methodology of dynamic calculation gives the opportunity to select the main parameters of hydro-mechanical system engineering based on providing the transient processes quality and stability conditions.

Key words: displacement, bearing, device, dynamic, flow, force, leaking, movement, regulator, rod, throttle, stability, unloading, amplitude-frequency characteristic, hydro-mechanical system, transient process

1. Introduction

Automatic balancing device (ABD) and other unloading devices can be used for the equilibration of the axial forces operating on a rotor of multistage centrifugal pumps. In case of usage of any unloading device, the presence of persistent bearings and system of consolidation leads to the complication of the system of the axial equilibration of rotor, reduction of its wear-resistance and economy decrease.

The reliability of ABD can be decreased in case of intensive wear of cylindrical throttle before and after the unloading device of the balancing disk. Therefore, the new design of ABD was offered – the locking automatic balancing device of the centrifugal pump, operating as the axial hydrostatic bearing with hyper bearing capacity and, simultaneously as the non-contact consolidation with self-adjustable leaking.
leaking. LABD provides the availability of the regulator of the pressure difference (RPD) supporting the excess of locking pressure over the discharge pressure.

In many branches of industry, where high-aggressive environments with hard dredges are transferred, the requirements for the construction of centrifugal pumps are highly specified: longevity, durability, simplicity and cheapness of exploitation [4]. Therefore in multistage centrifugal pumps with ABD we can observe axial rotor vibration, which is a result of resonance in the “rotor-ABD” system, or self-excited vibrations, which are caused by the loss of dynamic stability in the system. The definition of amplitude-frequency characteristics (AFC) and phase-frequency variations (PFV) of the balancing system and the dynamic stability control is of great importance to provide reliability of high-speed and high-pressure centrifugal machines [3].

1.1. The purpose and problems

The purpose of this paper is the research of the main differences of this device in comparison with the traditional model. In this connection the main problems are: the construction of the theory of dynamic analysis of LABD and its advantage in comparison with the traditional model of the unloading device, the analysis of dynamic characteristics and stability.

2. Experimental procedure

2.1. Traditional design of ABD

The basic scheme of the device of ABD is represented in Fig. 1.

![Fig. 1. The traditional design of ABD: 1 – unloading disk, 2, 7 – cylindrical throttles, 3, 6 – ABD cameras, 4 – axial throttle, 5 – release device.](image-url)
The advantages of ABD with the traditional design are the automatism of operation in wide range of change of axial force and comparative simplicity of a design. At the same time this construction has disadvantages connected with the growth of leaking through the knot of auto-unloading owing to inevitable deterioration of cylindrical throttle, and the impossibility to regulate the quantity of leaking.

2.2. The regulator of pressure difference

The pressure of the operating environment can vary. That is why there is a necessity to support the excess of locking pressure over the discharge pressure to prevent the leaking to atmosphere. Hence, LABD should provide the presence of RPD.

The regulator of direct action with the spring load and with one or two saddled regulating devices can be called one of the most reliable ones and at the same time simple in usage. The basic scheme of RPD is presented in Fig. 2.

2.3. The locking automatic balancing device

LABD represents the difficult hydro-mechanical system, which is a system of the automatic regulation consisting of two subsystems: ABD and RPD. The scheme of this device is presented in Fig. 3.

The advantage of given device is that LABD carries out functions of the axial hydrostatic bearing and the combined non-contact consolidation with self-adjustable leaking in a wide range of axial force change simultaneously.

Fig. 2. The basic scheme of RPD: 1 – RPD case, 2 – membrane, 3 – master control spring, 4 – rod, 5 – saddle, 6 – inlet camera, 7 – outlet camera, 8 – master control camera.
with the advantages of LABD, it has disadvantages, connected with the complexity of solving a problem of the hydro-mechanical system parameters establishment, uniting two systems of automatic regulation into one complete structure.

3. Results

When calculating dynamic characteristics we can consider the rotor in cooperation with ABD as lumped parameters system with axial oscillation relatively to static equilibrium position (Fig. 3). Stationary value of axial clearance, pressures and leaking can be defined through static calculations [6].

The dynamic calculation of LABD consists of the definition of “rotor-ABD” system dynamic characteristics, which is based on the equation of axial pump rotor and RPD rod movement and also equations of fluid flow balance through throttles taking into consideration compression and displacement flow.

The equation of rotor axial movement is based on d’Alambert’s principle:

\[(m + m_{Fz})\ddot{z}(t) + c_{Fz}\dot{z}(t) + k_{Fz}z(t) = F(t) - T(t) + F_{spr},\]  \( \text{(1)} \)
where \( m \) is rotor mass, \( m_{Fz}, c_{Fz}, k_{Fz} \) are added mass, damping and stiffness coefficients, \( T \) and \( F \) are axial forces, \( F_{spr} \) is release force, \( z \) is axial displacement of the pump rotor.

Further we can explore transient and amplitude-frequency characteristics depending on the character of axial force \( T(t) \) change.

The equation of the axial movement of the rod of RPD:

\[
m_0 \ddot{x}(t) + c_0 \dot{x}(t) + k_0 x(t) = F_{\text{reg}} - s[p_e(t) - p_1(t)],
\]

(2)

where \( m_0 \) is rod mass, \( c_0 \) and \( k_0 \) are damping and stiffness coefficients, \( F_{\text{reg}} \) is master control spring force, \( s \) is membrane area, \( p_1 \) is discharge pressure, \( p_e \) is locking pressure, \( x \) is axial displacement of RPD rod.

Fluid flow balance equations can be generated according to hydraulic path scheme of LABD (Fig. 4) taking into consideration the compression and the displacement flow:

\[
\begin{align*}
Q_{\text{in}}(t) &= Q_{\text{cam}}(t) + s_c \dot{x}(t) + \frac{V_{\text{cam}}}{E} \dot{p}_{\text{cam}}(t) = Q_e(t) + s \dot{x}(t) + \frac{V_M}{E} \dot{p}_e(t), \\
Q_e(t) + s \dot{x}(t) + \frac{V_M}{E} \dot{p}_e(t) &= Q_1(t) + s_2 \dot{z}(t) + \frac{V_2}{E} \dot{p}_2(t) + Q_T(t), \\
Q_T &= Q_3(t) - (s_2 + s_T) \dot{z}(t) + \frac{V_3}{E} \dot{p}_3(t),
\end{align*}
\]

(3)

where \( Q_{\text{in}}, Q_{\text{cam}}, Q_e, Q_1, Q_T, Q_3 \) are fluid consumptions through inlet throttle, RPD camera, locking throttle and three ABD throttles; \( s_c \) is axial face and saddle areas; \( s_2 \) is axial disk area; \( V_{\text{cam}}, V_M, V_2, V_3 \) are inlet and under-membrane camera volumes and volumes of cavities, which are situated before and after unloading disk; \( E \) is modulus of elasticity of locking environment; \( p_{\text{cam}}, p_2, p_3 \) are pressures in RPD and ABD cameras.

Fig. 4. The scheme of the hydraulic path of LABD.
It is possible to present the system of Eqs. (1) (3) in dimensionless form with dimensionless parameters

\[
\begin{align*}
  u &= \frac{z}{z_b}, \quad \xi = \frac{x}{x_b}, \quad \psi_1 = \frac{p_1}{x_b}, \quad \psi_{\text{cam}} = \frac{p_{\text{cam}}}{x_b}, \quad \psi_e = \frac{p_e}{x_b}, \\
  \psi_{in} &= \frac{p_{in}}{x_b}, \quad \psi_2 = \frac{p_2}{x_b}, \quad \psi_3 = \frac{s_M}{s_b}, \quad \sigma_M = \frac{s_M}{s_b}, \quad \chi = \frac{F_{\text{apr}}}{b_s b_b},
\end{align*}
\]

and also with time constants, damping and gain coefficients

\[
\begin{align*}
  T_c &= \frac{s_c x_b}{p_b g_1}, \quad T_{M1} = \frac{s_M x_b}{p_b g_1}, \quad T_{21} = \frac{s_2 z_b}{p_b g_1}, \quad T_{31} = \frac{(s_2 + s_T) z_b}{p_b g_1}, \\
  T &= \sqrt{\frac{m + n F_z}{k_{Fz}}}, \quad T_{\text{cam}} = \frac{V_{\text{cam}}}{E g_1}, \quad T_{M2} = \frac{V_2}{E g_1}, \quad T_{22} = \frac{V_2}{E g_1}, \\
  T_{32} &= \frac{V_3}{E g_1}, \quad T_0 = \frac{m_0}{k_0}, \quad K = \frac{k_{Fz} z_b}{p_b s_b}, \quad K_0 = \frac{k_0 x_b}{p_b s_b}, \\
  \zeta &= \frac{1}{2 K T} \frac{c_{Fz} z_b}{p_b s_b}, \quad \zeta_0 = \frac{1}{2 K_0 T_0} \frac{c_{0} x_b}{p_b s_b},
\end{align*}
\]

where \( z_b, x_b, p_b, s_b \) are the basic values of clearances, pressure and area.

Basing on the defined dependences we can generate the system of decision equations of dynamic calculation of LABD in dimensionless form:

\[
\begin{align*}
  \alpha_c \psi_e(t) \psi_2(t) + T_{M1} \dot{\xi}(t) + T_{M2} \dot{\psi}_e(t) &= \psi_2(t) - \psi_1(t) \\
  &+ \alpha_{Tb} u^3(t) [\psi_2(t) - \psi_3(t)] + T_{21} \ddot{u}(t) + T_{22} \dot{\psi}(t), \\
  \alpha_{Tb} u^3(t) [\psi_2(t) - \psi_3(t)] + T_{21} \ddot{u}(t) + T_{22} \dot{\psi}(t) &= \alpha_3 [\psi_3(t) - \psi_4] \\
  &- T_{31} \ddot{u}(t) + T_{32} \psi_3(t), \\
  K [T^2 \ddot{u}(t) + 2 \zeta T \dot{u}(t) + u(t)] &= \sigma_1 [\psi_2(t) - \psi_3(t)] - b \psi_1(t) + \chi, \\
  \alpha_{in} [\psi_{\text{in}} - \psi_{\text{cam}}(t)] &= \alpha_b \xi^3 [\psi_{\text{cam}}(t) - \psi_e(t)] + T_{\text{cam}} \dot{\xi}(t) + T_{\text{cam}} \dot{\psi}_{\text{cam}}(t), \\
  \alpha_b \xi^3 [\psi_{\text{cam}}(t) - \psi_e(t)] + T_{\text{cam}} \dot{\xi}(t) + T_{\text{cam}} \dot{\psi}_{\text{cam}}(t) &= \alpha_e [\psi_e(t) - \psi_2(t)] \\
  &+ T_{M1} \dot{\xi}(t) + T_{M2} \dot{\psi}_e(t), \\
  K_0 [T_0^2 \ddot{\xi}(t) + 2 \zeta_0 T_0 \dot{\xi}(t) + \xi(t)] &= \sigma_M [\delta \psi_0 - \psi_e(t) + \psi_1(t)].
\end{align*}
\]

The system of non-linear equations cannot be solved analytically. That is why we will pass to the system of linearized equalizations in variations:
\[ u(t) = u_0 + \delta u(t), \quad \xi(t) = \xi_0 + \delta \xi(t), \quad \psi_1(t) = \psi_0 + \delta \psi_1(t), \]
\[ \psi_{\text{cam}}(t) = \psi_{\text{cam}0} + \delta \psi_{\text{cam}}(t), \quad \psi_e = \psi_{e0} + \delta \psi_e(t), \quad \psi_2(t) = \psi_{20} + \delta \psi_2(t), \quad \psi_3(t) = \psi_{30} + \delta \psi_3(t), \]

where the subscript “0” means the stationary parameters; \( \delta \) is deviation sign.

The further analysis of dynamics of LABD is based on linearized expressions, which leads the non-linear mathematical model to the system of equations with variations:

\[
\begin{align*}
-\alpha_{\text{in}} \delta \psi_{\text{cam}} &= \delta \psi_2 - \delta \psi_1 + 3\alpha_{\text{TB}} u_0^2 (\psi_{20} - \psi_{30}) \\
&\quad + \alpha_{\text{TB}} u_0^2 (\psi_2 - \psi_3) + T_{11} \delta \dot{u} + T_{22} \delta \psi_2, \\
-\alpha_{\text{in}} \delta \psi_{\text{cam}} - \delta \psi_2 + \delta \psi_1 &= \alpha_3 \delta \psi_3 - T_{31} \delta \dot{u} + T_{32} \delta \dot{\psi}_3, \\
K(T^2 \delta \dot{u} + 2 \xi T \delta \dot{u} + \delta u) &= \sigma (\delta \psi_2 - \delta \psi_3) - b \delta \psi_1, \\
-\alpha_{\text{in}} \delta \psi_{\text{cam}} &= 3\alpha_{\text{b}} \xi_0^2 (\psi_{\text{cam}0} - \psi_{e0}) \delta \xi + \alpha_{\text{b}} \xi_0^3 (\delta \psi_{\text{cam}} - \delta \psi_e) \\
&\quad + T_{c} \delta \dot{\xi} + T_{\text{cam}} \delta \dot{\psi}_{\text{cam}}, \\
-\alpha_{\text{in}} \delta \psi_{\text{cam}} &= \alpha_e (\delta \psi_e - \delta \psi_2) + T_{M1} \delta \dot{\xi} + T_{M2} \delta \dot{\psi}_e, \\
K_0(T_0^2 \delta \dot{\xi} + 2 \zeta_0 T_0 \delta \xi + \delta \xi) &= -\sigma_M (\delta \psi_e - \delta \psi_1).
\end{align*}
\]

In case of necessity to use the numerical calculations we can take into consideration the input data of static analysis [6] with the next additional physics and geometric parameters:

- conductivity ratio: \( \alpha_{\text{in}} = 0.37, \quad \alpha_{\text{b}} = 0.12, \quad \alpha_e = 0.43, \quad \alpha_{\text{TB}} = 0.4, \quad \alpha_3 = 1.5; \)
- basic parameters: \( z_b = x_b = 9 \times 10^{-5} \text{m}, \quad s_b = 0.1 \text{m}, \quad p_{\text{B}} = 1 \text{MPa}; \)
- geometrical parameters: \( s_M = 6.4 \times 10^{-3} \text{m}^2, \quad s_c = 1.8 \times 10^{-4} \text{m}^2, \quad s_{\text{T}} = 0.1 \text{m}^2; \quad v_{\text{cam}} = 3.9 \times 10^{-5} \text{m}^3, \quad V_M = 1.3 \times 10^{-4} \text{m}^3, \quad V_2 = 1 \times 10^{-4} \text{m}^3; \)
- physical parameters: \( E = 2 \text{GPa}, \quad m = 250 \text{kg}, \quad c = c_0 = 1 \times 10^3 \text{kg} \cdot \text{s}^{-1}, \quad k = 1 \times 10^7 \text{kg} \cdot \text{s}^{-2}, \quad m_0 = 2.5 \text{kg}, \quad k_0 = 8.4 \times 10^3 \text{kg} \cdot \text{s}^{-2}; \)
- dimensionless geometrical ratio: \( \sigma_c = 1.1, \quad \sigma_M = 0.06; \)
- dimensionless physical ratio: \( \psi \in = 3, \quad \delta \psi_0 = 0.22, \quad B = 0.95, \quad \chi = 0.05. \)

Thus, the stationary values of parameters of LABD are corresponding to static calculation [6]. Geometry and parameters of RPD are corresponding to researches produced by Zahrulko on the experimental base of Joint-Stock Company SUMY FRUNZE NPO [7].
3.1. The amplitude – frequency characteristics and the phase – frequency variation

There are many types of disturbances that influence pump rotor. The predominant one is the axial sinusoidal vibration with operating frequency or multiply operating frequency. Therefore, the plotting of AFC and PFV is the crucial issue of ABD engineering. The frequency characteristics give us the full representation of dynamic and resonance behaviour, reactions to sinusoidal influence of the hydro-mechanical systems and their stability margins.

The transfer function of hydro-mechanical system is expressed by the next formula:

\[ W(p) = \frac{\delta u}{\delta \psi} \]  

(9)

After substitution \( p = i \omega \) (where \( i = \sqrt{-1} \) is the imaginary unit) into transfer function expression, we can take the frequency transfer function (FTF), with the module – AFC [5]

\[ A(\omega) = |W(i\omega)|, \]  

(10)

which is represented in Fig. 5; and the argument – PFV:

Fig. 5. AFC of LABD.
\[ \varphi(\omega) = \arg[W(i\omega)]. \] (11)

We can analyse the dynamic behaviour of its elements, analyse the stability and estimate the quality of transient processes by the frequency characteristics of system.

3.2. The analysis of the transient processes

During the multistage centrifugal pump work the step excitation is the lowest condition for the “rotor-LABD” system. Hence, it is necessary to analyse the transient processes. Thus, the dependences between the main parameters of LABD \((u, \xi, \psi_{\text{cam}}, \psi_{e}, \psi_{2}, \psi_{3})\) and external unit step action \(\psi_{1}\) with zero initial conditions (based on the solving of non-linear equation (4)) are represented in Fig. 6.

As it is seen from the figure, the regulation time is equal to the hundredth part of second. The transient characteristics show the over-control.

3.3. The dynamic stability analysis

Let us consider the stability of “rotor-LABD” system: it is its ability to involve the stationary value of the controlled parameter according to the given constant loading after the loss of time-varying excitations.
There are many criteria, which enable us to define the stability of linear system according to characteristic equation coefficients without finding roots. These criteria are subdivided into two groups: algebraic and frequency. For practical usage we can involve only algebraic criteria among which we can use the Gurvits criteria.

The characteristic polynomial of LABD is the 8-degree polynomial. That is why it is not possible to obtain the analytical expressions of stability criteria in an exact form. Therefore, we can lower the characteristic polynomial degree after entering the time scale factor $\theta = t/t_0$ for the control time $t_0 = 2 \times 10^{-3}$ (Fig. 6):

$$N(\bar{p}) = \frac{a_{0}}{t_0^8} \bar{p}^8 + \frac{a_{1}}{t_0^7} \bar{p}^7 + \ldots + \frac{a_{7}}{t_0^1} \bar{p}^1 + a_8$$

$$= \bar{a}_0 \bar{p}^8 + \bar{a}_1 \bar{p}^7 + \ldots + \bar{a}_7 \bar{p} + \bar{a}_8,$$

where $\bar{p} = d/d\theta$ is the operator of differentiation, $a_i/t_0^{8-i} (i = 0, 3)$ are dimensionless coefficients:

$$a_0/t_0^8 = 6.03 \times 10^{-11}, \quad a_1/t_0^7 = 3.01 \times 10^{-8}, \quad a_2/t_0^6 = 4.05 \times 10^{-5}, \quad a_3/t_0^5 = 1.29 \times 10^{-4},$$
$$a_4/t_0^4 = 2.83 \times 10^{-4}, \quad a_5/t_0^3 = 0.048, \quad a_6/t_0^2 = 8.79, \quad a_7/t_0 = 26.90, \quad a_8 = 8.75.$$

Numerical analysis shows that coefficients $\bar{a}_0 \ldots 4$ are more than 2 degree less than coefficients $\bar{a}_{5 \ldots 8}$. Therefore

$$N(p) \approx a_0 \bar{p}^3 + a_1 \bar{p}^2 + a_2 \bar{p} + a_3.$$  \hspace{1cm} (13)

As a result of regression of characteristic polynomial degree of dynamic system the Gurvits stability conditions are

$$\begin{cases}
    a_{1,2,3} > 0, \\
    a_1a_2 - a_0a_3 > 0
\end{cases}$$  \hspace{1cm} (14)

where expressions of coefficients $a_{0-3}$ are
\[
\begin{align*}
  a_0 &= K_0 T_0^2 T_{22} T_{\text{cam}} \alpha_e, \\
  a_1 &= K_0 T_0^2 \alpha_e \left( T_{\text{cam}} (1 + \alpha_{\text{Tb}} u_0^3) + T_{22} \alpha_{\text{in}} \right) \\
   &\quad - T_{22} T_{M1} T_{\text{cam}} \sigma_M, \\
  a_2 &= \alpha_{\text{in}} \left( K_0 T_0^2 \alpha_e (1 + \alpha_{\text{Tb}} u_0^3) + T_c T_{22} \sigma_M \right) \\
   &\quad - T_{M1} \sigma_M \left( T_{\text{cam}} (1 + \alpha_{\text{Tb}} u_0^3) + T_{22} \alpha_{\text{in}} \right), \\
  a_3 &= \sigma_M \alpha_{\text{in}} \left( T_c (1 + \alpha_{\text{Tb}} u_0^3) - T_{M1} \alpha_{\text{Tb}} u_0^3 \right). 
\end{align*}
\]

The safe operating area of LABD is determined through inequalities:

\[
\begin{align*}
  K_0 T_0^2 \alpha_e \left( T_{\text{cam}} (1 + \alpha_{\text{Tb}} u_0^3) + T_{22} \alpha_e \right) &> T_{22} T_{M1} T_{\text{cam}} \sigma_M, \\
  a_2 &= \alpha_{\text{in}} \left( K_0 T_0^2 \alpha_e (1 + \alpha_{\text{Tb}} u_0^3) + T_c T_{22} \sigma_M \right) \\
   &\quad - T_{M1} \sigma_M \left( T_{\text{cam}} (1 + \alpha_{\text{Tb}} u_0^3) + T_{22} \alpha_{\text{in}} \right), \\
  T_c (1 + \alpha_{\text{Tb}} u_0^3) &> T_{M1} \alpha_{\text{Tb}} u_0^3, \\
  \{ K_0 T_0^2 \alpha_e \left( T_{\text{cam}} (1 + \alpha_{\text{Tb}} u_0^3) + T_{22} \alpha_e \right) - T_{22} T_{M1} T_{\text{cam}} \sigma_M, \} \\
  \times \{ \alpha_{\text{in}} \left( K_0 T_0^2 \alpha_e (1 + \alpha_{\text{Tb}} u_0^3) + T_c T_{22} \sigma_M \right) \\
   - T_{M1} \sigma_M \left( T_{\text{cam}} (1 + \alpha_{\text{Tb}} u_0^3) + T_{22} \alpha_{\text{in}} \right) \}, \\
  > a_0 &= K_0 T_0^2 T_{22} T_{\text{cam}} \alpha_e \sigma_M \alpha_{\text{in}} \left( T_c (1 + \alpha_{\text{Tb}} u_0^3) - T_{M1} \alpha_{\text{Tb}} u_0^3 \right). 
\end{align*}
\]

Thus, to provide the conditions of LABD stability it is necessary to select correlations of the physical and geometric parameters by hydro-mechanical system engineering.

### 4. Conclusions

The main advantage of LABD (which is made as ABD with liquid supply and RPD usage) is that it is hydro-mechanical automatic control system which is subdivided into two systems and operates the non-contact bearing function and function of final-consolidation system with auto-regulated leaking simultaneously.

The methodology of dynamic calculation gives the opportunity to select the main parameters of LABD engineering based on providing the transient processes quality and stability conditions of hydro-mechanical system.
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