An Improved SPICE Model for MEMS Based Capacitive Accelerometers

C. Kavitha*, M. Ganesh Madhan†

Department of Electronics Engineering, Anna University, M.I.T Campus, 600 044 Chennai, India

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An improved electrical equivalent circuit for a capacitive MEMS accelerometer, incorporating temperature, pressure and squeezed film effects is reported. The circuit model corresponds to a single degree of freedom (SDOF) vibrating system, including dominant micro physical mechanisms. Static, transient and frequency response analysis are carried out at temperature and pressure ranges of 100 K to 400 K and 30 to 3000 Pa respectively. The effect of these parameters on the resonance frequency, peak displacement and settling time of the accelerometer are determined. Simulations are performed using PSpice® circuit simulator.

Keywords: Squeezed film, Capacitive accelerometer, Thermal effect, Damping, Electrical attractive force.

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1. INTRODUCTION

MEMS (Micro Electro Mechanical Systems) technology involves integration of mechanical, fluidic, optical and electrical components. The advantages are low cost and reduced size. Accelerometers are classified as piezoresistive, piezoelectric and capacitive types. Among all, the capacitive type is mostly preferred in commercial applications. Due to its high sensitivity, low noise and low power dissipation characteristics. In MEMS based accelerometer, the mechanical vibrations are converted to electrical signal by the sensing elements. The device performance depends on the elastic properties of silicon, capacitance and voltage measurements by electronic interface circuits. In MEMS accelerometers, gas is often employed as a damping medium, which has a significant impact on the accelerometer behavior. The compression of gas due to moving plates has to be carefully adjusted to obtain the desired response [1]. Usually the compressible gas films are analyzed using Reynolds equation [2]. This approach has been used by many authors to study the frequency response of micro mechanical pressure sensors [3-5]. Timo Veijola et. al. [6] has developed a circuit model that involves damping and spring forces created by squeezed film. They have determined the response of MEMS accelerometer using linear and nonlinear frequency dependent components, under different pressures. They have used a self-developed circuit simulation program APLAC [7] for their analysis. In this paper, we report an improved circuit model based on Timo Veijola’s approach, to study the effect of temperature and pressure on capacitive MEMS accelerometer. In our approach, the parallel resonator circuit model is replaced by variable inductor and resistor implemented as controlled current sources. This scheme also includes temperature and pressure dependent squeezed film and thermo elastic damping models. Based on this model the DC, Transient and frequency response analysis are carried out at different pressures and temperatures, using a commercial circuit simulator PSpice.

2. MEMS ACCELEROMETER MODELING

A typical micromachined capacitive accelerometer [1, 8] consists of a mass suspended with a small cantilever beam which is located inside the cavity of a small silicon block. In the capacitive measurement scheme, the movement of the cantilever is measured as a change of capacitance between the plates above and below the cantilever. Acceleration acting on the cantilever alters the air gap dimensions leading to increase of capacitance in one side and decrease in the other side. The sensitivity of the capacitive accelerometers is determined by the Young’s modulus, thermal coefficients of the materials and electronic interface circuit connected to the sensing element. Generally this accelerometer type is modeled as mass-spring-damper system and its corresponding schematic diagram is shown in Fig. 1. It consists of a proof mass, which is supported in such a way that it can move only in the x-direction, a spring, and a dashpot damper.

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* kaviphd2011@yahoo.co.in
† mganesh@annauniv.edu
The force \( F \) is the sum of an external mechanical force \( F_{ext} \) and an internal electrical attractive force \( F_d \). The electrical attractive force on the mass is caused by the potential difference across the capacitor plates. Assuming the motion perpendicular to the plate surfaces, the electrostatic force is given as
\[
F_d = \frac{\varepsilon AU^2}{2d^2}
\]
where \( \varepsilon \) is the dielectric constant of the gas, \( A \) is the plate area, \( U \) is the capacitor voltage and \( d \) is the gap width. The mechanical accelerometer model is realized by the equivalent electrical model of a parallel resonator and is given by the differential equation as
\[
\frac{C}{L} \frac{d^2 \phi}{dt^2} + \frac{1}{R} \frac{d \phi}{dt} + \frac{1}{L} \phi = I_{ext}
\]  
(3)

where \( C \) is the capacitance, \( \phi \) is the flux, \( R \) is the resistance, \( L \) is the inductance and \( I_{ext} \) is the external current. The displacement equals, \( L_{i_L} \), where \( i_L \) is the current through the inductor.

### 2.1 Squeezed Film Effect

A Simple linear MEMS capacitive accelerometer model with viscous gas flow is considered. The structure consists of a mass with cantilever beams and two fixed electrodes [8]. The region between the electrodes may be filled by the damping medium such as gas or liquid. Due to vibrations of proof mass the gas flow in a narrow air gap will cause damping [9, 10]. The gas pressure inside the sensor can be changed to control the squeezed film effect. The distribution of the gas pressure in the gap between two moving plates is governed by a fluid model with viscous gas flow is considered. The structure consists of a mass with cantilever beams and two fixed electrodes [8]. The region between the electrodes may be filled by the damping medium such as gas or liquid. Due to vibrations of proof mass the gas flow in a narrow air gap will cause damping [9, 10]. The gas pressure inside the sensor can be changed to control the squeezed film effect. The distribution of the gas pressure in the gap between two moving plates is governed by the equivalent electrical model of a parallel resonator and is given by the differential equation as
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where \( C \) is the capacitance, \( \phi \) is the flux, \( R \) is the resistance, \( L \) is the inductance and \( I_{ext} \) is the external current. The displacement equals, \( L_{i_L} \), where \( i_L \) is the current through the inductor.

The above effects are presented in Fig. 2. and Fig. 3.

#### 2.2 Thermo Elastic Damping

As the temperature increases, Young’s modulus and thermal expansion coefficient of silicon material (111) decreases and increases respectively [12, 13, 14 and 15]. This results in change of cantilever length, spring constant and resonant frequency.

Table 1 – Physical parameters of accelerometers [1]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass ( M )</td>
<td>4.9 µg</td>
</tr>
<tr>
<td>Width of the moving mass ( w )</td>
<td>2.96 mm</td>
</tr>
<tr>
<td>Length of the moving mass ( l )</td>
<td>1.78 mm</td>
</tr>
<tr>
<td>Length of the cantilever beam</td>
<td>520 µm</td>
</tr>
<tr>
<td>Gap widths ( d_{a} ) and ( d_{b} )</td>
<td>3.95 ± 0.05 µm</td>
</tr>
<tr>
<td>Mean free path ( \lambda ) at 1 atm</td>
<td>70.0 ± 0.7 nm</td>
</tr>
<tr>
<td>Viscosity coefficient ( \eta )</td>
<td>22.6 ± 0.2 µNs m⁻²</td>
</tr>
</tbody>
</table>

The increase in length of cantilever beam due to thermal expansion coefficient of silicon at different temperature is calculated by
\[
\delta = L \delta \Delta T
\]
(10)

The above effects are presented in Fig. 2. and Fig. 3.
Thermal strain and stresses can be estimated as

\[ \varepsilon_T = \frac{\delta}{L} \]  
\[ \sigma_T = E \varepsilon_T \]  

where \( \varepsilon_T \) is the Strain, \( \sigma_T \) is the stress, \( E \) is the Young’s Modulus of silicon material. Moment of inertia of the cantilever is obtained by

\[ I = \frac{bh^3}{12} \]  

where \( b \) is the width and \( h \) is the thickness of the cantilever beam. As temperature increases, the spring constant reduces as per the following equation [9].

\[ K = \frac{3EI}{L^3} \]  

The elasticity coefficient variation with temperature also affects the resonance frequency by the relation

\[ f_r = \frac{1}{2\pi} \sqrt{\frac{K}{M}} \]  

The variation is shown in Fig. 4.

Viscosity (\( \eta \)) of a gas is altered by temperature as follows

\[ \eta = \eta_0 \left( \frac{0.555T + C}{0.555T_0 + C} \right)^{3/2} \]  

where \( \eta_0 \) is the reference viscosity at reference temperature \( T_0 \), \( C \) is the Sutherland constant for the gas. The mean free path equation is

\[ \lambda = \frac{RT}{\sqrt{2\pi d_a^2 LP}} \]  

where \( R \) is a gas law constant, \( L \) is the Avagadro’s number, \( T \) is the temperature in Kelvin, \( d_a \) is the collisional cross section, \( P \) is the Pressure. The mean free path dependence on temperature and pressure is shown in Fig. 5. The damping coefficient is evaluated by

\[ D = \frac{2\eta L b}{d} \]  

3. EQUIVALENT CIRCUIT MODEL

The electrical equivalent circuit is normally developed for the capacitive accelerometer [1] using parallel RLC circuit with two air gap sections in the form of parallel RL sections (squeezed film damping effects) [4, 6, 8 and 9]. In the proposed model temperature, pressure and bias voltage applied to the parallel plates are fixed as one unit and the equivalent circuit of accelerometer is kept as another unit, with mutual interaction between them. The block diagram is shown in Fig. 6.
As temperature increases, the length of cantilever beam gets elongated which changes the inductance and resistance in the equivalent circuit, which represents spring constant and damping coefficient. The temperature is modeled as a voltage source in PSpice and all other effects are developed as voltage dependent voltage sources (VCVS). Temperature is varied in the range of 100 K to 400 K for the device under study. The thermal expansion coefficient and young's modulus values are evaluated by a second order polynomial, using curve fitting technique. They are implemented as a Voltage controlled voltage source (VCVS). The inductance and resistance values are found to vary in the range of 3.43 mH to 3.49 mH and 23.06 MΩ to 5.423 MΩ respectively, for temperature range of 100 K to 400 K. The pressure of the argon gas, considered as the damping medium, is also varied in the range of 30 Pa to 3000 Pa. The pressure parameter is assumed as a voltage source which affects the Knudsen number, dynamic viscosity and effective viscosity. The complete equivalent circuit developed is shown in Fig. 7.

\[
C_1 = \frac{\varepsilon A}{d_a - x} \\
C_2 = \frac{\varepsilon A}{d_b - x}
\]

where \(C_1\) and \(C_2\) are capacitance between movable electrode and fixed electrode of two sides, \(\varepsilon\) is the dielectric constant of the gas in the air gap, \(A\) is the plate area, \(d_a\) and \(d_b\) are gap widths.

Capacitance depends on air gap distance between plates and the bias voltage. Under symmetrical bias condition \(V_{\text{bias1}} = V_{\text{bias2}}\), the nominal capacitance value is obtained as 11.81PF.
4.2 Transient Response Analysis

A step input current equivalent to an acceleration of 0.5g is applied to the movable mass and the simulation is carried out under pressures of 30 Pa, 300 Pa and 3000 Pa. Capacitance variations due to different pressures at two different temperatures 100 K and 400 K are shown in Fig. 10 and Fig. 11. The capacitance variation due to different temperatures at constant pressure is shown in Fig. 12 and Fig. 13. The transient analysis is carried out in the temperature range of 100 K and 400 K with different pressures.

Displacement of the proof mass under different pressure and temperature are shown in Fig. 14 and Fig. 15. At lower pressures, the displacement exhibits large oscillations. At 300 Pa, oscillations are reduced and reach steady state within few milli seconds. For 3000 Pa a large damping and increase rise time is observed. This increase damping, at high pressures is in accordance with the results of Timo Veijola and thus validates our model.

In the case of 100 K temperature and 30 Pa pressure, the displacement is found to be around 120 nm and settling time is around 30 ms. At pressures of 300 Pa and 3000 Pa, the displacement and settling time are observed to be 88 nm, 80 nm and 3 ms, 12 ms, respectively. For a constant pressure of 30 Pa with varying temperature of 100 K, 280 K and 400 K, the transient response shows a decreasing displacement and settling time. However, for 3000 Pa, the displacement and settling time is found to increase.
4.3 Frequency Response Analysis

An external $\pm 3 \text{ g}$ force is applied to the mass of accelerometer and the displacement at each frequency is evaluated. If the bias voltages across the two capacitances are equal, the mass is restored to the original position. The analysis is repeated for different pressures and temperatures. The response is plotted in Fig. 16 and Fig. 17. Higher pressures lead to increased damping. Increase in temperature is found to affect the frequency response at higher pressures. The $3 \text{ dB}$ bandwidth of the device is found to decrease with increasing pressure. The frequency response is more or less constant with temperature.

![Fig. 15](image)

**Fig. 15** – Transient analysis on displacement without bias for 30 Pa, 300 Pa and 3000 Pa

![Fig. 16](image)

**Fig. 16** – Frequency analysis on displacement for 30 Pa, 300 Pa and 3000 Pa at 100 K

![Fig. 17](image)

**Fig. 17** – Frequency analysis on displacement for 30 Pa, 300 Pa and 3000 Pa at 280 K

The peak Displacement variation with temperature is shown in Fig. 18.

![Fig. 18](image)

**Fig. 18** – Peak displacement for 30 Pa, 300 Pa and 3000 Pa at different temperatures

5. CONCLUSIONS

An improved circuit model for capacitive MEMS accelerometer incorporating thermo elastic damping and squeezed film effects is developed. Based on this model temperature and pressure effects on the performance of accelerometer are studied, the developed model is compatible with PSpice circuit simulator.

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