

Using of multi-objective optimization in financial portfolios

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The main goal of this work was the improvement of calculation and graphing for analysis of different dimension financial portfolios, which will increase Matlab-tools work accuracy and efficiency.

Was made comparative analysis of Matlab instruments for working with financial portfolios, and was offered methods to improve graphing and increase the accuracy of the calculation.

INTRODUCTION

Nowadays there are in various branches of science are commonly used various tools for analysis and optimization problems. One of these is Matlab.

Matlab is a high-performance language for technical computing. It integrates computation, visualization, and programming in an easy-to-use environment where problems and solutions are expressed in familiar mathematical notation.

Optimization is a discipline used for searching extremum of a function and generally refers to mathematical problems where the goal is to minimize or maximize an objective function subject to some constraints.

Multi-objective optimization (Pareto optimization) is an area of multiple criteria decision-making, which is associated with the mathematical optimization problem involving more than one objective function to optimize simultaneously.

MULTI-CRITERIA OPTIMIZATION IN FINANCE

Optimization is one of the primary techniques for financial decision making. Wide use of optimization techniques in finance includes such classes of problems as portfolio selection, risk management, regression problems, pricing and hedging of derivatives, and asset liability management.

In finance, the general and the main problem is to choose a portfolio when there are two conflicting goals: to have the expected value of portfolio returns as high as possible and to have risk, measured by the standard deviation of portfolio returns, as small as possible. This problem is often represented as a graph in which the efficient frontier shows the best combination of risk and expected return that are available, and in which indifference curves show the investor's preference for different combinations of risk and expected return.

The problem of multi-objective optimization finance portfolios is formulated as follows (1):

$$\begin{cases} \{f_1(\mathbf{w}), f_2(\mathbf{w})\} \rightarrow \min, \\ \mathbf{w} \in \Omega. \end{cases} \quad (1)$$

Here Ω is a set of feasible values, which is included in the domains $D_1(f_1) \subset R^n, D_2(f_2) \subset R^n$ of functions $f_1(\mathbf{w}), f_2(\mathbf{w})$.

The complexity of the financial markets and their operations require using of the models with more complex constraints and objective functions than Markowitz model.

The standard Markowitz mean-variance portfolio problem is to select assets to minimize the variance of the portfolio profit while giving a specified expected return, given historical data of mean and covariance of stock returns.

$$\begin{cases} f = (\mathbf{w}'Q\mathbf{w}, -w'm) \rightarrow \min, \\ \mathbf{w}'i = 1, i = (1,1,\dots,1) \end{cases} \quad (2)$$

There are two ways to optimize this problem:

1. To minimize objective function $\mathbf{w}'Q\mathbf{w}$ and to assign $-w'm$ and impose restriction on it, \mathbf{w} and $\mathbf{w}'i$.

2. To minimize objective function $-w'm$ and to assign to $w'Qw$ and impose restriction on it, w and $w'i$.

After research of this problem the main goal of this work was to optimize Markowitz mean - variance finance portfolio problem with use of different approaches in work with Financial Portfolio Optimization Toolbox in Matlab.

In practice we made detailed analysis of multi-parametric optimization and methods of its realisation; compute multi-objective optimization problem for two examples of financial portfolios with Matlab tools, such as YALMIP and MPT. To make this tools more effective and functional for portfolios with different dimension.

We had two experimental finance portfolios with which the main work was conducted and which have assets of some unknown company, first portfolio for 64 assets and second - 1000 assets

To compute the standard Markowitz mean-variance portfolio problem we should use YALMIP, which can be used to calculate explicit solutions of linear and quadratic programs by interfacing the Multi-Parametric Toolbox (MPT)

We have the same problem as standard Markowitz mean-variance portfolio problem but we needed to use one more parameter - transaction cost ($t = c'w$). Also we had two different way to solve the problem – when x is the fixed value and when x is the value of the certain set

$$f(w'Qw, -m'w, c'w) \rightarrow \min, \quad (3)$$

$$\begin{cases} w'Qw \rightarrow \min, \\ -m'w \leq E_1, \\ c'w \leq E_2, \\ w'i = 1' \\ w \geq 0. \end{cases} \quad x = [E_1, E_2], \quad (4)$$

We have experimental finance portfolios which have assets of some unknown company, for 64 assets.

There are two ways to optimize our problem:

1. To minimize objective function $w'Qw$, maximize the profit $-m'w$ with considering

2. To minimize objective function $-w'm$ maximize the profit $-m'w$ with considering t . But we can use x - which is not fixed.

CONCLUSIONS

Implementing parametric optimization into optimization software packages remains one of the challenges. Unfortunately, available software for parametric optimization is very limited.

In this work was solved linear and multi-parametric problem of financial portfolios method of Markowitz. Was analyzed different approaches to the portfolio risk optimization problem within a conditional independence framework and compared optimization of risk measures with the traditional Markowitz mean-variance optimization formulation.

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