FRACTAL ANALYSIS OF THE MEDICAL INSURANCE MARKET

Małgorzata Mańka-Szulik, ORCID: https://orcid.org/0000-0002-5328-8736
PhD, Department of Management, Faculty of Organization and Management, Silesian University of Technology, Poland

Svitlana Kolomiiets, ORCID: https://orcid.org/0000-0002-7832-8415
PhD, associate professor, Sumy State University, Ukraine

Dariusz Krawczyk, ORCID: https://orcid.org/0000-0003-1823-0309
PhD, Department of Applied Social Sciences, Faculty of Organization and Management, Silesian University of Technology, Poland

Kateryna Olihnenko, ORCID: https://orcid.org/0009-0000-0756-777X
Master’s degree, Netcracker, Sumy, Ukraine

Correspondence author: Svitlana Kolomiiets, s.kolomiiets@biem.sumdu.edu.ua

Type of manuscript: research paper

Abstract: The latest experience obviously reflects that social-economic systems increasingly demonstrate an unforecastable behaviour. Financial markets are characterised by extremely quick change and crisis phenomena. Use of traditional modelling methods (in particular, the effective market hypothesis) provides no opportunity to obtain efficient forecasts about state and development of financial markets. One of promising methods in researching and modelling financial markets is the fractal analysis. Various natural and social phenomena possess fractality properties, namely there are similar structures on different scales. Generality of the fractal analysis methods allows their applying to study systems of any nature – from physical to economic and social ones. Fractality attributes of time series provide its dynamics pre-forecast and assess the time series predictability. The fractal analysis consists in establishing the extent of time series similarity to the fractal ones and defining a relation between the trend line and fractal dimension. The Hurst exponent assessment (persistence or anti-persistence of time series) predicts the further process development on the preliminary data basis. The fractal analysis efficiency during the unstable market periods is applicable for studying different social-economic systems (in particular, the Ukrainian medical insurance market). The research object is time series of gross insurance rewards on the Ukrainian medical insurance market. The research topic is the time series fractal analysis. The paper deals with peculiarities of the time series fractal analysis, the R/S analysis for time series of gross insurance rewards on the Ukrainian medical insurance market (via the MS Excel software). The Statistica 10.0.228.2 application generated the ARIMA model to predict dynamics of the gross insurance rewards on the Ukrainian medical insurance market. The obtained results may be used to conduct the R/S analysis of financial time series. In particular, that concerns those time series that describe the insurance market. Also, dynamics of the financial time series may be forecasted via these results as well.

Keywords: health insurance; ARIMA model; Hurst indicator; R/S analysis; fractal analysis; time series.

JEL Classification: G19, G22, C32, C53

Received: 13 October 2023   Accepted: 17 December 2023   Published: 31 December 2023

Funding: There is no funding for this research

Publisher: AR&P

Introduction
Healthcare is one of the main indicators of humanity development within both a separate country and the whole world. Without opportunities of population health recovery, it is impossible to promote the social-economic and cultural development of any country. Public healthcare is an extremely important economic resource. Here, investments produce both social and economic effects.

The worldwide practice shows that today’s healthcare can not be supported without a proper introduction of insurance mechanisms. The medical insurance market influences the healthcare sector via increase in medical service affordability, decrease in financial obstacles for providing disease treatment and prevention. It enhances the population awareness of healthy lifestyle, raises the medical service quality, etc. Research of the medical insurance market dynamics is very important to understand the extent of insurance influencing the public healthcare.

In the 21st century, the permanent change proves the idea that the world is full of chaos, disasters and unforeseen consequences. Financial markets are characterised by extremely quick change and crises. Traditional modelling methods (for example, the efficient market hypothesis) do not give any opportunity to forecast state and development of financialy markets accurately. One of the promising methods to research and model financial markets is the fractal analysis. Its efficiency during unstable market periods determines its popularity to assess various social-economic systems (in particular, the medical insurance market). The medical insurance market modelling is being quite relevant. The reasons are the Ukrainian healthcare reforming, the road map of medical insurance (approved in 2023), the medical insurance development as a potential source of healthcare funds.

Literature review
Many researchers pay attention to issues of the medical insurance market as an efficient tool of medicine financing. There are several research directions. Zhuravka et al. (2020) reviews the experience of foreign countries and investigates the voluntary insurance development in Ukraine as an extra-budgetary source of healthcare funding. The calculations made it possible to conclude that the voluntary insurance market in Ukraine has a potential for development. Kuzior et al. (2022a) indicate a deficit in medical spending and insufficient development of the voluntary health insurance market.

The COVID-19 pandemic had a significant influence over dynamics of many social-economic systems (Kuzior et al. 2021, 2022b; Ober & Karwot, 2023). Changes also apply to the medical insurance market. Research of the COVID-19 impact on the global insurance market was conducted by the Lloyd’s Insurance Corporation (2020). According to the Lloyd’s experts, systemic risks (such as pandemics) that cause large economic and societal losses are unlikely to be covered in their entirety by the global insurance industry. The reason is the total economic loss would exceed its financial resources. High-quality insurance protection is possible only in partnership between insurers and governments. As emphasised in the document, much of this work can be carried out by the global insurance industry itself, working collaboratively to design and develop new products, services and structures. However, to overcome the challenges of offering protection for systemic risks at scale, governments should combine (re)insurance capital with capital market resources and sovereign funds to provide the necessary security and capacity to pay claims.

Suprun et al. (2020) analysed the COVID-19 impact on the insurance sector in 2019-2020. They evaluated direct and indirect losses, new possibilities and tasks among insurers. Also, the COVID-19 insurance in Ukraine was assessed. Besides, prospects of insurance protection from the COVID-19-like risks were observed. From the authors’ perspective, the insurance protection from pandemics is possible only due to partnerships between state and private entities. Here, new models of insurance management must be engaged. They should be based on digitalisation and remote access.

Zhuravka et al. (2022) studied main tendencies of the Ukrainian volunteer medical insurance market. It is explained that medical insurance may become a promising source of extra funds to support the Ukrainian healthcare system. The Ukrainian share within the European insurance market was analysed with its relevance and demand for the last years.

Theoretical and practical issues of compulsory insurance efficiency were considered in the work by Barannyk et al. (2023). The world experience makes it obvious that compulsory insurance is an important tool in economic development of any country. Experts offer regarding the compulsory insurance not only as a financial system element but also as a public protection component. It ensures social security in the country.

Borysiuk (2020) analysed the existing healthcare models, foreign medical insurance experience and its adaptation in Ukraine. The volunteer and compulsory insurance is argued to play a positive role in raising medical service quality, attracting new funds and protecting human rights.
Pazieieva (2021) described the modern status of Ukrainian medical insurance and its importance in social security. The author represents methods, approaches, conclusions and offers to upgrade the Ukrainian medical insurance system. The main worldwide tendency of medical insurance development is use of IT technologies and support of insurance companies. Also, the state economy influence on financial resilience of the medical insurance sector was researched.

Most works have provided a deep insight into dynamics of the medical insurance market. However, many theoretical and practical questions remain non-answered. In particular, that concerns the medical insurance forecasting in critical cases.

Since the Ukrainian Healthcare Ministry has adopted the medical insurance road map, there is an increasing urgency in modelling dynamics of the medical insurance market. Within permanent changes, modelling dynamics of any social-economic system requires applying non-linear methods. One of them is the fractal analysis.

The main fractal analysis idea consists in the fact that many natural and social phenomena possess fractal properties (that is similar structures on various scales). Time series comprise different components: trends, cycles with different periods, fluctuations, etc. Usually, such series are fractals. So, behaviour of time series remains the same on different scales.

The fractal analysis was formed via the fractal market theory. It states that the future market development (including future values of time series) depends on retrospective changes. According to the fractal market theory, the pricing process is globally determined. It depends on initial conditions. Locally, it is random.

The fractal market hypothesis is based on the fractal theory by B. Mandelbrot whose fractal geometry turned out to be a new tool of risk assessment. He described five main assumptions concerning the financial market dynamics (Mandelbrot et al., 2004):

- Huge price variations are normal for the financial market. Markets are risky;
- Financial disturbance occurs by groups;
- Market dynamics is determined by outer and inner factors;
- Famous schemes of market functioning are “deluding financial mirages”;
- Market time is relative. It can slow down in case of stability and accelerate in case of unstability.

Peters (1994, 2008) confirmed that the financial market nature has fractal features. Real time series of economic indexes (quoting of shares, currencies, enterprise accountability, etc.) reflect a complex irregular behaviour. Here, trends and flats change their random walk.

On financial markets, the quoting series of financial tools possess fractal properties. Therefore, they may be described via the fractal geometry methods. The fractal theory use for financial market studies was elaborated in works by Aslam et al. (2020a, 2020b), da Silva Filho et al. (2018), Ghosh et al. (2019), Muhammad et al. (2023), Radu et al. (2022), Wang et al. (2018), Xu et al. (2022).

Among the main series properties, there is the long-term memory when past prices influence the future ones. To explain fractal processes, experts apply various indexes (for example, the fractal dimension). Different values of fractal dimension correspond to various processes (trend, flat, random walk). They inform on possible presence of “critical points” – significant deviations in series values. The fractal analysis of time series aims at clarifying how much the researched series are similar to fractal ones. Also, relations between trend lines and the fractal dimension \( D \) should be defined. Via mathematical techniques, it was proved that fractal dimension satisfies the inequation \( 1 < D < 2 \). The fractal dimension value may indicate the further behaviour of time series and its predictability.

Our paper is going to research dynamics of the Ukrainian medical insurance market. It will be conducted via the fractal analysis of time series of gross insurance rewards on the Ukrainian medical insurance market.

**Methodology and research methods**

The fractal analysis of time series will be implemented through the following stages:

1. Establishing the initial research base. We will identify the quantity assessment features of Ukrainian medical insurance dynamics. Here, we will define time series levels of gross insurance rewards on the Ukrainian medical insurance market.

2. Conducting the pre-forecast time series analysis of gross insurance rewards on the Ukrainian medical insurance market. For this aim, we will use the R/S analysis to estimate the Hurst exponent.

The Hurst exponent characterises relations between the trend force (a series determinancy factor) and the noise level (a randomness factor). Many natural phenomena (river move, precipitation, temperature, sun spots, etc.) correspond to a shifted random walk. In other words, they are described by trend with overlaid noise. In
empirical data, the Hurst exponent $H$ can clarify the index change, presence of periodical and non-periodical cycles, process propensity to trend.

The Hurst exponent is as an empirical formula logarithm:

$$ R_S = \left( \frac{n}{2} \right)^H $$

$H$ – the Hurst exponent; $R$ – range of cumulative difference series $\sum_{i=1}^{n} (y(t_i) - \bar{y}(t))$; $S$ – mean-square deviation of levels $n$ in the time series $y(t)$.

There are various ways to calculate the Hurst exponent. Usually, algorithms by Mandelbrot et al. (2004) and Peters (1994) are involved. However, calculation results via these techniques may differ.

To conduct the fractal analysis for time series of gross insurance rewards, we apply the algorithm by Feder (2013).

Let we have the time series $y(t) = \{y_1(t), y_2(t), ..., y_n(t)\}$, with the amount of levels $n$. The R/S analysis algorithm:

1) Calculating the average $\bar{y}(t)$ and the mean-square deviation $S$;
2) Determining deviation from the average via the formula $y_i(t) - \bar{y}(t)$;
3) Through the deviation sequence and via a gradual sum cumulation, establishing the cumulation series via the formula:

$$ Z = \sum_{i=1}^{n} (y_i(t) - \bar{y}(t)) $$

4) Calculating the variation range for the cumulation series (2):

$$ R = maxZ - minZ $$

5) Due to the logarithm formula (1), defining the Hurst exponent:

$$ H = \frac{ln \left( \frac{R}{S} \right)}{ln \left( \frac{n}{2} \right)} $$

6) Thanks to the Hurst exponent, concluding how persistent or anti-persistent the time series is.

There are three time series classifications as to the Hurst exponent:

- $0 \leq H \leq 0.5$ ($1.5 < D \leq 2$). The time series is anti-persistent or ergodic (“pink noise”). The economic system tends to constant change. If there is a system increase in the previous period, the future one will be likely to drop (and vice versa). Resilience of such an anti-persistent behaviour depends on the Hurst exponent proximity to zero. The closer Hurst exponent is to zero, the more changeable and volatile the series is. Such a system is called “return to the average”.

- $H = 0.5$ ($D = 1.5$). The time series is random (“white noise”). Events are uncontrollable. The current process state does not affect the future one. There will be no correlation between retrospective and predictable data (random behaviour of economic indexes).

- $0.5 < H \leq 1$ ($1 < D < 1.5$). The time series is persistent and resilient to trends (“black noise”). Events are not random. There is a trend, a constant tendency to index rise or fall – both in the past and future. The closer Hurst exponent is to 0.5, the less evident the series trend is. The closer the index value is to 1, the more often we have either a growth sequence or a recession sequence.

The Hurst exponent deviation from 0.5 is the development of fractal properties in time series.

3. Generating the dynamics forecast model of time series of gross insurance rewards on the Ukrainian medical insurance market.

The fractal dimension value is an indicator of time series behaviour and its dynamics predictability. Actually, the fractal dimension is replaced with the Hurst exponent. They are related in the equation:
\[ D = 2 - H \] (5)

\( D \) – fractal dimension; \( H \) – Hurst exponent.

At \( 1 < D < 1.5 \), the time series have “a long-term memory”. The market is persistent. If \( D \) is close to 1, the current trend can finish quickly. Such series dynamics is described via the long-term memory models (ARIMA).

At \( D = 1.5 \pm 0.05 \), the system behaviour is stochastic. Series dynamics is explained via the ARIMA models.

At \( 1.5 < D < 2 \), the time series becomes more non-linear. If \( D \) is close to 2, there can be an anti-persistent state. You should analyse fundamental economic factors or reject any forecasts.

To provide a time series model of gross insurance rewards on the Ukrainian medical insurance market and to obtain a short-term forecast, the ARIMA technique is applied. The ARIMA models describe both stable and unstable time series. They explicate the time series behaviour through its previous values.

The Box-Jenkins method of the ARIMA model selection for time series comprises three stages:
1) Model identification;
2) Model assessment and its adequacy check;
3) Forecast.

On the first stage, we should check the series stability. For this aim, we apply the visual diagram analysis, the ACF and PACF analysis, the Dickey-Fuller test. An unstable series has to be converted into a stable one via separation of trends, season components or differences.

Later, the ACF and PACF activities are assessed with hypotheses for the values \( p \) and \( q \).

On the second stage, parameters are estimated for the selected models. They are used for the adequacy check (the Ljung-Box Q test). If several models are adequate, the AIC and SIC criteria are included for the final model selection.

On the third stage, forecast is implemented.

**Results**

For the fractal analysis of the Ukrainian medical insurance dynamics, we engage the value of gross insurance rewards in 2000-2023.

<table>
<thead>
<tr>
<th>Year</th>
<th>Gross insurance rewards</th>
<th>Year</th>
<th>Gross insurance rewards</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000</td>
<td>214.8</td>
<td>2012</td>
<td>3,153.5</td>
</tr>
<tr>
<td>2001</td>
<td>281.5</td>
<td>2013</td>
<td>3,627.1</td>
</tr>
<tr>
<td>2002</td>
<td>257.6</td>
<td>2014</td>
<td>3,229.0</td>
</tr>
<tr>
<td>2003</td>
<td>368.2</td>
<td>2015</td>
<td>1,488.8</td>
</tr>
<tr>
<td>2004</td>
<td>414.6</td>
<td>2016</td>
<td>1,718.3</td>
</tr>
<tr>
<td>2005</td>
<td>520.1</td>
<td>2017</td>
<td>5,485.9</td>
</tr>
<tr>
<td>2006</td>
<td>723.4</td>
<td>2018</td>
<td>6,021.2</td>
</tr>
<tr>
<td>2007</td>
<td>1,021.2</td>
<td>2019</td>
<td>6,627.1</td>
</tr>
<tr>
<td>2008</td>
<td>1,451.3</td>
<td>2020</td>
<td>7,078.7</td>
</tr>
<tr>
<td>2009</td>
<td>1,442.3</td>
<td>2021</td>
<td>8,912.1</td>
</tr>
<tr>
<td>2010</td>
<td>1,679.1</td>
<td>2022</td>
<td>9,214.8</td>
</tr>
<tr>
<td>2011</td>
<td>2,329.1</td>
<td>2023</td>
<td>9,679.1 (9 months)</td>
</tr>
</tbody>
</table>

**Table 1: Input data**

Sources: Developed by the authors based on Forinsurer.com (2023)

Analysis of the gross insurance reward dynamics provides information on development tendencies of the personal volunteer insurance market. The gross reward rise or fall may indicate the variable demand for insurance services, economic situation and other factors that influence the medical insurance market.
For the given time series, we calculate the Hurst exponent as $H \approx 0.935$ via formulas (2) – (4). This value shows that the gross insurance reward series is persistent. The future values depend on the past ones. If the series increases or decreases in the previous period, the same tendency is likely to be in future.

To generate the ARIMA model, we use the Statistica software (2023). The model generation is based on the time series stability principle. Analysis of the input data diagram represents an evident growth tendency with the unstable time series (Figure 1).

Figure 1: The $Y$ series

Source: Developed by the authors using the Statistica software (2023)

For the initial series, let us apply the first difference method. That is the difference between adjoining series levels (Figure 2).

Figure 2: Diagram of the first differences

Sources: Developed by the authors using the Statistica software (2023)
Visual analysis of the first difference series diagram allows stating that trends and seasonality are absent. Therefore, the series is stable. Stability of the first difference series is confirmed via the DF test.

To define the parameter $p$ and $q$, we use the PACF and ACF functions, respectively. If ACF is cut off and PACF goes to zero exponentially, the summands $AR(p)$ must be present in the model. If PACF is cut off and ACF goes to zero exponentially, the summands $MA(q)$ must be present in the model. If ACF and PACF go to zero, both summands are included. The $AR(p)$ model order corresponds to the number of the PACF last significant coefficient. The $MA(q)$ model order corresponds to the number of the ACF last significant coefficient. The ACF and PACF analysis (Figures 3-6) provided an opportunity to obtain the ARIMA model parameters $(1; 1; 0)$.
To generate the ARIMA model, let us set and assess its parameters through the maximum likelihood estimation. Results are represented on Figure 7.

**Figure 5: The partial autocorrelation function (PACF)**

Sources: Developed by the authors using the Statistica software (2023)

**Figure 6: The partial autocorrelation function (PACF) values**

Sources: Developed by the authors using the Statistica software (2023)

To generate the ARIMA model, let us set and assess its parameters through the maximum likelihood estimation. Results are represented on Figure 7.

**Figure 7: Main parameters of the ARIMA model**

Sources: Developed by the authors using the Statistica software (2023)
Column 1 – parameter estimation. Column 2 – asymptomatic standard error. Column 3 – the criterion \( t \). Column 4 – significance levels. Columns 5 and 6 – lower and upper limits of the 95% confidence intervals for unknown model parameters. Table of Figure 7 shows that the \( p(1) \) assessment is statistically significant. Consequently, the ARIMA model equation (1; 1; 0) is:

\[
Y_t - Y_{t-1} = 0.42696 \cdot (Y_{t-1} - Y_{t-2}) + \varepsilon_t
\]

(6)

\( Y_t \) – current time series value; \( Y_{t-1} \) – previous time series value; \( \varepsilon_t \) – random disturbance.

Now, we are going to check the model adequacy. That is done via analysis of the ACF and PACF residuals and the Kolmohorov-Smirnov test.

Analysis of the ACF and PACF residuals confirms the absence of autocorrelation residuals (Figures 8-9). The residual histogram shows their normal distribution (Figure 10). Therefore, the generated ARIMA model is adequate.

![Figure 8: The ACF residuals](source)

Source: Developed by the authors using the Statistica software (2023)

![Figure 9: The PACF residuals](source)

Source: Developed by the authors using the Statistica software (2023)
The forecast was conducted for the next five years. Its results are shown on Figures 11-12.

Source: Developed by the authors using the Statistica software (2023)
Figure 11 demonstrates the forecast values with lower and upper limits and standard errors. The forecast values start from 9877.34 and finish in 10020.13. It indicates an expectable positive trend in future. The confidence intervals show a range of possible further values.

Conclusions
Natural and social phenomena possess fractal features, that is similar structures on various scale. The fractal analysis is applicable for investigation of any system: physical, economic, social, etc. Fractality attributes of time series provide its dynamics pre-forecast and assess the time series predictability. The fractal analysis consists in establishing the extent of time series similarity to the fractal ones and defining a relation between the trend line and fractal dimension. The Hurst exponent assessment (perception or anti-persistence of time series) predicts the further process development on the preliminary data basis.

The paper deals with the R/S analysis for time series of gross insurance rewards on the Ukrainian medical insurance market (via the MS Excel software). The Statistica application generated the ARIMA model to predict dynamics of the gross insurance rewards on the Ukrainian medical insurance market.

The obtained results may be used to conduct the R/S analysis of financial time series. In particular, that concerns those time series that describe the insurance market. Also, dynamics of the financial time series may be forecasted via these results as well.


Conflicts of interest: Authors declare no conflict of interest.
Data availability statement: Not applicable.
Informed consent statement: Informed consent was obtained from all subjects involved in the study.

References
19. Radu, V. et al. (2022). Analysis of the Romanian capital market using the fractal dimension. Fractal and Fractional, 6(10), 564. [CrossRef]